

# Experimentation of Centroid Stability and Simulated Noise for Video Based Position Monitoring, Statics and Dynamics

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The staff scientists at CHESS (Cornell High Energy Synchrotron Source) have developed an X-ray beam position monitor (BPM) method that utilizes He gas fluoresced by the X-ray beam as it traverses the cavity. A CCD camera acquires images of the fluorescence and calculates the centroid of the beam. This paper presents results from a variety of experiments using this Video BPM on an LED Light Source on an optics bench compared against Monte Carlo software developed to compare effects of CCD noise and background effects on the centroid of the beam. Comparisons will be made between the experiment and the simulation while changing a variety of parameters that alter both the settings of the camera and the video acquisition software itself. Further experiments and simulations will be performed on a dynamic light source, comparing these results to that of a static LED. to study various aliasing effects.

The static measurements yielded very solid results, showing an inverse-square root relationship between the stability of the centroid and two of the image acquisition parameters — Frame Average number and Shutter time. The experiments attained a sigma (standard deviation of the trace of the centroid) as low as  $0.07\mu\text{m}$ . As low as the static measurements would get, though, a degree of instability was always present in dynamic trials, due to an effect known as aliasing. Certain attributes of the experiment contributed to the aliasing, specifically the accuracy of the CCD Camera and the finite pixel size, but ultimately an instability of tenths of microns was added to the static stability in moving situations.

Using Monte Carlo simulations to model the LED light source proved successful. The same trend relating to these acquisition parameters was present when simulating the static image experiments. Also, when simulating the dynamic situation, these aliasing effects were noticeable as in experiments.

## I. INTRODUCTION

Synchrotron Radiation, created by radially accelerated charged particles, has a wide range of application in the sciences. From protein crystallography to analyzing paintings, the x-rays emitted from CHESS have led to numerous discoveries since its inception in 1980. The x-rays are created in both the electron and positron beamlines of CESR (Cornell Electron Storage Ring) by being run through bending magnets (dipoles), and wigglers. The x-rays are then sent through a monochromator and then into an experimental hutch, where users from outside of CHESS run experiments. Because the experiments are usually performed at a great distance from the x-ray source, the stability of the beam line is key. The most minute shifts in the beam may have drastic negative consequences. Thus, the video beam position measurement system has been implemented to track the stability of the centroid of the beam over time. The Video BPM is able to track the centroid position of the beam on the order of microns, the degree of precision required for the experiments performed at CHESS.

USBChameleon, a program developed at CHESS, is used to track the centroid. It will also assist in making various measurements related to the light source being used, particularly its profile and the standard deviation ( $\sigma$ ) of the trace of the centroid.

Several parameters are highly influential on the stability of the centroid, especially the Frame Average value, Shutter Time, and Gain. Frame Average is the number of frames that the CCD camera acquires before making any calculations. The higher the number, the longer it takes to process each image, but the centroid is more stable, resulting in better accuracy at the expense of measurement time.. Shutter Time is the length of exposure time for the camera, which will range between 25ms and 100ms. Gain is the camera's amplifier setting to make an image artificially brighter. This is helpful when decreasing exposure times, where the image will darken due to fewer photons striking the CCD chip in the shorter timespan. This project will analyze these parameters and how they affect the centroid.

## II. EFFECTS ON STABILITY

A large variety of parameters have an influence on the overall stability of the x-ray beam. In performing experiments on the LED light source, many of the factors can be controlled, adjusted, and analyzed to study their effect on the stability. However, because there are so many parameters, most of them must be fixed and only a few will be varied. Some of these parameters are the following:

### **Optical:**

- Lens
- Distance between Camera and Light Source
- F-stop (or aperture)

### **Light Source:**

- Intensity (brightness)
- Stability
- Symmetry

### **Acquisition Parameters:**

- Shutter
- Gain
- Frame Averaging
- Camera Resolution

#### **Experimental Conditions:**

- Vibrations
- Airflow

#### **Computational Factors:**

- Offset/Cutoff of Background
- Filtering
- Centroid Calculation (Normal v. Squared)

The optical settings have to do with the setup of the camera, the experiment, and the lens, and the optics associated with them. The light source is pertaining to the LED test light source. The brightness can be changed throughout experiments, while stability is hoped to be minimized. Acquisition Parameters are controlled by the computer and are the main variables that will be changed in the experiments. Experimental conditions, similarly to the LED stability, are not directly controlled, but hoped to be minimized through several methods. The computational factors are also effected by the computer program.

Note: the method of centroid calculation, either normal or squared, has a significant effect on the stability of the centroid.

The formula for calculating the position of the centroid using the normal method is

$$X_c = \frac{\sum x \cdot I(x, y)}{\sum I(x, y)} \quad (1)$$

where  $I(x, y)$  is the pixel intensity value at the pixel  $(x, y)$ .

The formula for calculating the position of the centroid using the squared method is

$$X_{c^2} = \frac{\sum x \cdot I^2(x, y)}{\sum I^2(x, y)} \quad (2)$$

The squared centroid calculation, then, gives more weight to higher intensity values, and therefore typically results in a more stable centroid [1].

### **III. SIGMA**

The main focus of this project is on Sigma, or the standard deviation of a number of values measured. The two main types of sigma being measured are of the position of the centroid of the image over time and the standard deviation of a given pixel's intensity over time.

For the position trace of the centroid, ideally, given unchanging conditions, the trace is constant. However, the measured centroid data have a noise value from statistical error, that is expressed as the standard deviation of the trace while parameters remain unchanged.

The noise measured while analyzing a given pixel’s intensity over time arises from instability in the CCD chip. One major source of this is what is called Read Noise, or an error in converting between the signal picked up by the chip and a digital signal being read into the computer. The other major sources of sigma due to the CCD chip are Photon noise, a “statistical variation in the arrival rate of photons incident on the CCD,” and Dark noise, “from statistical variation in the number of electrons thermally generated within the silicon structure of the CCD” [5].

Other contributors to sigma arise during the experiment, with the majority of these come from the experimental parameters listed above, most notably the experimental conditions and mechanical instability of the setup.

## IV. HARDWARE

### A. CCD Camera

The camera used for experiments is a Chameleon brand USB 2.0 camera from Point Grey research (See Fig. 1). The chip is a 1.3MP  $\frac{1}{3}$ ” ICX445 EXview HAD CCD from Sony. The resolution is  $1296 \times 964$  pixels. The camera resolution can be changed to  $640 \times 480$ ; this is done by a  $2 \times 2$  pixel binning (averaging), resulting in better signal-to-noise performance. This process results in one higher bit of accuracy, as explained by the camera’s manufacturer. It is mounted to the optics bench facing the LED light source, also mounted to the bench on a motor-controlled table.

