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Spin Dynamics

Georg Hoffstätter
Institute of applied Physics, TH–Darmstadt

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Polarized Protons have never been accelerated to more than about 25GeV/c. Here some of the methods used for the analysis of polarized proton beams in high energy accelerators such as RHIC (250GeV), HERA (820GeV), and the TEVATRON (900GeV) are outlined. The talk will start with the spin equation of motion and its transformation to the curvilinear coordinate system defined by the accelerators design trajectory. However, since we are not interested in the motion of a single spin but rather in the general solution for all possible initial spins, it is far more sensible to compute the rotation matrix that transports initial spins to there final position in the accelerator. This rotation matrix can be represented by a four dimensional rotation vector. The equation of motion for this rotation vector is formulated and serves as a basis for the analysis of spin dynamics.

This equation of motion for the rotation vector in the accelerator coordinate frame can be solved by various approximations. Starting with an initial phase space coordinate for a particle, numerical integration of the equation of motion leads to the rotation vector. However, often one is much more interested in the rotation vector as a function of the initial phase space coordinates. This function can be very complicated. However, the Taylor expansion of the rotation vector with respect to the initial phase space variables can easily be computed with Differential Algebra codes. To speed up long term computations, one can also find analytical approximations for the transport of phase space variables and for the rotation vector which rotates arbitrary initial spins during there travel through an optical element. Then one applies these transport functions successively for each element of an accelerator. This procedure is known as element by element tracking.

The rotation vector describing the action of a single optical element can be obtained in a nonlinear expansion with respect to the initial phase space coordinates. The simplest approximation is often already quite useful. Here one linearizes with respect to the initial phase space coordinates and additionally with respect to the deviation of the spin from a preferred spin direction. Another approximation which has proven to be very useful is the single resonance model. The computation of resonance strength as well as some common results obtained from this model will be mentioned.

For very high energy polarized proton beams the equilibrium polarization direction becomes very important. This equilibrium direction can vary substantially across the beam in the interaction region of a high energy experiment when no countermeasure is taken. Such a divergence of the polarization direction would not only diminish the average polarization available to the particle physics experiment, but it would also make the polarization involved in each collision analyzed in a detector strongly dependent on the phase space position of the interacting particle. Apart from that, there is also a lot of theoretical interest in the equilibrium direction since this field of spin directions is required for the computation and understanding of the spin tune. The spin tune describes the angle of spin rotation during one turn around an accelerator and depends on the amplitudes or action variables with which particles move in phase space but not on the angle variables of there motion. Furthermore, it will be shown that there is a unique equilibrium spin field whenever there is no orbit and no spin orbit resonance.

In order to analyze and compensate these effects, methods for computing the equilibrium polarization direction are needed. Here the computation of this quantity in linear approximation will be illustrated and an analytical solution of the single resonance model will be derived. Furthermore the Taylor polynomial of the phase space dependent spin direction which is in equilibrium under particle motion can be computed by an order by order normal form transformation. Finally the method of stroboscopic averaging, which computes this direction in a very efficient numerical way, is illustrated. Since only tracking data is needed, this method can be implemented easily in existing spin tracking programs. Several examples demonstrate the importance of the equilibrium spin direction and the applicability of stroboscopic averaging.