Accelerator Physics - Homework 3
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Exercise (Midplane symmetry of Hamiltonians)
What property does the Hamilton function $H(x, a, y, b, \tau, \delta, s)$ have when the motion is mid-plane symmetric?

Exercise (Canonical momentum)
Show that the Lorentz-force equation can be derived from the Hamiltonian

$$H = c\sqrt{[\vec{p}_c - q\vec{A}(\vec{r}, t)]^2 + (mc)^2} + q\Phi(\vec{r}, t)$$

where the canonical momentum $\vec{p}_c$ is related to the classical momentum by $\vec{p} = \vec{p}_c - q\vec{A}$.

Exercise (Symplecticity)
(a) A matrix $\underline{M}$ is symplectic when it satisfies $\underline{M}\underline{J}\underline{M}^T = \underline{J}$. Using $\underline{J}^{-1} = -\underline{J}$ and $\underline{J}^T = -\underline{J}$, show that the following properties are also satisfied:

$$\underline{M}^{-1} = -\underline{J}\,\underline{M}^T\underline{J}\ , \quad \underline{M}^T\underline{J}\,\underline{M} = \underline{J}\ . \tag{1}$$

(b) The linear transport map of a quadrupole is given by

$$\begin{pmatrix} x \\ p_x \end{pmatrix} = \begin{pmatrix} \cos(\sqrt{k}s) & \frac{1}{\sqrt{k}}\sin(\sqrt{k}s) \\ -\sqrt{k}\sin(\sqrt{k}s) & \cos(\sqrt{k}s) \end{pmatrix} \begin{pmatrix} x_0 \\ p_{x0} \end{pmatrix} \tag{2}$$

when k is the strength of the quadrupole field. Derive the generating function $F_1(x, x_0, s)$ that represents this map.

Exercise (Synchrotron Radiation)

(a) What is the fundamental wavelength of an undulator with $\lambda_u = 22$mm and $K = 0.5$ for a beam of 5GeV?

(b) An electron with 1MeV approaches a proton at rest in a central collision. Assume the proton remains at rest, how close will the electron approach the proton, how strong is the electric field it experiences, and how much power will the electron radiate at the location with the largest field? Use the following formula for the power P when the acceleration is parallel to the particles velocity:

$$P = \frac{2r_e q^2}{3mc}\vec{E}^2 \ . \tag{3}$$

Exercise (Radiation power in a transfer line)

Imagine you have to design a transfer line that bends a 5GeV electron beam of 100mA by 180°

(a) How many 4m long bending magnets do you need if the magnetic field should be smaller than 1T?

(b) How much synchrotron power will be produced per meter of magnet length?

(c) Approximately how much power per area hits the wall of a vacuum chamber of 2.5cm diameter?

(d) Approximately how many photons are radiated per meter of magnet length? Assume that the energy of photons is well described by the characteristic frequency

$$\omega_c = \frac{3}{2}\frac{c\gamma}{\rho} \ . \tag{4}$$

(e) How much energy does each electron loose in that transfer line?

Exercise (Undulator radiation)

An electron beam of 5GeV is sent through a long bending magnet of 2T field strength and an undulator with 5T pole field and 5mm gap and 25mm period. **(a)** What is the characteristic wavelength and photon energy of the bending magnet radiation.

(b) What photon energies can be produced by the undulator in forward direction?