### B. Experimental Setup

For all experiments, the camera was mounted down to an optics bench, as seen in Fig. 1. Set up opposite it 50cm away on the bench is a mount with a vice holding the motor-controlled table, to which the LED light source is mounted. The LED is fit into an aluminum plate that has been wrapped in black aluminum foil to eliminate reflections. See Fig. 2 for more detail.

## V. USBCHAMELEON

The software being used, USBChameleon, has a variety of functions that are very useful for tracking the centroid and profile of the beam over time. The outputs display position, full width at half-max (FWHM) of the intensity profile, the overall intensity luminescence integral, and  $\sigma$  calculated from the last twenty values saved of the position of the centroid. The GUI displays the image with the Region-of-Interest (ROI) surrounding it, images of the intensity profile of the image within the ROI, and the centroid (position) trace. Also on screen are a variety of inputs for settings — calibration options, inputs for various settings (frame average, etc.), and an option to open up the Camera Settings [2][3].



FIG. 1. The CCD Camera used for image acquisition.

## VI. EXPERIMENTS

The project consisted of three major experiments. Experiment 1 was measuring the sigma of the centroid position of a stationary light source. Experiment 2 consisted of measuring sigma as a function of pixel intensity, also with a stationary light source. For Experiments 1 and 2, the main variables adjusted were Frame Average and the Shutter Time. Experiment 3 analyzed a different effect of image digitalization. Rather than measuring sigma, the goal was to observe and understand an effect known as aliasing.

When an object moves, because of the conversion to a digital image for the computer to process, it will sometimes become indistinguishable, and lead to a slight distortion of the image.

For each experiment, a blurred (unfocused) LED was used. This choice was made for two major reasons. First, the blurring led to a much smoother, continuous distribution of pixel intensities throughout the ROI, instead of jumping between a number of intensity values. Second, the profile of the blurred LED strongly resembles a Gaussian Distribution (See Fig. 4), which is highly advantageous for simulation — the mathematical description of the profile will allow simulations to be calculated smoothly.

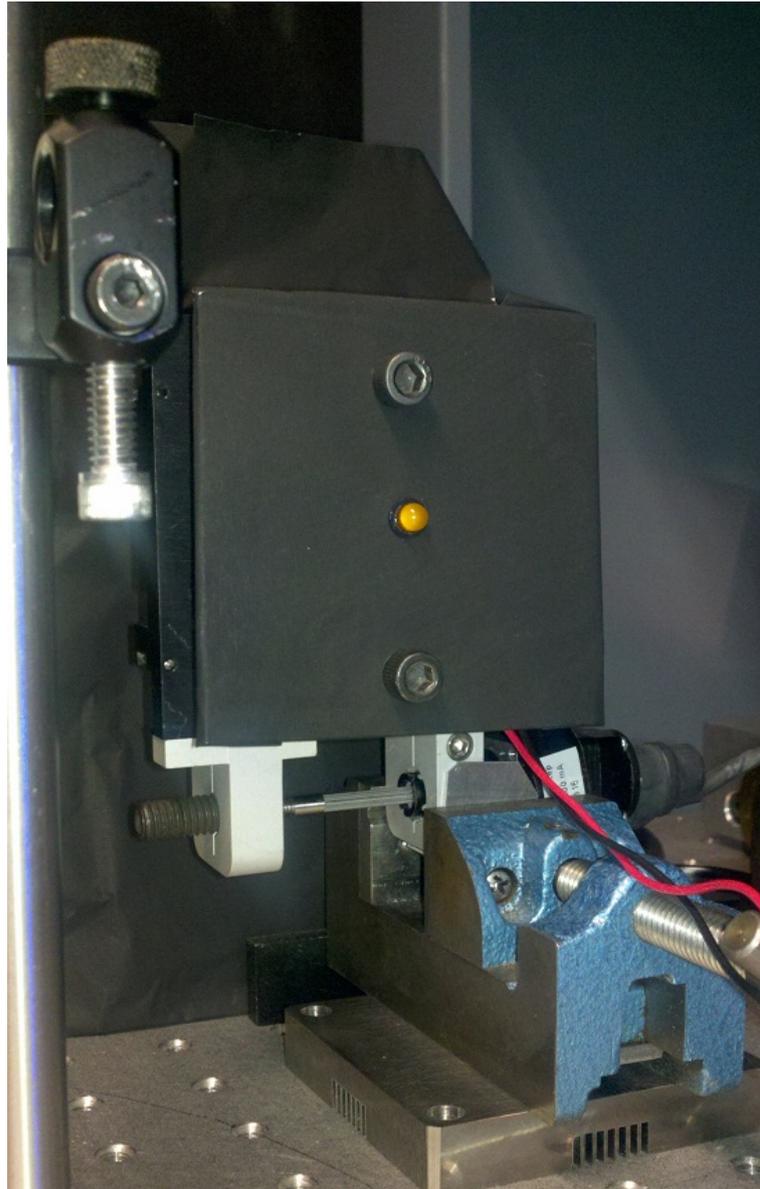


FIG. 2. The LED light source mounted to the Motor-Controlled Table.

#### A. Experiment 1 — Sigma of Centroid: Static Light Source

The CCD Camera and LED Light source were set up on the optics bench, stabilized, and covered with a metal “tunnel.” The metal, spraypainted black on the inside, is meant to reduce airflow through the experiment and to eliminate any background light from entering the experiment. The goal of this is to reduce the error noted above as “experimental conditions.” For each trial, 200 saves were taken — meaning a variety of length of trial between the lower frame average/shorter shutter time to the higher frame average/longer shutter time. The data containing 200 points each were saved in separate files and analyzed afterwards.

