Magnetism

Essentially quantum mechanical (classically: no magnetic moment for system in therm. equilibrium) – probably most complicated subject in solid state physics.

Main driving forces: spin/orbital momentum and Coulomb interaction + Pauli

① Macroscopic response

$$\vec{B} = \mu_0 \vec{H} + \mu_0 \vec{M}$$

$\vec{B}$: magnetic induction (inside material)

$\vec{H}$: magnetic field strength (external currents)

$\vec{M}$: magnetization

$$\vec{M} = \frac{\vec{\mu}_{tot}}{V}$$

$\vec{\mu}_{tot}$: total magnetic dipole moment

$\vec{M}$: magn. dipole moment per unit volume

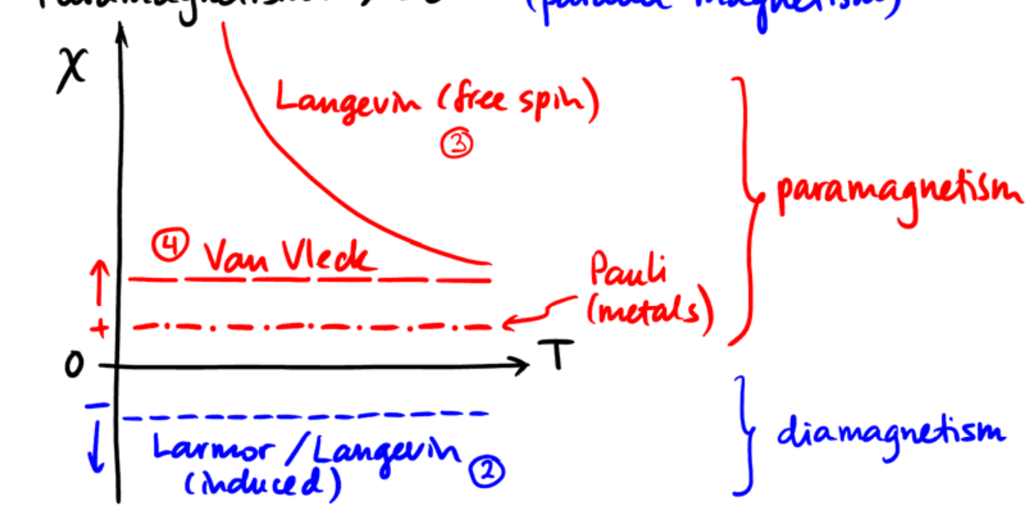
For small fields and certain materials

$$\vec{M} = \chi \vec{H}$$

χ: magnetic susceptibility

Diamagnetism : $\chi < 0$ (opposite magnetism)

Paramagnetism : $\chi > 0$ (parallel magnetism)



② Diamagnetism

* $\chi < 0$, $|\chi| \sim 10^{-5}$ for solids (max $\chi_{Bi} = -16.6 \times 10^{-5}$)

$\sim 10^{-8} - 10^{-7}$ for gases

* weak and common
* notable exception: perfect diamagnetism of superconductors $\chi = -1$ } different physics

* easily overpowered by para-, ferro- magnetism
  => only systems with no net dipole moment

* basic idea: precessing angular momentum in external magnetic field
  => induced orbital moments

Lenz's law: induction fights its cause
=> weakens the field

Recall: $\vec{\mu}_{atom} = I \cdot area$, classical picture

$$\mu_{atom} = \left(\frac{e}{T}\right)(\pi r^2) = \left(\frac{ev}{2\pi r}\right)\pi r^2 = \frac{e}{2m}\underbrace{mvr}_{L}$$

$\vec{\mu}_{atom} = \frac{e}{2m}\vec{L}$, also true QM-cally: $\hat{\vec{\mu}} = \frac{e}{2m}\hat{\vec{L}}$ (no spin!)

$$\vec{M} = \frac{N_i}{V}\frac{e}{2m}\langle \vec{L}_i \rangle$$ for i^{th} e^- of density $\frac{N_i}{V}$

Even number of e^-'s in molecules/atoms tend to be paired with $\pm L_z$
=> no net magnetic moment

$\pm L_z$

Or systems with closed shells: i.e. inert gases
$\langle \vec{L} \rangle = 0$

External B_z: change in angular freq.

