Accelerator Physics - Homework 4
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Exercise (Curvi-linear system)
Given a reference trajectory that is a helix around the z-axis with

$$\vec{R}(z) = r\cos(kz)\vec{e}_X + r\sin(kz)\vec{e}_Y + z\vec{e}_Z, \tag{1}$$

with the Cartesian coordinate vectors $\vec{e}_X$, $\vec{e}_Y$ and $\vec{e}_Z$.
(a) Show that z is not the pathlength s with which the reference trajector is parametrized. Then compute the path length $s(z)$ and specify $\vec{R}(s)$ so that $|d\vec{R}| = ds$ and compute $\vec{e}_s$, $\vec{e}_\kappa$, and $\vec{e}_b$.
(b) Compute $\vec{e}_x$ and $\vec{e}_y$ of the curvilinear system and check that $\frac{d}{ds}\vec{e}_x$ and $\frac{d}{ds}\vec{e}_y$ are what they are specified to be in the handouts.

Exercise (Time, energy, and symplecticity)
Let the linearized particle transport from initial phase space coordinates $\vec{z}_i$ to final phase space coordinates $\vec{z}_f$ be:

$$\begin{pmatrix} x_f \\ a_f \\ \tau_f \\ \delta_f \end{pmatrix} = \begin{pmatrix} M_{11} & M_{12} & 0 & D_x \\ M_{21} & M_{12} & 0 & D_a \\ T_x & T_a & 1 & R_{56} \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x_i \\ a_i \\ \tau_i \\ \delta_i \end{pmatrix} . \tag{2}$$

The zeros in the matrix show that the particle motion is independent of the starting time and that the energy is independent of the starting conditions.
(a) Describe the meaning of the coefficients D_x, D_a, T_x, and T_a.
(b) Use the fact that this 4×4 matrix is symmplectic to show that the top left-hand 2×2 sub-matrix is symplectic.
(c) Show how T_x and T_a can be computed when this top left-hand sub-matrix and the dispersion D_x and its slope D_a are known.

Exercise (Combined function magnets)
Using the relativistic approximation $\gamma^2 \gg 1$, derive the 6 dimensional transport matrix for
A) a quadrupole magnet with length L and quadrupole coefficient k. Make a separate calculation for positive and for negative k.

B) a combined function magnet with length L, curvature κ and quadrupole coefficient k. Use $K = k + \kappa^2$ and distinguish the cases $k > 0$, $k < -\kappa^2$, and $-\kappa^2 < k < 0$.
C) Check that the time of flight could have been computed from the condition $\vec{T} = -\mathbf{J}_4\mathbf{M}_4^{-1}\vec{D}$ due to symplecticity.

Exercise (Time of flight spectrometer)
A time of flight spectrometer takes all particles that come from a collision point regardless of their initial slopes x' and y' and transports them to a point in a detector plane. The time of flight should depend only on the energy, not on the initial position or the initial angle of the particles in the collision plane. Write the most general form that the transport matrix from the collision plane to the detector plane can have.

Exercise (Path length and momentum coordinates)
If not the time of flight $\tau = (t_0 - t)\frac{E_0}{P_0}$ and the relative energy change $\delta = \frac{\Delta E}{E}$ had been chosen as phase space variables, but the deviation in path length Δl and the relative momentum deviation $\frac{\Delta P}{P}$, how would the transport matrix look like and how could it be computed from the transport matrix in exercise 4?
Exercise (Thin lens approximation)
Determine the thin lens approximation for the 6×6 matrix of a combined function magnet with quadrupole strength k and curvature κ. Use that the thin lens approximation is linearized in the length L.

Exercise (Solenoid matrix)
(a) Determine the transport matrix for a solenoid in the sharp cutoff limit. It has the length L, and the longitudinal field strength on axis is B_z for $x \in [0, L]$ and 0 outside this region. As solenoid strength you can use the parameter $g = \frac{qB_z}{2p}$.
(b) Determine the thin lens approximation of this solenoid, which is linearized in L.
Exercise (Phase space distribution):
(a) Given the Twiss parameters α, β, γ: specify the transformation from the amplitude and phase variables J and Ψ to the Cartesian phase space variables x and x'.
(b) Specify the inverse transformation.
(c) Given the Gaussian beam distribution in amplitude and phase variables, $\rho(J, \phi) = \frac{1}{2\pi\epsilon}e^{-\frac{J}{\epsilon}}$. What is the projection $\rho(x)$ of this distribution on the x axis. Check that the rms width of this distribution leads to $\sqrt{<x^2>} = \sqrt{\beta\epsilon}$.