Throughout each of these trials, the gain was set at 0.0. This was decided because adding in the factor of gain, which amplifies the signal artificially, added an extra variable into the

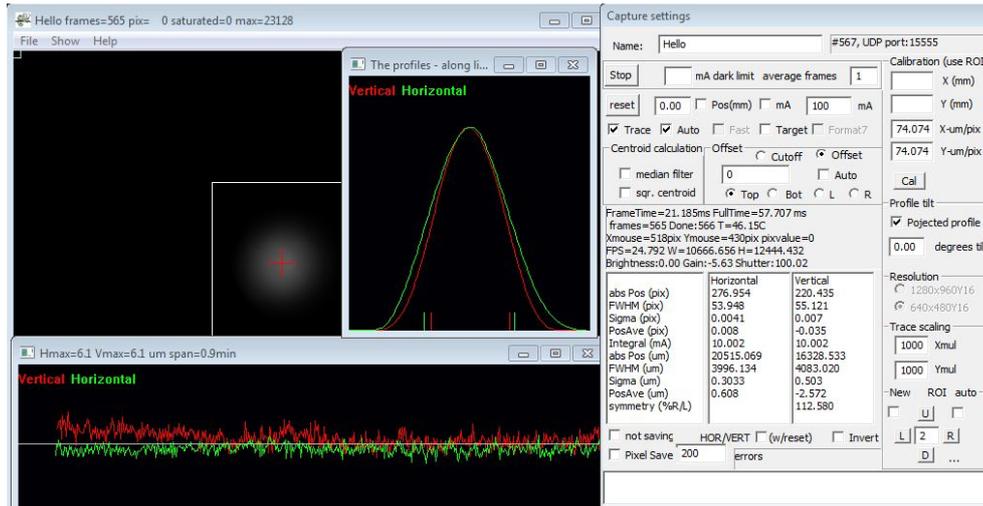


FIG. 3. A screenshot of USBChameleon.

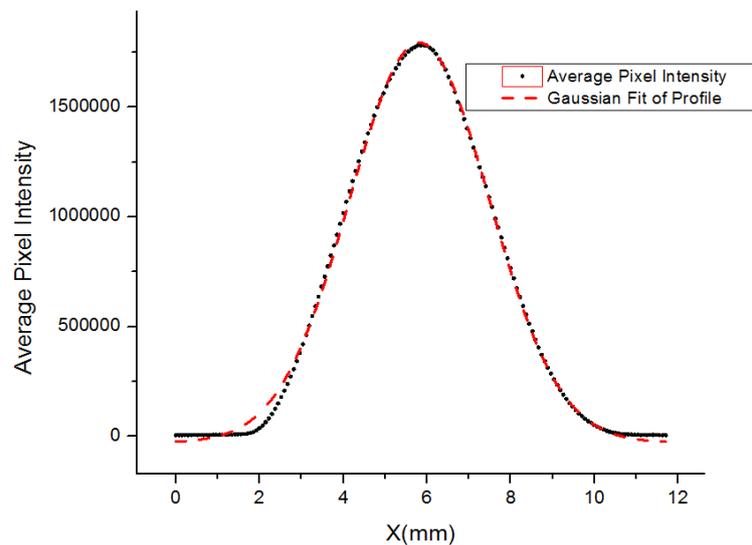


FIG. 4. A comparison of the Intensity profile of a Blurred LED to the fitted Gaussian Distribution.

experiment. Instead, the maximum intensity of the trials varied due to the different shutter values.

The value for maximum intensity for each trial was recorded as well. Because it varies with shutter value, it was necessary to keep track of this for later simulation.

Sigma-x ( $\sigma_x$ ) was determined by calculating the standard deviation of the residual (with a second-order polynomial) of the position of the centroid over time. Even while using constant experimental parameters, a slow drift of the centroid position occurred throughout a number of the trials. The effect of this slow drift on the statistical analysis can be eliminated by using a polynomial fit and using the residual to determine the standard deviation. The source of this slow drift is still unknown and has been hypothesized as being due to the CCD chip warming up and reaching thermal equilibrium.

The FWHM of the (hypothetically) Gaussian profile of the blurred LED was calculated

by averaging the FWHM measured by USBChameleon over the 200 trials. This, as well as the value converted from microns to pixels, was recorded in addition to the maximum intensity for the simulation to follow.

Data was taken for 60 combinations of Frame Average and Shutter Value. Frame Averages measured were 1, 2, 3, 4, 5, 6, 7, 8, 9, and 10. Shutter Values measured were 25ms, 40ms, 55ms, 70ms, 85ms, and 100ms.

### **B. Experiment 2 — Experimental Measurement of Pixel Intensity Noise: Static Light Source**

This experiment’s aim was to determine the statistics of the pixel intensity of a stable light source. To do this, an additional process was written and added to USBChameleon called “PixelSave.” With this option, the pixel intensities along a line can be saved each time the centroid (and other values) are calculated. The process is repeated a number of times (200) so that statistical error on each pixel’s average intensity can be experimentally obtained. The PixelSave function records this data each time it reaches the number input for Frame Average, so the frame average and shutter time both influence this standard deviation.

As before, the trials were performed for 60 combinations: Frame Averages of 1, 2, 3, 4, 5, 6, 7, 8, 9, and 10; and Shutter Values of 25ms, 40ms, 55ms, 70ms, 85ms, and 100ms.

Also as in Experiment 1, the Gain was kept at 0.0 throughout the trials. This was done for consistency, but as the shutter value was changed, the brightness of the LED had to be adjusted using the power source attached to it.

For each trial, the data was sorted into a matrix, and for each column (pixel), the average of the 200 rows (trials) was calculated for an average pixel intensity. The standard deviation of these 200 values was calculated as well, and a table was organized as Sigma v. Average Pixel Intensity. These values were then sorted in increasing fashion and saved in text files to be used for simulation.

The simulation code was later changed so that only 6 of these tables are now required — one at each shutter value, with a frame average value of 1. The difference between these two methods of simulation, as discussed later, was marginal.

### **C. Experiment 3 — Aliasing of a Dynamic Light Source**

Experiment 3 focuses on the effects of a moving LED light source, in contrast with the static analysis of experiments 1 and 2. The major effect that is hoped to be observed is aliasing, which comes from two major sources. The first arises in the image digitalization. Because the camera has 12 bits of accuracy, it must “round” to a pixel intensity value. The second is a result of the pixel having a finite size. A smoothly-distributed image stretched over this finite size results in a “jagged” image profile, as each pixel may only have one intensity value. Moving this rounded, jagged image across on the motion-controlled slide, the effect of the aliasing is observable.

The experiment consisted of using USBChameleon to manipulate the slide by controlling an actuator and moving it in micron-sized steps. The trials were taken in both high and low resolution, using normal and squared centroid calculations, and moving in steps of  $5\mu\text{m}$ ,

10 $\mu\text{m}$ , and 15 $\mu\text{m}$ . The centroid is calculated and recorded at each step, and then the slide is moved again.