$$\Delta\omega = \frac{eB_z}{2m}$$ Larmor frequency

=> induces change in L_z in $-z$ direction

$$\Delta L_z = -m r_{xy}^2 \Delta\omega = -\frac{eB_z}{2} r_{xy}^2,$$

$$\mu_z = -\frac{e}{2m}\Delta L_z = -\frac{e^2}{4m} B_z r_{xy}^2$$

$\langle r^2 \rangle = \langle x^2 \rangle + \langle y^2 \rangle + \langle z^2 \rangle$, for spherical sym. each
$\langle r_{xy}^2 \rangle = \langle x^2 \rangle + \langle y^2 \rangle = \frac{2}{3}\langle r^2 \rangle$ component is as likely

$$\Rightarrow \quad \vec{M} = -\frac{NZ}{V}\frac{e^2}{6m}\langle r^2\rangle \vec{B} \leftarrow \mu_0\vec{H}$$

$$\Rightarrow \quad \boxed{\chi = -\frac{N}{V}\frac{e^2 Z}{6m}\langle r^2\rangle \mu_0}$$ Langevin's diamagnetism

good agreement for inert gases & dielectric solids

③ Paramagnetism (Langevin)

* molecules/atoms have net magnetic moment
* tend to align themselves along magn. field
  minimizing $U = -\vec{\mu}\cdot\vec{B}$
* $\vec{M} \uparrow\uparrow \vec{H}$, $\chi > 0$ $(10^{-3}$ to $10^{-5})$

Recall : $$\hat{\mu} = -g\,\mu_B \frac{\hat{J}}{\hbar}$$

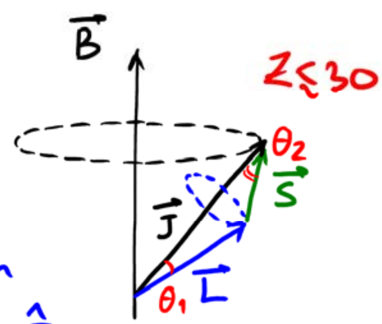
magn. moment ; g-factor ; Bohr magneton $\mu_B = \frac{e\hbar}{2m}$; total angular momentum in atom

if $\hat{J}$ is entirely due to orbital angular momentum $\hat{L}$, then $g_L = 1$ (classical)

if $\hat{J}$ is entirely due to spin $\hat{S}$, then $g_S = 2 + \frac{\alpha}{\pi} \simeq 2$

Dirac eq. ; QED correction

Generally: $1 \leq g_J \lesssim 2$, g_J is known as Landé's factor



Semiclassical picture of LS coupling in ext. magn. field

* $\vec{J}$ precesses about $\vec{B}$ at ω_{Larmor}
* $\vec{L}$ and $\vec{S}$ precess about $\vec{J}$ more rapidly due to spin-orbit interaction

$$\hat{\mu} = -\frac{\mu_B}{\hbar}\left(\underset{=1}{g_L}\,|\hat{L}|\cos\theta_1 + \underset{=2}{g_S}\,|\hat{S}|\cos\theta_2\right)\frac{\hat{J}}{|\hat{J}|}$$

$$\left.\begin{aligned}\hat{L}\cdot\hat{J} &= |\hat{L}||\hat{J}|\cos\theta_1 \\ \hat{S}\cdot\hat{J} &= |\hat{S}||\hat{J}|\cos\theta_2\end{aligned}\right\} \Rightarrow \hat{\mu} = -\frac{\mu_B}{\hbar}\left\langle \frac{\hat{L}\cdot\hat{J}}{|\hat{J}|^2} + 2\frac{\hat{S}\cdot\hat{J}}{|\hat{J}|^2}\right\rangle \hat{J}$$

Next, using $\hat{S} = \hat{J} - \hat{L}$, or $\hat{S}^2 = \hat{J}^2 + \hat{L}^2 - 2\hat{L}\cdot\hat{J}$

$$\Rightarrow \left\langle \frac{\hat{L}\cdot\hat{J}}{|\hat{J}|^2}\right\rangle = \frac{J(J+1)+L(L+1)-S(S+1)}{2J(J+1)}$$

similarly, $\left\langle \frac{\hat{S}\cdot\hat{J}}{|\hat{J}|^2}\right\rangle = \frac{J(J+1)+S(S+1)-L(L+1)}{2J(J+1)}$

$\Rightarrow \hat{\mu} = -\frac{\mu_B}{\hbar} g_J \hat{J}$, with Landé g-factor

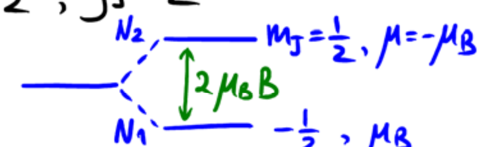
$$g_J = 1 + \frac{J(J+1)+S(S+1)-L(L+1)}{2J(J+1)}$$

