Advanced Accelerator Physics and Accelerator Simulation Homework 8

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Exercise 1:
Find the general form of the beta function in a drift:
(a) by solving the differential equation for $\beta(s)$ with the initial conditions $\beta(0) = \beta_0$ and $\alpha(0) = \alpha_0$.
(b) by propagating the matrix of initial Twiss parameters by the transport matrix of a drift according to

$$\begin{pmatrix} \beta & -\alpha \\ -\alpha & \gamma \end{pmatrix} = \underline{M} \begin{pmatrix} \beta_0 & -\alpha_0 \\ -\alpha_0 & \gamma_0 \end{pmatrix} \underline{M}^T \ . \tag{1}$$

(c) Find the general form of a beta function in a quadrupole of focusing strength k. Do not use the thin lens approximation.

Exercise 2:
a) Use the transport matrix from s_0 to s written in terms of Twiss parameters at s_0 and s to show that the one turn matrix of a ring at s can be written as

$$\mathbf{M} = \mathbf{1}\cos\mu + \begin{pmatrix} \alpha & \beta \\ -\gamma & -\alpha \end{pmatrix} \sin\mu \tag{2}$$

when α, β, and γ are the Twiss parameters that are periodic with the length L of the ring and $\mu = \Psi(L) - \Psi(0)$ is the one turn phase advance.
b) Show that the matrix before $\sin\mu$ in this equation has a characteristic of the complex i in that squaring it leads to $-\mathbf{1}$.
c) Use this to compute $\mathbf{M}^n$.

Exercise 3:
If the one turn matrix

$$\mathbf{M} = \begin{pmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{pmatrix} \tag{3}$$

is known, specify how the periodic Twiss parameters and the one turn phase advance can be computed. Under what conditions is the one turn phase advance real? What does this mean for the long term motion in phase space which is described by $\mathbf{M}^n\vec{z}_0$ for large n.

Exercise 4:

(a) Find the Twiss parameters of the phase space ellipse that is periodic for a FODO Cell in thin lens approximation, i.e. all particles that enter a FODO cell on this phase space ellipse exit the cell on the same ellipse. Let the focusing and defocusing quadrupole have the strength k and $-k$. Furthermore, let the cell start with half a focusing quadrupole, and let the distance between quadrupoles be $L/2$ so that the transport matrix of the cell is given by

$$\underline{M} = \underline{Q}(\frac{k}{2})\underline{D}(\frac{L}{2})\underline{Q}(-k)\underline{D}(\frac{L}{2})\underline{Q}(\frac{k}{2}) \ . \tag{4}$$

(b) Characterize how this periodic phase space ellipse changes along the FODO cell by drawing ellipses in phase space at various points along the cell. Do this for the horizontal and the vertical plane separately.

(c) Compute the periodic dispersion (η, η') for this FODO cell, assuming that there is a thin lens dipole with bending angle ϕ in the center between both quadrupoles.

(d) For what betatron phase advance (in degree) along the FODO is the maximum beta function in the FODO the smallest?

(e) Let the FODO have quadrupoles of 1m length, 5cm bore radius, and a pole tip field of 1T for protons of 40GeV. How long does the FODO have to be to obtain 60° phase advance?

Exercise 5:

Characterize Twiss parameters by $\{\beta(s), \alpha(s), \psi(s)\}$. Imagine two sections of a beam line where the first section transports Twiss parameters $\{\beta_0, \alpha_0, 0\}$ to $\{\beta_1, \alpha_1, \psi_1\}$ and the second transports $\{\beta_1, \alpha_1, 0\}$ to $\{\beta_2, \alpha_2, \psi_2\}$. Show that the total beam-line transports $\{\beta_0, \alpha_0, 0\}$ to $\{\beta_2, \alpha_2, \psi_1 + \psi_2\}$.

Exercise 6:

Compute the luminosity of an accelerator with bunch repetition frequency f, N_1 and N_2 particles in bunches of the first and second beam respectively, Gaussian bunch profiles in both beams with rms width $\sigma_x^(1)$, $\sigma_y^(1)$, $\sigma_z^(1)$, $\sigma_x^(2)$, $\sigma_y^(2)$, $\sigma_z^(2)$, when the two beams are parallel, but their trajectories miss each other by Δx in the horizontal and Δy in the vertical. (Note: example solutions are needed, if you do a good job with this question, feel free to type it up in LaTeXand email the solution to me.)

Exercise 7 (bonus question):

Compute the luminosity of an accelerator with bunch repetition frequency f, N_1 and N_2 particles in bunches of the fist and second beam respectively, Gaussian bunch profiles in both beams with rms width $\sigma_x^(1)$, $\sigma_y^(1)$, $\sigma_z^(1)$, $\sigma_x^(2)$, $\sigma_y^(2)$, $\sigma_z^(2)$, when the two beams have a crossing angle θ, but meat on center, i.e. at some time during the collision their centers coincide. Feel free to use a formula manipulating program for this excercise. (Note: example solutions are needed, if you do a good job with this question, feel free to type it up in LaTeXand email the solution to me.)