Advanced Accelerator Physics and Accelerator Simulation Homework 7

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Exercise 1:
Determine the thin lens approximation for the 6×6 matrix of a combined function magnet with quadrupole strength k and curvature κ. Use that the thin lens approximation is linearized in the length L.

Exercise 2:
(a) Determine the transport matrix for a solenoid in the sharp cutoff limit. It has the length L, and the longitudinal field strength on axis is B_z for $x \in [0, L]$ and 0 outside this region. As solenoid strength you can use the parameter $g = \frac{qB_z}{2p}$.
(b) Determine the thin lens approximation of this solenoid, which is linearized in L.

Exercise 3:
Derive the equation of motion for Twiss parameters, $\alpha' + \gamma = K(s)\beta$ with $K = [\kappa^2(s) + k(s)]$ from the linearized equation of motion $x'' = -K(s)x$. Use $x = \sqrt{2J\beta(s)}\sin(\Psi(s) + \Phi)$, $\alpha = -\frac{1}{2}\beta'$ and $\Psi'(s) = \beta^{-1}$.

Exercise 4:
(a) Given the Twiss parameters α, β, γ: specify the transformation from the amplitude and phase variables J and ϕ to the Cartesian phase space variables x and x'.
(b) Specify the inverse transformation.
(c) Given the Gaussian beam distribution in amplitude and phase variables, $\rho(J, \phi) = \frac{1}{2\pi\epsilon}e^{-\frac{J}{\epsilon}}$. What is the projection $\rho(x)$ of this distribution on the x axis. Check that the rms width of this distribution leads to $\sqrt{< x^2 >} = \sqrt{\beta\epsilon}$.