In UM-HE-99-05

Spin motion at high energy in HERA-p

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0.1 Introduction

Intrinsic depolarizing resonances at the high energies of HERA-p can be very strong[1] and the strengths of the first order resonances can approach unity at the top end of the HERA energy range. For example figure 1 shows the strengths of first order resonances for the momentum range from 500GeV/c to 1000GeV/c with flattening snakes [2] for the 1996 optical configuration used for acceleration up to full energy. The orbital tunes are $Q_x = 31.27899$, $Q_y = 32.27253$ and $Q_s = 0.0006389$ at the start. The strengths were evaluated on a vertical phase space ellipse with an invariant emittance of 16π mm. mrad. (2σ). The very strong resonances occur

Figure 1: Resonance strengths in HERA-p with flattening snakes from 500 to 1000GeV/c for an invariant amplitude of 16π mm.mrad.(2σ).

whenever the spin phase advance and the orbit phase advance are in resonance in the FODO cells of the arc. Such very strong resonances can occur more than 29 times during the ramp from 40GeV/c to 820GeV/c.

0.2 Acceleration of a polarized beam through resonances

The equilibrium state of a polarized beam is described by the invariant spin field (the $\vec{n}$-axis) [3]. The upper limit of the average polarization at equilibrium $P_{\lim}$, which is given by the average of $\vec{n}$ over the phase space, is small when the energy is close to that of a very strong depolarizing resonance [4]. In this article the particles are distributed *on* phase space torii. The situation can be improved by the installation of Siberian Snakes but even with snakes residual resonance structures (RRS's) at or near the energies of the original resonances can remain so that it is necessary to choose the snake scheme carefully as described below. In particular, the choice of the snake types and their parameters must be optimized. In the following we study the effects of 'horizontal' snakes, i.e. snakes whose spin rotation axes ('snake axes') lie in the horizontal plane. Similar kinds of detailed investigations by Balandin and Golubeva are described in [5].

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0.2.1 Filtering

In HERA-p there is space for Siberian Snakes in four straight sections and, with a significant increase of cost, in the centers of the four arcs. To find the best choice for the orientations of the snake axes we use the snake filtering algorithm [1] to maximize the equilibrium polarization averged over the momentum range at the chosen amplitude. In particular, we tested all four and eight snake schemes for snakes with snake axes which lie at angles which are multiples of 22.5 degrees (for four snakes) and 45 degrees (for eight snakes) with respect to the radial direction.

0.2.2 Filtering with four snakes

Figure 2 (top) shows the variation of $P_{\rm lim}$ with momentum for the optics used in figure 1 with four snakes after filtering with particles on a vertical phase space ellipse with an invariant emittance of 4π mm. mrad. (1σ). Figure 2 (bottom) shows the corresponding $P_{\rm lim}$ for an amplitude of 16π mm. mrad. (2σ). Beginning in the East the snake axes are at 90, -45.0, 0 and + 45 degrees degrees from the radial direction respectively. For this article $P_{\rm lim}$ is calculated with the new version of the SODOM algorithm [6] which is now embedded in the program SPRINT [4]. The $\vec{n}$-axis obtained from SODOM agrees with that obtained using stroboscopic averaging in SPRINT. Even with filtering, RRS's remain and as can be seen in figure 2 their positions

Figure 2: $P_{\rm lim}$ from 500 to 1000GeV/c in HERA-p with flattening snakes and four additional filtered snakes for a vertical invariant amplitude of 4π mm.mrad.(1σ) (top) and 16π mm.mrad.(2σ) (bottom)

are strongly correlated with the positions of the original resonances. But the strengths of the RRS's need not be strongly correlated to the strengths of the original resonances. According to figure 2 (top), for a beam in which the particles only execute vertical betatron motion on the 1σ ellipse, the equilibrium polarization could reach almost 100 percent at the highest momentum.

However, the full energy can only be reached by acceleration through the RRS's and that can lead to depolarization if the acceleration is not sufficiently adiabatic. The result of a simulation of acceleration up to full energy is shown in figure 3 where $P_{\rm lim}$ (dotted curve in this and other figures) is plotted together with the ramped polarization $P_{\rm ramp}$ (crosses).

The same set of optics is used as in figures 1 and 2 and at the beginning of the simulation the particles were on the surface of the phase space torus corresponding to horizontal, vertical and longitudinal invariant emittances of 4π mm.mrad., 4π mm.mrad. and 1.78 10^{-2} m.rad. respectively corresponding to '1σ' in all three modes. A representative acceleration rate of 13KeV per turn was used. The acceleration was simulated within the SPRINT program and each spin was set initially parallel to the $\vec{n}$-axis correponding to the position of the particle in phase space. In spite of the fact that all three modes of orbital motion are involved, the ramped polarization in figure 3 (left) is very similar to the $P_{\rm lim}$, indicating that for particles inside 1σ in all three modes, the acceleration rate is sufficiently adiabatic and that the horizontal and longitudinal motions have little effect. However only about 6 percent of the beam is contained within that boundary so that acceleration at larger amplitudes must be investigated.

