Coupling Resonances

$$\tfrac{d}{d\vartheta}J_x=\cos(\tilde{\psi}_x+\varphi_x)\sqrt{2J_x\beta_x}\Delta f_x\tfrac{L}{2\pi}\quad,\quad \tfrac{d}{d\vartheta}\varphi_x=\upsilon_x-\sin(\tilde{\psi}_x+\varphi_x)\sqrt{\tfrac{\beta_x}{2J_x}}\Delta f_x\tfrac{L}{2\pi}$$

$$\tfrac{d}{d\vartheta}J_y=\cos(\tilde{\psi}_y+\varphi_y)\sqrt{2J_y\beta_y}\Delta f_y\tfrac{L}{2\pi}\quad,\quad \tfrac{d}{d\vartheta}\varphi_y=\upsilon_y-\sin(\tilde{\psi}_y+\varphi_y)\sqrt{\tfrac{\beta_y}{2J_y}}\Delta f_y\tfrac{L}{2\pi}$$

$$\tfrac{d}{d\vartheta}\vec{\varphi}=\vec{\partial}_J H\quad,\quad \tfrac{d}{d\vartheta}\vec{J}=-\vec{\partial}_\varphi H\quad,\quad H(\vec{\varphi},\vec{J},\vartheta)=\vec{\upsilon}\cdot\vec{J}-\tfrac{L}{2\pi}\int_0^{\vec{x}}\Delta\vec{f}(\hat{\vec{x}},s)d\hat{\vec{x}}$$

The integral form can be chosen since it is path independent. This is due to the Hamiltonian nature of the force: $\Delta f_{x,y}(x,y,s)=-\partial_{x,y}\Delta H(x,y,s)$

Single Resonance model for two dimensions means retaining only the amplitude dependent tune shift and one term in the two dimensional Fourier expansion:

$$H(\vec{\varphi},\vec{J},\vartheta)=\vec{\upsilon}\cdot\vec{J}+H_{00}(\vec{J})+H_{n\vec{m}}(\vec{J})\cos(n\vartheta+m_x\varphi_x+m_y\varphi_y+\Psi_{n\vec{m}}(\vec{J}))$$

For $n+m_x\upsilon_x+m_y\upsilon_y\approx 0$

$$m_x\varphi_x+m_y\varphi_y=\vec{m}\cdot\vec{\varphi}$$

Sum and Difference Resonances

$n + m_x \upsilon_x + m_y \upsilon_y \approx 0$ means that oscillations in y can drive oscillations in x in

$$x'' = -K\,x + \Delta f_x(x, y, s)$$

The resonance term in the Hamiltonian then changes only slowly:

$$H(\vec{\varphi}, \vec{J}, \vartheta) = \vec{\upsilon} \cdot \vec{J} + H_{00}(\vec{J}) + H_{n\vec{m}}(\vec{J}) \cos(n\vartheta + \vec{m} \cdot \vec{\varphi} + \Psi_{n\vec{m}}(\vec{J}))$$

$$\tfrac{d}{d\vartheta}\vec{\varphi} = \vec{\partial}_J H \quad , \quad \tfrac{d}{d\vartheta}\vec{J} = -\vec{\partial}_\varphi H$$

$$J = \vec{m} \cdot \vec{J}$$

$$J_\perp = m_x J_x - m_y J_y = \vec{m} \times \vec{J} \quad \Rightarrow \quad \tfrac{d}{d\vartheta} J_\perp = 0$$

Difference resonances lead to stable motion since:

$$n + |m_x|\upsilon_x - |m_y|\upsilon_y \approx 0 \Rightarrow |m_x| J_x + |m_y| J_y = const.$$

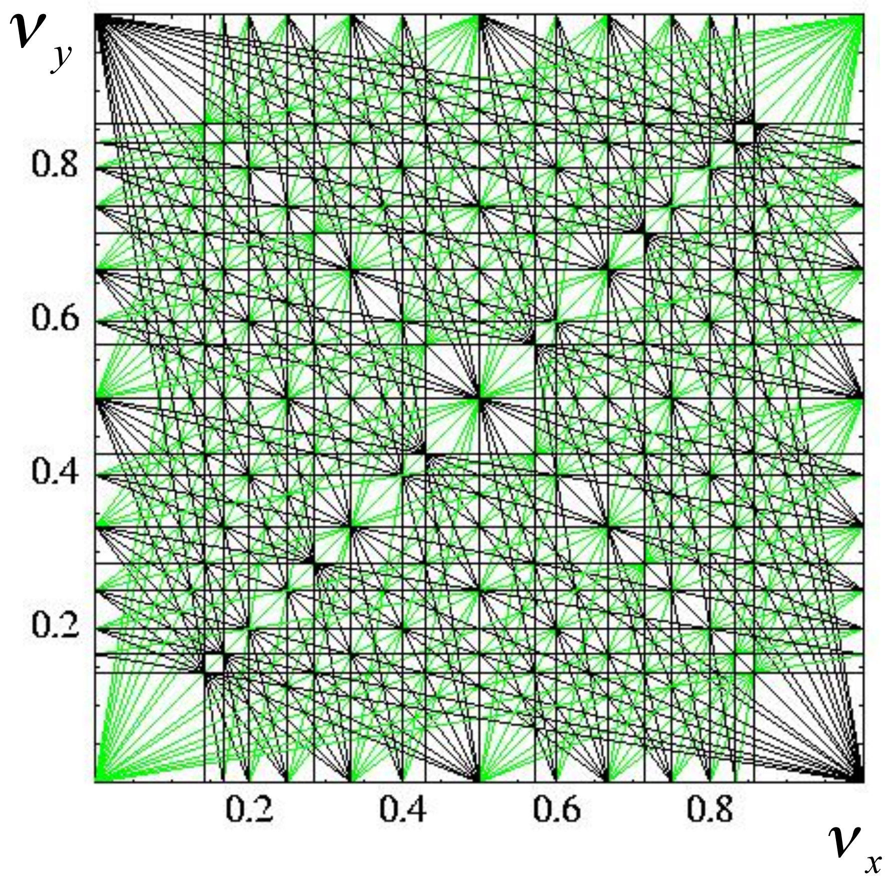
Sum resonances lead to unstable motion since:

$$n + |m_x|\upsilon_x + |m_y|\upsilon_y \approx 0 \Rightarrow |m_x| J_x - |m_y| J_y = const.$$

Resonances Diagram

$n + m_x \upsilon_x + m_y \upsilon_y \approx 0$ means that oscillations in y can drive oscillations in x in

$$x'' = -K\,x + \Delta f_x(x, y, s)$$



All these resonances have to be avoided by their respective resonance width.

The position of an accelerator in the tune plane s called its Working Point.