A delay was placed between moving the slide and recording the centroid of between 4 and 6 seconds. This was to ensure that the slide had enough time to stabilize before measuring data.

Each trial consisted of the LED being moved several millimeters, typically 8mm. These step sizes were chosen due to their small size in relation to the pixel conversion size. In low camera resolution, that conversion was 74.38 $\mu\text{m}$ .

The data was then analyzed to see a trend, comparing the position of the centroid to a straight line fit (the actual motion of the slide), and a Fourier Transform was used to characterize the data.

## VII. EXPERIMENTAL RESULTS

### A. Experiment 1

From the 60 measurements made of sigma, a trend arose in analysis. As expected, with a higher frame average number and longer shutter time, sigma reduced significantly.

The dependance of sigma in low resolution on both frame average and shutter followed an inverse-square root relationship. The formula for sigma from these experimental values for Low Resolution is the following:

$$\sigma = \frac{6.897}{\sqrt{F \cdot S}}, \quad (3)$$

where  $F$  is the Frame Average Value and  $S$  is the shutter time in milliseconds.

In high resolution, the dependence had the same inverse-square root trend but with a slightly different coefficient. Though the coefficient is higher for the Low-Resolution, it seems from observation that the High-Resolution sigmas tend to be greater than the Low-Resolution ones.

$$\sigma = \frac{6.823}{\sqrt{F \cdot S}} \quad (4)$$

### B. Experiment 2

For experiment 2, the data tended to follow a square-root relationship between average pixel intensity and standard deviation, as expected from typical statistical distributions. The fit held relatively well for each trial, but it was ultimately decided that an interpolation method would be best for simulation, as discussed later.

$$\sigma_{pix-int} \propto \sqrt{I(x, y)} \quad (5)$$

It also was seen that  $\sigma_{pix-int}$  depended on frame average value and shutter value with an inverse-square root relationship, similarly to what was measured in experiment 1.

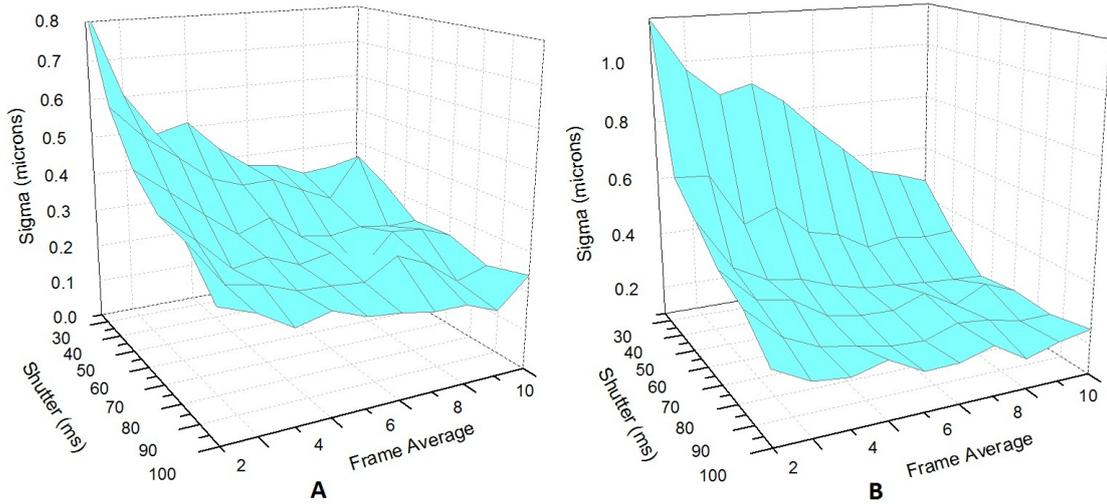


FIG. 5. A plot of Sigma v. Frame Average and Shutter Time, A. Low Camera Resolution, B. High Camera Resolution

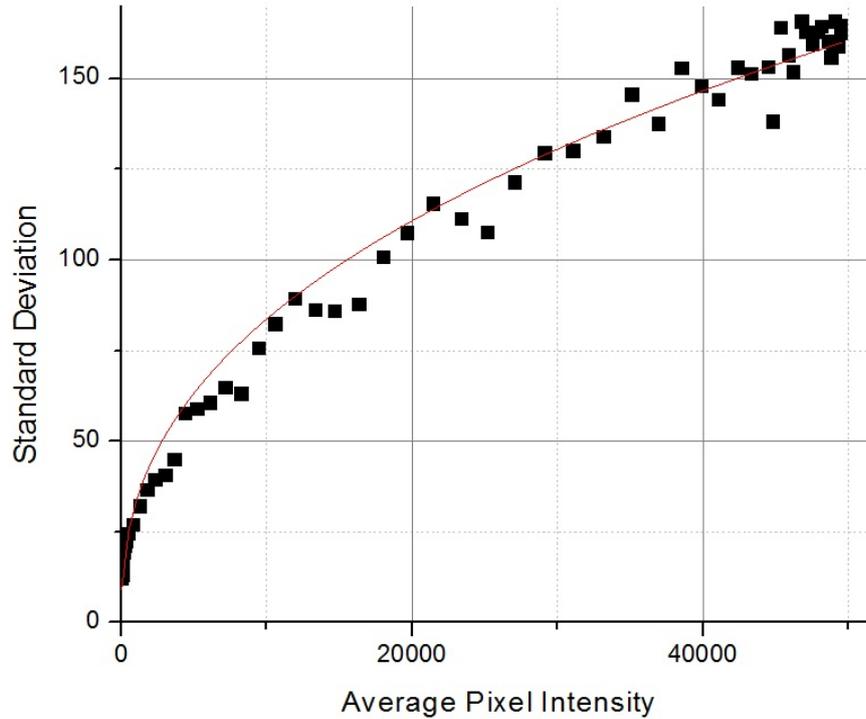


FIG. 6. Average Pixel Intensity v. Standard Deviation from Experiment 2

### C. Experiment 3

The aliasing provided interesting results. The typical trend when analyzing the FFT of an experiment measured in Low Camera Resolution was a large peak at 0.026, which is the reciprocal of  $38.4\mu\text{m}$ , roughly half of the conversion value of 74 microns per pixel. When data was taken in High Camera Resolution, peaks were measured at 0.026 again, as well as