Figure 2 (bottom) shows the dependence of $P_{\rm lim}$ on momentum for the same conditions as in figure 2 (top) except that the particles are on the 2σ vertical ellipse. Now the variation

Figure 3: P_{lim} and P_{ramp} from 785GeV/c to 825GeV/c in HERA-p with flattening snakes and four additional filtered snakes for invariant amplitudes of 1σ,1σ,1σ (left) and 2σ,2σ,2σ (right).

of P_{lim} with energy is much more prominent and, as expected at larger amplitude, the RRS's are stronger. At the highest energy the equilibrium polarization would not exceed about 93 percent. The outcome of accelerating for a small range around 805GeV/c for particles at 2σ in all three planes is shown in figure 3 (right) where we see that the polarization is lost above 795GeV/c. We are thus led to investigate spin stability with an eight snake scheme. However, experience has shown that eight snake schemes are not neccessarily better than filtered four snake schemes. So the eight snake schemes must be selected by filtering too.

0.2.3 Filtering with eight snakes

In the following we show a series of comparisons of P_{lim} and the polarization P_{ramp} surviving after acceleration for a filtered eight snake scheme. Beginning in the East the orientations of the snake axes are as follows: 90, 90, -45, 0, 0, 0, +45 and 90 degress respectively. The conditions are given in the captions. Comparison of figures 4 (left) and 3 (left) shows that,

Figure 4: P_{lim} and P_{ramp} from 785 to 825GeV/c in HERA-p with flattening snakes and eight additional filtered snakes for an invariant amplitude of 1σ (left) and 2σ (right) in all three planes.

as expected, eight filtered snakes do a better job of supressing variations of the polarization during acceleration than four such snakes. However, figure 4 (right) shows that at 2σ in each plane, and with eight filtered snakes the RRS near 805GeV/c is still very strong and that polarization cannot be maintained when accelerating through that region. In contrast, figure 5 (top) demonstrates that for particles executing only vertical motion on the 2σ ellipse, eight filtered snakes are perfectly adequate for maintaining polarization during acceleration through the RRS at 805GeV/c. So the strong depolarization seen in figure 4 (right) is due in some way to the additional effects of horizontal and longitudinal motion. It is apparent

Figure 5: P_{lim} and P_{ramp} from 785 to 825GeV/c in HERA-p with flattening snakes and eight additional filtered snakes for: (top) an invariant vertical amplitude of 2σ; (bottom left) a vertical amplitude of 2σ together with a longitudinal amplitude of 1σ; (bottom, right) a vertical amplitude of 2σ together with a longitudinal amplitude of 2σ.

from figure 5 (bottom,left) that energy oscillations on the (1σ) longitudinal ellipse cause little disturbance beyond that due to vertical motion on the 2σ ellipse. However on moving out to the 2σ longitudinal ellipse (figure 5 (bottom,right)) strong additional RRS's appear. Although polarization can be maintained on acceleration through the RRS at 805GeV/c, it is then lost at 810GeV/c. What about the effects of horizontal betatron oscillations? Figure 6 (top,left) shows that for 1σ vertical motion, additional 2σ horizontal motion has some effect but that it is not very pronounced. Moreover, although for 2σ vertical motion and 1σ horizontal motion

Figure 6: $P_{\rm lim}$ and $P_{\rm ramp}$ from 785 to 825GeV/c in HERA-p with flattening snakes and eight additional filtered snakes for: (top,left) a horizontal amplitude of 2σ and a 1σ vertical amplitude; (top, right) a horizontal amplitude of 1σ and a 2σ vertical amplitude; (bottom, left) a horizontal amplitude of 1σ, a 2σ vertical amplitude and a 1σ longitudinal amplitude; (bottom,right) a horizontal amplitude of 2σ and a 2σ vertical amplitude.

the RRS near 805Ge/c is very strong, a polarization of about 30 percent survives acceleration (figure 6 (top,right)). But when longitudinal motion at (1σ) is added (figure 6 (bottom, left)) only about 50 percent of the polarization survives although above 805GeV/c $P_{\rm lim}$ has not changed much. Finally, in figure 6 (bottom,right) we show the effect on the polarization of 2σ horizontal motion and 2σ vertical motion. In this case only about 25 percent survives the RRS at 805GeV/c. The results presented above have shown that for a perfectly aligned HERA and for pure vertical betatron motion, a snake angle configuration can be found by filtering for which residual resonance effects are so weak that almost no polarization is lost during acceleration through them. However the snake angle configuration needed is not very intuitive. In the following sections we analyse the reasons for this.

From figures 5 and 6 it is clear that the effects of horizontal and longitudinal motion cannot be ignored. This is no surprise since rotations around different axes do not commute so that spin rotations due to vertical, horizontal and longitudinal motions are not independent and the resonance spectrum becomes richer. Naturally, at high energy noncommutation effects can be particularly pronounced. However, other simulations show that for a perfectly aligned HERA and by careful choice of betatron tunes, acceleration up to 820GeV/c without significant loss of polarization *is* possible for particles within the 2σ, 2σ, 2σ torus. These results will be presented elsewhere. Note that orbital coupling, nonlinear fields or nonlinear motion are not prerequisites for the occurence of high order resonances.

Acknowledgement

We thank N. Golubeva and V. Balandin for useful discussions and extensive computations.

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