Accelerator Physics - Homework 5
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Exercise (Beta functions)
Find the general form of the beta function in a drift:
(a) by solving the differential equation for $\beta(s)$ with the initial conditions $\beta(0) = \beta_0$ and $\alpha(0) = \alpha_0$.
(b) by propagating the matrix of initial Twiss parameters by the transport matrix of a drift according to

$$\begin{pmatrix} \beta & -\alpha \\ -\alpha & \gamma \end{pmatrix} = \underline{M} \begin{pmatrix} \beta_0 & -\alpha_0 \\ -\alpha_0 & \gamma_0 \end{pmatrix} \underline{M}^T \ . \tag{1}$$

(c) Find the general form of a beta function in a quadrupole of focusing strength k. Do not use the thin lens approximation.

Exercise (Periodic Twiss parameters)
a) Use the transport matrix from s_0 to s written in terms of Twiss parameters at s_0 and s to show that the one turn matrix of a ring at s can be written as

$$\underline{M} = \underline{1}\cos\mu + \begin{pmatrix} \alpha & \beta \\ -\gamma & -\alpha \end{pmatrix} \sin\mu \tag{2}$$

when α, β, and γ are the Twiss parameters that are periodic with the length L of the ring and $\mu = \Psi(L) - \Psi(0)$ is the one turn phase advance.
b) Show that the matrix before $\sin\mu$ in this equation has a characteristic of the complex i in that squaring it leads to $-\underline{1}$.
c) Use this to compute $\underline{M}^n$.

Exercise (One turn matrix)
If the one turn matrix

$$\underline{M} = \begin{pmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{pmatrix} \tag{3}$$

is known, specify how the periodic Twiss parameters and the one turn phase advance can be computed. Under what conditions is the one turn phase advance real? What does this mean for the long term motion in phase space which is described by $\underline{M}^n \vec{z}_0$ for large n.

Exercise (Propagation of Twiss parameters)
Characterize Twiss parameters by $\{\beta(s), \alpha(s), \psi(s)\}$. Imagine two sections of a beam line where the first section transports Twiss parameters $\{\beta_0, \alpha_0, 0\}$ to $\{\beta_1, \alpha_1, \psi_1\}$ and the second transports $\{\beta_1, \alpha_1, 0\}$ to $\{\beta_2, \alpha_2, \psi_2\}$. Show that the total beam-line transports $\{\beta_0, \alpha_0, 0\}$ to $\{\beta_2, \alpha_2, \psi_1 + \psi_2\}$.