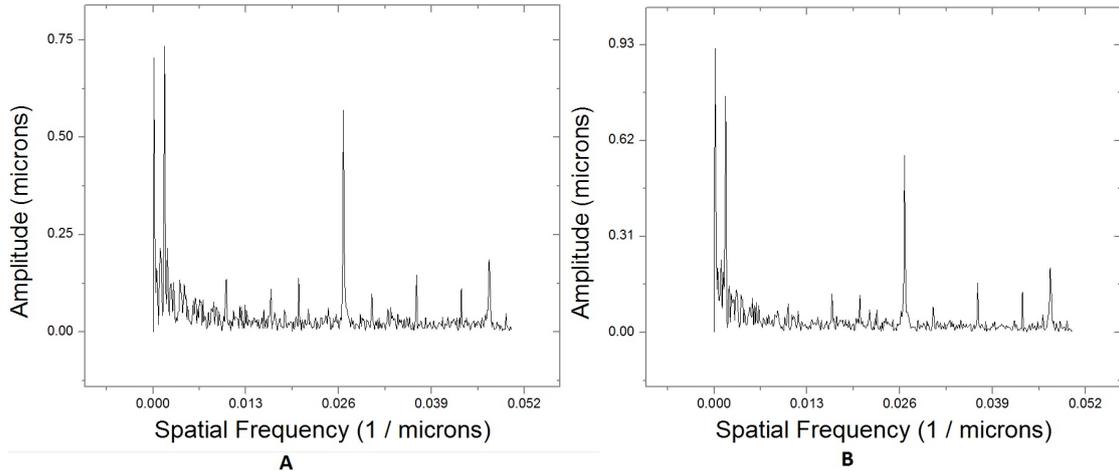


FIG. 7. FFT of measured Aliasing at 10 micron steps. A. Normal Centroid, B. Squared Centroid.

at other harmonics of the conversion factor in that case, 37 microns per pixel. Because the slide is moving at distances much smaller than that of a pixel, it requires one half-pixel's distance of shifting for the centroid to be measured at that new pixel location, so the position oscillates at that frequency.

In comparing squared centroid calculations against normal, there seems to be no difference, if any at all. See Fig. 7 for comparison.

The residual of the data (compared to the linear fit) had the characteristic frequency of 0.026 as noted above and its harmonics, but each of the trials also had an overarching oscillation in addition. Analyzing the residual graph as well as the FFT, the oscillations tended to have a wavelength of roughly 600 to 620 $\mu\text{m}$ , with no obvious explanation of its source. Looking into the details of the motor-controlled slide used for this experiment, it was discovered that the linear actuator used has a measured value for "Linear Motion Per Motor Rev" of 0.6096 m, therefore explaining this oscillation[6].

## VIII. METHODS TO REDUCE ALIASING EFFECTS

After performing several experiments to analyze aliasing, methods were hypothesized and implemented into the USBChameleon program in an attempt to reduce the oscillatory motion seen in Experiment 3. The overall goal of these was to better understand the characteristics of a dynamic light source and the digitalization, as well as to improve stability in general.

### A. Enlarge

The first idea was to convert the ROI used into a larger version, twice the length and width, so 4 times the number of pixels. The intensity values of the original ROI would be filled into this new array, with the points between being averaged in between the existing values. The hope for this was to create a smoother, larger profile to move across and potentially reduce the aliasing effects.

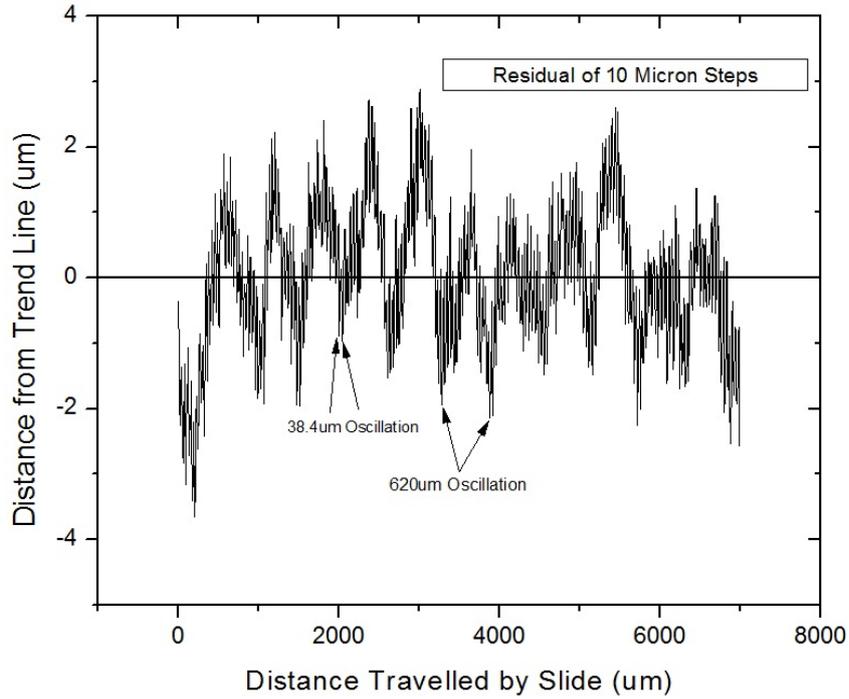


FIG. 8. Residual of measured Aliasing at 10 micron steps. The residual has characteristic oscillations with wavelength  $38.4\mu\text{m}$  and  $620\mu\text{m}$ .

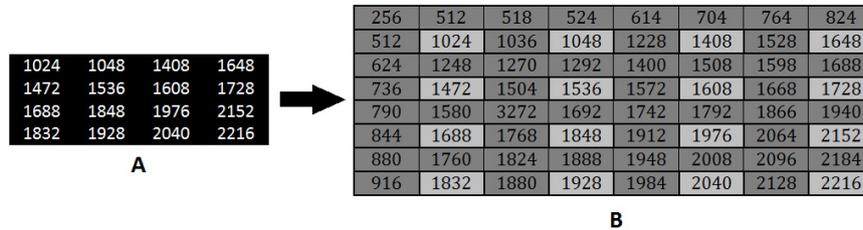


FIG. 9. An example of the enlarging. The original ROI is in A, and the enlarged is in B, with the original values in lighter grey.

When in low-camera resolution, the enlarging failed to eliminate the aliasing effects. However, it made the peaks well-defined, and the major peak (frequency at  $0.0267$  — corresponding to  $37.5\mu\text{m}$ ) at a height of  $0.56\mu\text{m}$  was much higher than each of the harmonic frequencies' peaks, each at around  $0.1\mu\text{m}$ . The high resolution trial resulted in less well-defined peaks with similar frequencies as the low resolution. Each peak ranged between  $0.5$  and  $1.0\mu\text{m}$ .

