Accelerator Physics - Homework 1
USPAS 2010 (hosted by MIT)

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June 20, 2010

Exercise (Isocyclotron)
Find the radial dependence of the magnetic field $\mathrm{B}_z(r)$ in an isocyclotron with angular frequency ω_z.

Exercise (Microtron)
Consider a microtron with one accelerating cavity ($l = 1\text{m}$, $g = 30\text{MV/m}$) and $\omega_{RF} = 2\pi \cdot 1.3 \times 10^9\text{Hz}$. What is the proper value of the magnetic field B?

Exercise (Drift tube linac)
In a Wideroe linear accelerator, what is the limit of the drift tube's length as the speed of particles $v \to c$?

Exercise (LEP at CERN)
The main dipole magnets of the Large Electron Positron (LEP) collider had a bending radius of 3096 m.
(a) How strong was their magnetic field when LEP accelerated electrons to 105 GeV?
(b) This field strength is relatively small, why was the field not increased to increase the energy?
(c) The LEP tunnel was about 26.6km long. What fraction of it was used for bending the beam?

(d) LEP produced about 20MW of synchrotron radiation when it stored electrons at 100GeV. How much would the same number of electrons have radiated at 200GeV?

Exercise (The Earth Accelerator)

(1) Assume that the earth has an exact dipole magnetic field which is oriented parallel to the rotation axis. The magnetic field at the poles is about $2 \cdot 10^{-5}$T.

(a) Protons of what kinetic energy could you store in a bare vacuum pipe around the equator. What energy of electrons could you store?

(b) How much of their energy would these electrons lose during one turn?

(2) Consider a storage ring built around the 40 Mm circumference of the earth, where 100% of the tunnel were used for bending particles on a circular trajectory.

(a) How large would the energy be for protons when the LHC magnets with a magnetic filed of 8.7 T were used? Could one produce the highest proton energies of the universe in this way?

(b) How much energy would such a proton loose per turn, i.e. how much energy would accelerating sections have to provide per turn for this partice? What accelerating field would be required if it were continuous around the equator?

(c) How much power of synchrotron radiation would they approximately produce for the same current as in LEP (scaled from the LEP data given above)? Do not forget to scale so that the current stays the same, the number of particles in the ring is then not the same as in LEP.

(d) How large would the electron energy in this tunnel be if its synchrotron radiation load per length of the tunnel should be the same as that in LEP when the same current is stored (scaled from the LEP data given above)? Again, the number of particles in the ring is not the same as in LEP.

Exercise (Energy in Rings and Linacs)

A circular accelerator with dipoles of 100m bending radius stores an electron current of 0.1A at 5GeV. How much power is required to compensate for the emission of synchrotron radiation? How much power would be required to accelerate this electron current to 5GeV in a linear accelerator?

Exercise (Colliders)

(1) The PEP-II asymmetric B-Factory at SLAC stores electrons with an energy of 9.0 GeV and positrons with 3.1 GeV.

(a) How much energy is in the center of momentum system when a positron and an electron collide?

(b) What energy would positrons need to have in order to create the same energy in the center of momentum during a fixed target collision with electrons?

(2) Check that the LHC with its 7TeV protons should be listed for 10^{17}eV on the Livingston Chart. Where should the LEP collider with 100GeV electrons be on this chart? For any collision experiment with center of momentum energy E_{cm}, the Livingston chart shows how much energy a proton or electron would need to create the same center of momentum energy in a collision with a proton at rest.

Exercise (Doppler effect)

(a) In the lecture we studied that the matrix for a Lorentz transformation of a boost in the x-direction is

$$\underline{L} = \begin{pmatrix} \gamma & -\gamma\beta & 0 & 0 \\ -\gamma\beta & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} .$$

Derive the matrix $\underline{L}(\vec{\beta})$ for a boost in an arbitrary direction.

(b) Given that $K^\mu = \{\frac{\omega}{c}\, ,\ k_x\, ,\ k_y\, ,\ k_z\}$ is a 4-vector, derive the relativistic Doppler shift formulas.

(c) For non-relativistic velocities, there is no transverse Doppler shift, i.e. a frequency is not shifted if its emitter moves transverse to the wave propagation. Is this also true for relativistic speeds?