

Applications of Symplectic Scaling

G. H. Hoffstätter and M. Berz

The particle transport through beamlines, isotope separators, spectrographs, and other devices often depends strongly on the fringe fields of the optical elements involved. A method to compute those effects efficiently was developed; its use will be demonstrated for several properties of the A1200 isotope separator.

In last year's annual report¹, it was discussed that the computation of transfer maps can be performed very quickly using differential algebra evaluation of the propagator of motion in the case that the maps describe particle motion through the main-field region of optical elements². However, if the fields change along the central trajectory, very time consuming numerical integration has to be used. We developed an efficient approximation method using the scaling properties of maps. To ensure the symplecticity of the approximated maps, we analyzed which implications these scaling properties have on symplectic representations of the map, namely the generating functions and the Lie exponents^{3,4}. This approach is usually very accurate and much faster than numerical integration. Meanwhile this method was tested on several examples, some of which will be presented here. An analysis of accuracy and speed was made with the result that symplectic scaling is now included in the most recent version 6 of COSY INFINITY⁵, which was distributed to the users at the end of 1992.

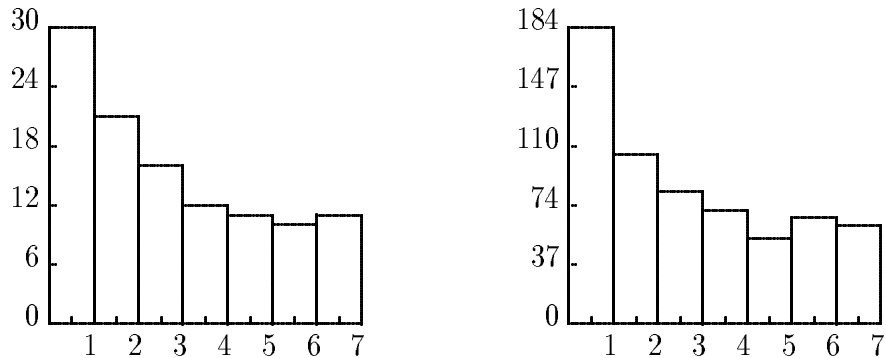


Figure 1: Factor of time advantage of SYSCA to numerical integration with as a function of the order of the map. Left: Quadrupole, Right: Dipole.

The symplectic scaling (SYSCA) approximation is especially helpful in the design of a realistic system after approximate parameters of the elements have been obtained by neglecting fringe fields. These values can be used to create a reference file for symplectic scaling. In this way, a very high accuracy almost

equivalent to accurate but time intensive numerical integration of the map can be obtained. The time advantage of this method is illustrated in figure 1.

Fringe fields do have noticeable effects already in first order. In the example of the A1200⁶ isotope separator at the NSCL, the effect of the fringe fields on the calculated setting of the field strength is shown in figure 2. The fringe fields were described by Enge functions, and the Enge coefficients had been fitted to measured field data. Here the time advantage of the proposed approximation in the fit is three minutes versus two hours. As a measure of accuracy, we study

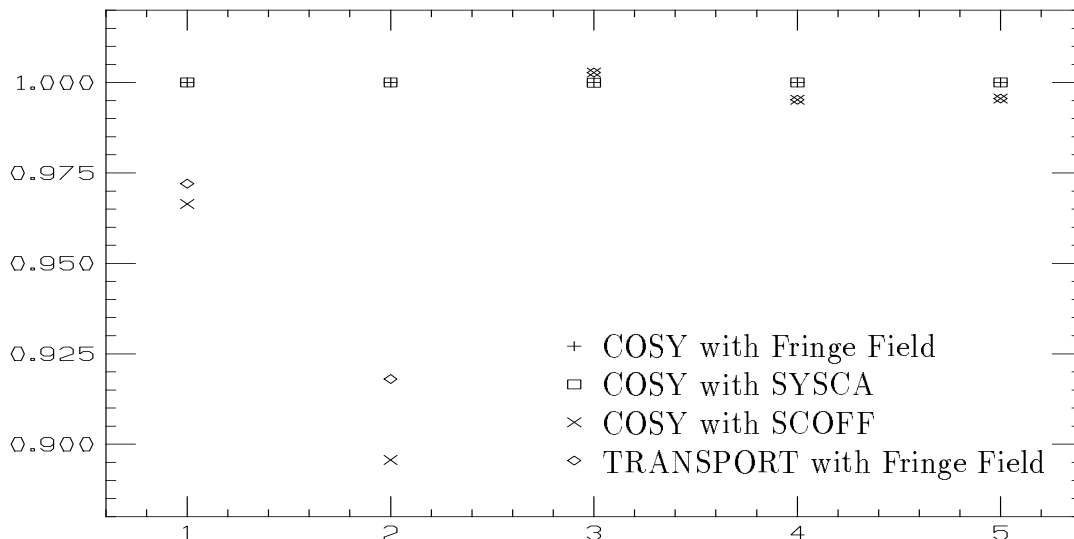


Figure 2: Relative deviation of predicted field settings with SCOFF and SYSCA from the correct settings for five quadrupoles. The standard fringe field approximation of TRANSPORT is given as a reference; the deviation is mainly due to the approximation of quadrupole fringe fields.

the tilt angle Θ of the dispersive image plane and the opening aberration C_0 for various approximation methods. In the discussed device the coefficient $(x|aa)$ vanishes because of symmetry of the axial ray and anti symmetry of the dipole fields; therefore $(x|aaa)$ is the relevant opening aberration,

$$\Theta = -\frac{(x|a\delta)}{(a|a)(x|\delta)} , \quad C_0 = (x|aaa) . \quad (1)$$