1024	1232	1352
1848	1408	1608
2096	2416	2808

→

1024	1232	1352
1848	1798	1608
2096	2416	2808

FIG. 10. An example of the  $3\times 3$  averaging process, left before averaging, right after.

### B. Average

A second method was introduced to attempt to smooth out the profile. It takes each pixel and the surrounding eight, and calculates the average of those eight, setting that average to be the pixel’s intensity value. This makes the profile much smoother in general, and reduces the jagged nature of it, because the average is not necessarily a multiple of eight or sixteen (the result of the camera’s ADC).

The averaging did not eliminate the aliasing effects, but after performing an FFT on the data’s residual, the peaks were noticeably well-defined. In low camera resolution, the peak at 0.0266 (coresponding to  $37.5\mu\text{m}$ ) had an amplitude of  $0.564\mu\text{m}$ . In high camera resolution, the peaks were less distinct but had similar amplitudes, ranging between 0.5 and  $0.6\mu\text{m}$ .

### C. “Jigsaw”

The third method to analyze aliasing was not developed necessarily to reduce the aliasing effects, but to analyze them in a different context. Instead of the motor-controlled slide moving in one direction, it now has an option to switch directions every 100 steps. This now consists of the light source moving (typically) in  $10\mu\text{m}$  steps, so 1mm, and then reversing direction, resulting in a back-and-forth motion.

Analysis of the jigsaw motion was performed using an FFT as before. For low camera resolution, the peaks and their harmonics were readily present. In high camera resolution, the peaks were more widely distributed, but still located about the same spatial frequency as in the typical aliasing experiment. In both cases, the oscillations tended to have an amplitude of roughly  $0.5\text{-}0.6\mu\text{m}$ .

### D. 45-Degree Tilt

Another idea proposed was a 45-degree rotation so that the light source would be moving in both the  $x$  and  $y$  directions throughout trials. Originally, the camera itself was rotated 45 degrees, but it was discovered that this method was not entirely stable, and that it would be more efficient to have the motor controlled slide rotated. The same method was used as in the typical aliasing experiment, where the slide would move in 10 micron increments over the course of 8 millimeters. Because of the tilt, however, it would be moving 10 microns total each step, which accounted for  $7.071\mu\text{m}$  in each direction. This also effected the pixel-to-micron conversion by the same factor of  $\sqrt{2}$ .

Measurements were taken at both high and low resolution with the slide moving forward and backwards for a total of four trials. The results were analyzed, as before, using an FFT



FIG. 11. The motor-controlled slide rotated at a 45-degree angle.

of the residual of the data. There was no difference in the analysis between the graphs in the  $x$  and  $y$  directions, as predicted, and there was also no difference in when the slide was moving forwards or backwards.

When in low resolution, the peaks of the FFT had an amplitude of roughly  $0.3\mu\text{m}$ , which is significantly lower than the  $0.5\text{-}0.6\mu\text{m}$  from before. In high resolution, the peaks were around  $0.5\mu\text{m}$ , which is not much lower than the previous peaks, but they were much more distinct than in previous trials.

The peaks occur at a frequency of  $0.0375$ , which relates to a wavelength of  $26.63\mu\text{m}$ , which is half of the diagonal distance across one of the pixels, which is expected, as the image is now moving diagonally, instead of just laterally across pixels.

This lower amplitude, and therefore reduction of aliasing, is likely due to the fact that when moving in a  $45^\circ$  line, the transition across each pixel occurs over a greater distance, and this transition is then “smoothed” out. The result of this is the lower amplitude.

## IX. SIMULATION

To perform a simulation of the centroid of a light source, Peter Revesz proposed the idea to create a 2D artificial intensity profile of the LED, specifically a Gaussian Distribution, and to add randomized noise to each pixel in the profile, and then to calculate the centroid of that profile with the randomized noise added.

Modelled similarly to USBChameleon, the Monte Carlo program creates this randomized profile and calculates its centroid a number of times, given a frame average value. It then averages these centroid positions together, using that as the calculated centroid, as USBChameleon would.

The simulation program repeats this process a number of times, typically 200. After calculating the centroid of the randomized profile each time, it records the positions in a text file which can be analyzed. From that, the standard deviation of this trace is calculated to be compared against the standard deviation of the trace of the experimental data from Experiment 1.

### A. Creating the Profile

The profile, as seen in USBChameleon and Fig. 4, strongly resembles a Gaussian, a two-dimensional Gaussian was used to model the ideal profile to be randomized. The equation for this is the following:

$$I(x, y) = M \cdot e^{-\left(\frac{(x-x_o)^2}{2\sigma_x^2} + \frac{(y-y_o)^2}{2\sigma_y^2}\right)}, \quad (6)$$

where  $I(x, y)$  is the pixel intensity at point  $(x, y)$ ,  $M$  is the maximum intensity value of the profile,  $(x_o, y_o)$  is the position of the maximum (center), and  $\sigma_x$  and  $\sigma_y$  are the values of sigma on the profile (calculated from the FWHM) in the  $x$  and  $y$  directions, respectively. The values for  $M$ ,  $\sigma_x$ , and  $\sigma_y$  are obtained from the experiment and input into the simulation.

### B. Randomizing Noise

The results from Experiment 2 have been indexed in a directory referenced by the Monte Carlo simulation.

Originally, the program read in a table from experiment 2, of average pixel intensity v. standard deviation, given frame average, shutter value, and camera resolution. Now, instead, it takes the shutter value and camera resolution and uses the table measured at a frame average of 1 for every situation.

Using this, the value for standard deviation at  $(x, y)$  is interpolated from the table between the two nearest points for Pixel Intensity.

To add noise to the ideal Gaussian Profile, the numbers for Pixel Intensity ( $I(x, y)$ ) and the interpolated standard deviation are entered into a normal distribution that is randomized, returning a value (positive or negative) to be added to the ideal value, resulting in a Randomized Profile.

The process utilized by the simulation is known as a Box-Muller Transformation. Code for this transformation was implemented and used to determine the normal distribution used to randomize the Ideal Gaussian [4].

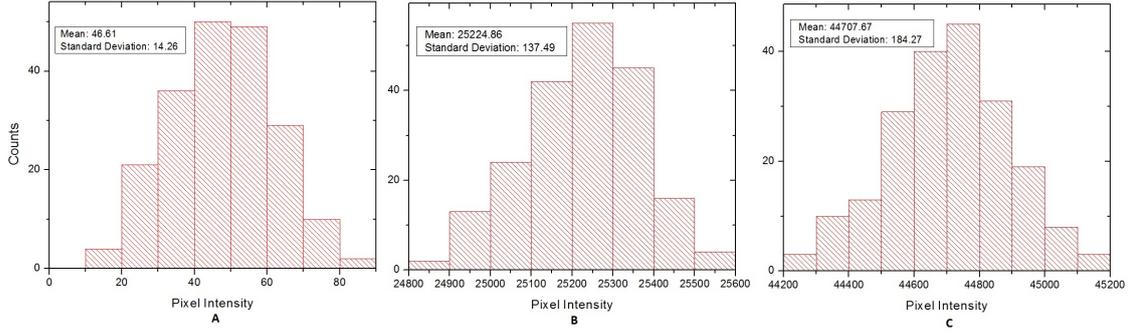


FIG. 12. Sigle-Pixel statistics on three pixels in a trial from Experiment 2.

In Fig. 12 it is evident that the distribution of a given pixel's intensity tends to be a Gaussian Distribution. Thus, the Box-Muller Transformation was used to determine a randomization method for these pixel intensities.

With using the tables for frame average 1 only, the randomized profile must be constructed  $n$  times, where  $n$  is the frame average value. The program then calculated the profile's centroid for each of the  $n$  profiles and then uses the average as the result for the trial. This method is similar to how USBChameleon acquires data and the results are similar to the original version but now has a more consistent method.

### C. Aliasing

One effect of the use of a digital camera, as mentioned above, is aliasing. Programmed into the simulation code is an option to move the source slightly throughout the simulation at a user-input step size and track both the centroid of the ideally calculated Gaussian and the Randomized profile. The expected outcome for the ideal profile is as follows:

$$IP = S \cdot T, \quad (7)$$

where  $IP$  is the Ideal Centroid Position,  $S$  is the step size, and  $T$  is the step number. However, the result is a slight oscillation about this line. The resulting equation appears as follows:

$$IP = S \cdot T + A \cdot (f(T)), \quad (8)$$

where  $A$  is an amplitude of oscillation and  $f(T)$  is an oscillating function with a period of  $\frac{m \cdot N}{2}$ , where  $m$  is an integer and  $N$  is the number of steps it takes for the image to shift one whole pixel.

## X. SIMULATION RESULTS

### A. Experiment 1

Simulating the same number of trials (200) with the same parameters as experiment 1 (maximum intensity, average FWHM, ROI size), the simulation had measured sigma at about 1/3 of that measured in the experiments. For both low resolution and high, the

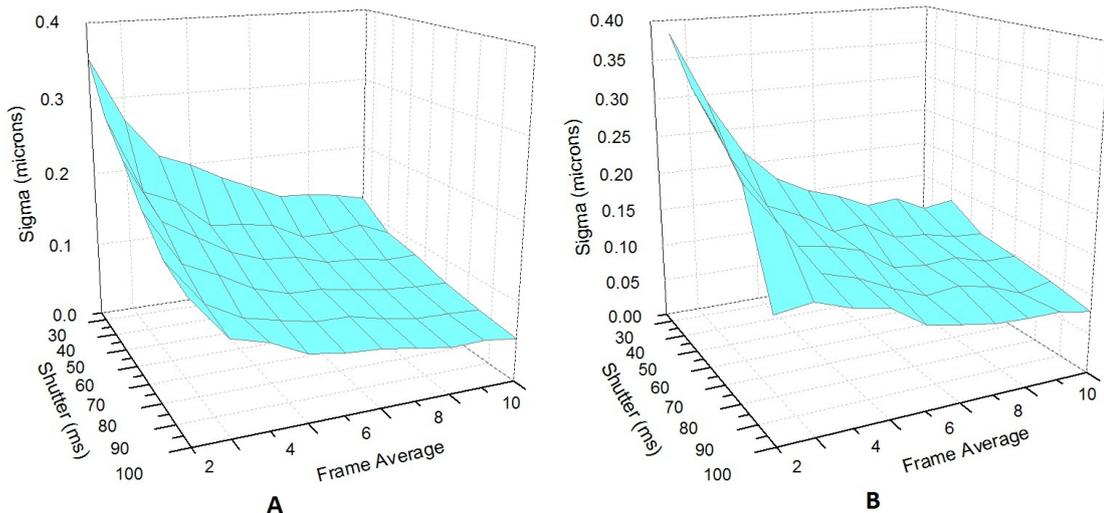


FIG. 13. The simulated plots of Sigma v. Frame Average and Shutter, A. Low Camera Resolution, B. High Camera Resolution

trend seemed to follow an inverse-square root relationship for frame average and shutter, the trends being as follows:

$$\sigma_L = \frac{1.91655}{\sqrt{F \cdot S}} \quad (9)$$

$$\sigma_H = \frac{4.32634}{\sqrt{F \cdot S}}, \quad (10)$$

where  $\sigma_L$  is the low-resolution sigma and  $\sigma_H$  is the high-resolution sigma. Comparing these results to those in Experiment 1, it is evident that the noise measured in Experiment 2 and utilized in the simulations results in the sigmas simulated here. This noise also results in the trend relating the frame average value and shutter time to sigma, that is, an inverse-square root relationship.

### B. Experiment 3

The simulation of a moving light source, as expected, still had a degree of an aliasing effect. It resembled the same frequencies as those in the experiment, without the characteristic oscillation of  $620\mu\text{m}$  due to the moving table. Also like the experiment, the difference between high and low resolution for these trials was marginal.

FFTs of the data resulted in several distinct frequency peaks that, when analyzed, are observed as harmonic frequencies of the same wavelengths observed in Experiment 3.

The ideally calculated gaussian exhibited these peaks strongly in the FFT, as seen in Fig. 15

The amplitude of the oscillations in the randomized profiles were typically between  $0.1$  and  $0.2\mu\text{m}$ , compared to the experimental values of  $0.5$  to  $0.6\mu\text{m}$ .