Table 1 shows Θ and C_0 for various fringe-field models. The values of Θ with and without fringe fields differ by 0.5% for the first dispersive image plane in the A1200; the third order aberration, however, is completely wrong if fringe fields are disregarded. This comparison also shows that quadrupole fringe fields, although often disregarded, can have effects which dominate over dipole fringe

| | | |
|---|----------|----------|
| Θ and C_0 with SCOFF approximation | 80.8840° | -65.96m |
| Θ and C_0 with dipole fringe fields only | 81.1696° | -65.96m |
| Θ and C_0 with quad fringe fields only | 81.2694° | -682.68m |
| Θ and C_0 with SYSCA approximation | 81.2701° | -687.10m |
| Θ and C_0 with actual fringe fields | 81.2702° | -687.10m |

Table 1: Tilt angle and opening aberration for various fringe-field models.

fields. Nonlinear effects can be seen by sending a cone of particles through the 7th order A1200 map. The images with SCOFF and SYSCA approximation are shown in figure 3. The maximum angle used is 15mrad.

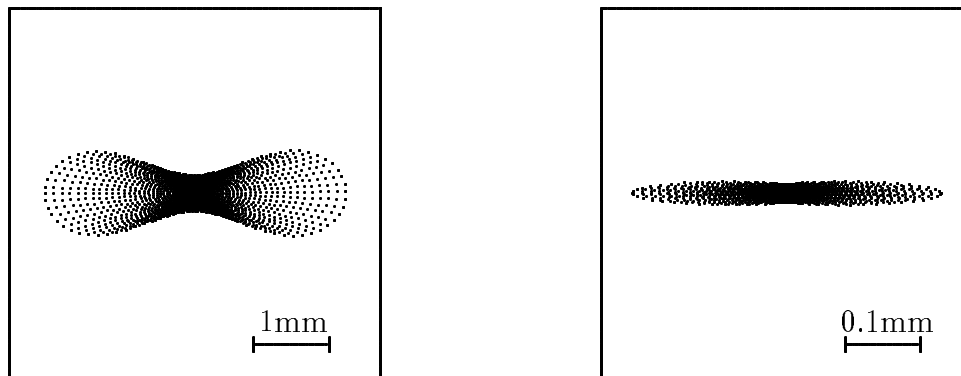


Figure 3: Beam spots with SYSCA (left) and SCOFF (right) approximation. The plot produced with the exact fringe fields can not be distinguished from the plot produced with SYSCA.

The effort involved in generating a symplectic approximation is rewarded when repetitive tracking is being performed. The example lattice of choice is the proposed PSR II Ring at Los Alamos National Laboratory. The 9th order 5000 turn tracking pictures are displayed in figure 4. The tracking was performed with the described standard numerical integration, SYSCA, and a nonsymplectic fringe-field approximation obtained by low accuracy numerical integration. Nonsymplectic tracking rapidly destroys the phase space. SYSCA yields more stable results than the numerical integration since the limited accuracy of the numerical integrator slightly violates symplecticity. The corresponding 9th order maps were produced with the SYSCA mode in COSY INFINITY in 30 minutes, whereas the standard numerical integration took 15 hours, and the nonsymplectic approximation took 44 minutes on a VAX 4000-90 computer.

REFERENCE

1. G. H. Hoffstätter, M. Berz, NSCL Annual Report, 236 (1991)

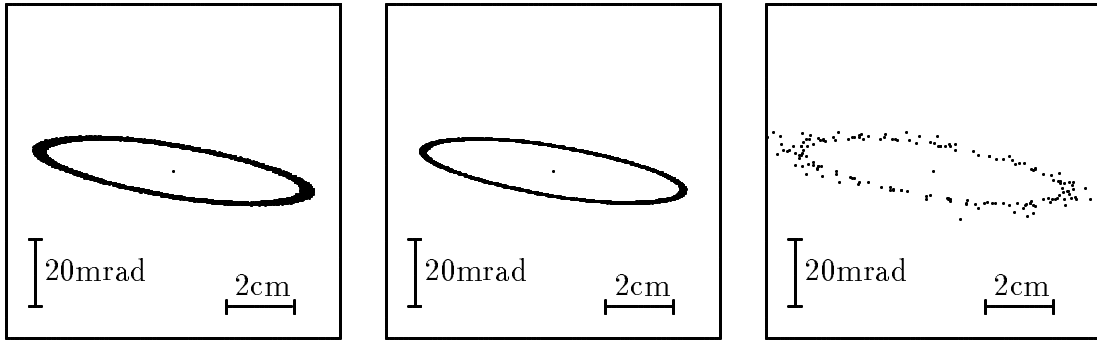


Figure 4: 5000 turn tracking with fringe fields obtained by numerical integration (left), SYSCA (middle), and a nonsymplectic fringe field approximation (right). The initial position of the particle is $(x, y) = (3\text{cm}, 3\text{cm})$ with no initial inclination.

2. M. Berz, Nucl. Instr. and Meth. A298, 426 (1990)
3. G. H. Hoffstätter, M. Berz, Proc. to Comp. Accel. Phys., to be publ., (1993)
4. G. H. Hoffstätter, M. Berz, Proc. to Part. Accel. Conf., to be publ., (1993)
5. M. Berz, COSY INFINITY User's Manual, MSUCL-869 (1993)
6. B. M. Sherrill et al. The First Intern. Conf. on Radioactive Nucl. Beams, Berkeley, World Scientific Publishing (1990)