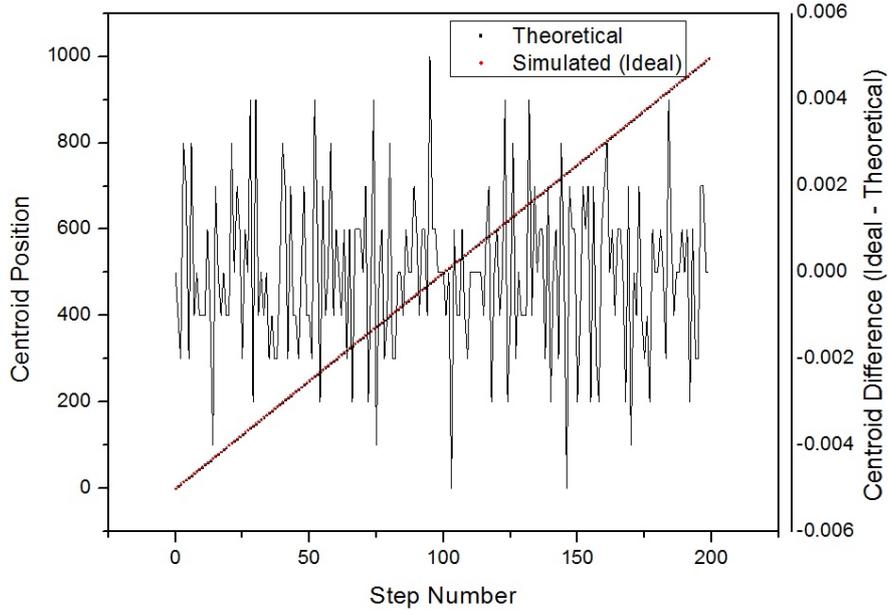


FIG. 14. The linear portion of the graph is of both the theoretical centroid position ( $IP = S \cdot T$ ) and the simulated ( $IP = S \cdot T + A \cdot (f(T))$ ). The oscillatory portion is of the difference between the two —  $A \cdot (f(T))$ .

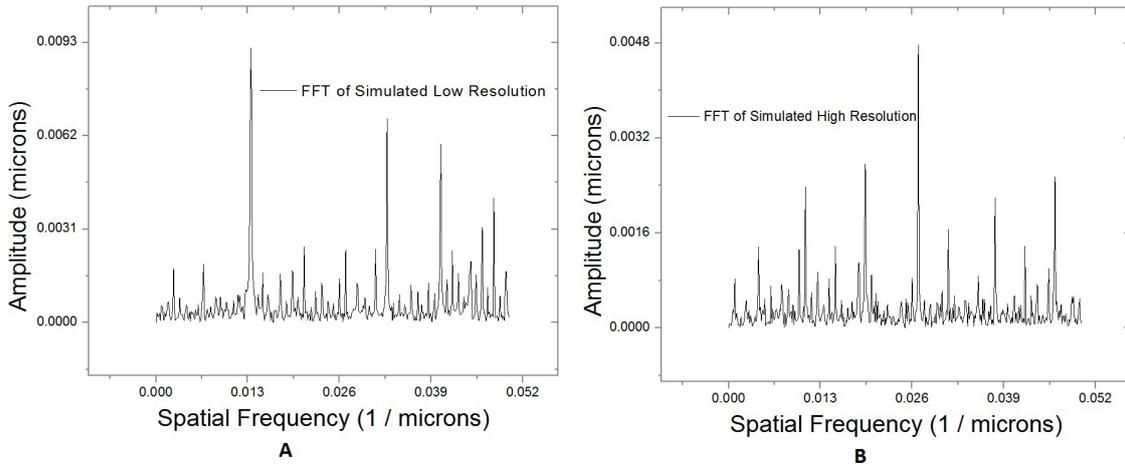


FIG. 15. FFTs of Dynamic LED Light Source, Simulated.  $10\mu\text{m}$  steps across  $8\text{mm}$ . A. Low Resolution, B. High Resolution

## XI. CONCLUSION

For the static LED light source, analyzing the stability of the image over time revealed that two parameters, frame average number and shutter time, had a significant effect on the stability. Increasing either of these values led to a drop in the standard deviation of the trace of the centroid ( $\sigma$ ). However, with an increase of either of these parameters comes an increase in measurement time, so there is an ideal value for both parameters to have a stable centroid of the image without sacrificing too much time.

Also regarding statics, the analysis of pixel noise statistics showed that the fluctuations in pixel intensity over time represented a normal distribution, and that the higher the average pixel intensity, the greater the standard deviation of this distribution is. Applying these measurements to the simulation, it was apparent that this pixel noise contributed to a significant factor of the measured instability of the centroid, and that the same trend was noticeable between frame averaging, shutter time, and sigma. This trend was an inverse-square root relationship.

In comparing the high camera resolution results with the low camera resolution, it is notable that the low-resolution measurements are better in that  $\sigma$  is typically lower in these, in the majority of the experimental measurements and simulations. This likely arises from the “extra bit” of precision that arises when the camera does its  $2 \times 2$  binning process in the ADC (as explained by the camera’s manufacturer).

As mentioned above, the two different effects of aliasing, resulting in a “jagged,” “blurred” image profile, were noticeable in moving the LED light source at distances much smaller than that of the conversion from microns to pixels by the camera. The simulation confirmed this effect, both in the ideally generated Gaussian distribution and the distribution with randomized noise added. All of these aliasing trials, be it experimental or simulated, displayed spatial frequencies that related to “wavelengths” at a fraction of the micron-to-pixel conversion factor. In the FFT results of the measured and simulated data, these harmonics were present in a variety of frequency peaks.

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- [1] P. Revesz, J. A. White. Nuclear Instruments and Methods in Physics Research A 540 (2005) 470-479.
  - [2] P. Revesz, A.B. Temnykh, A.K. Pauling. Nuclear Instruments and Methods in Physics Research 821 (2010).
  - [3] P. Revesz, A.B. Temnykh, A.K. Pauling. Nuclear Instruments and Methods in Physics Research A 649 (2011) 94-96.
  - [4] E.F. Carter Jr. Box-Muller Code. Available: <ftp://ftp.taygeta.com/pub/c/boxmuller.c>
  - [5] Hamamatsu Learning Center: CCD Noise Sources and Signal-to-Noise Ratio. Available: <http://learn.hamamatsu.com/articles/ccdsnr.html>

- [6] Zaber Products: Linear Actuator, NEMA Size 08, 16 mm travel, 4 lb Force. Available: [http://www.zaber.com/products/product\\_detail.php?detail=NA08A16](http://www.zaber.com/products/product_detail.php?detail=NA08A16)