In magnetic field B ($\uparrow\uparrow$ z dir.): $U = -\mu_z B$

$$U = -m_J g_J \mu_B B, \quad m_J = -J, \dots, J$$

For a free electron, $L=0$, $S=\frac{1}{2}$, $g_J = 2$ $\quad m_J = \pm\frac{1}{2}$,

$U = \pm\mu_B B$



anomalous Zeeman effect

$$M = \frac{N_1\mu_B - N_2\mu_B}{V}$$

$$N_2 \propto \overset{1}{\text{degeneracy}} \times e^{-\frac{2\mu_B B}{kT}} \quad \frac{2\mu_B B}{kT} \equiv 2x$$

$$N_1 + N_2 = N = \text{const}\left(1 + e^{-2x}\right); \quad \text{const} = \frac{N}{1+e^{-2x}}$$

$$M = \frac{\mu_B}{V}\frac{N}{1+e^{-2x}}\left(1 - e^{-2x}\right) = \frac{N\mu_B}{V}\frac{e^x - e^{-x}}{e^x + e^{-x}}$$

$$M = \frac{N\mu_B}{V}\tanh\left(\frac{\mu_B B}{kT}\right); \quad \text{for small } B: \tanh x \approx x$$

$$\Rightarrow M \simeq \frac{N\mu_B^2}{VkT}B \quad \text{or} \quad \boxed{\chi = \frac{N}{V}\frac{\mu_B^2}{kT}\mu_0}$$ Langevin paramagnetism

$\chi = \frac{C}{T}$ – Curie-Brillouin law

Curie const.

④ Temperature independent paramagnetism

Van Vleck paramagnetism (Nobel prize, 1977)

Consider 2-state system

—— $|1\rangle$ excited state

—— $|0\rangle$ ground state

e.g. $m_J = 0$

No magnetic moment in unperturbed states ($B=0$) :

$\langle 0|\hat{\mu}_z|0\rangle = 0$

$\langle 1|\hat{\mu}_z|1\rangle = 0$

But, off-diagonal $\underbrace{\langle 1|\hat{\mu}_z|0\rangle = \langle 0|\hat{\mu}_z|1\rangle^*}_{\text{Hermitian matrix } \hat{\mu}_z} \neq 0$

In magnetic field, the system is perturbed

$$|0'\rangle = |0\rangle + \frac{B\langle 1|\hat{\mu}_z|0\rangle}{E_1 - E_0}|1\rangle$$

$$|1'\rangle = |1\rangle - \frac{B\langle 0|\hat{\mu}_z|1\rangle}{E_1 - E_0}|0\rangle$$

2nd order perturbation theory

Now, perturbed states have magn. moments :

$$\langle 0'|\hat{\mu}_z|0'\rangle = \frac{2B|\langle 1|\hat{\mu}_z|0\rangle|^2}{E_1 - E_0}$$

$$\langle 1'|\hat{\mu}_z|1'\rangle = -\frac{2B|\langle 1|\hat{\mu}_z|0\rangle|^2}{E_1 - E_0}$$

Often, $E_s - E_0 \gg kT$, i.e. most of the population in $|0'\rangle$ state, then

$$\boxed{\chi = 2\frac{N}{V}\frac{|\langle 1|\hat{\mu}_z|0\rangle|^2}{E_1 - E_0}\mu_0}$$

Van Vleck paramagnetism

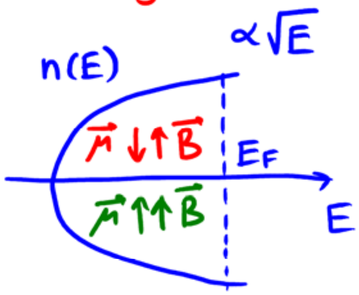
T- independent

Pauli paramagnetism

In non-magnetic metals (= most of metals), Langevin's paramagnetism (free spin) does not apply.

$$\chi \neq \frac{N}{V}\frac{\mu_B^2}{kT}\mu_0$$

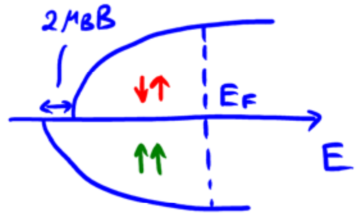
Q: why?
A: Fermi degenerate gas!
very small fraction of e^-'s gets magnetized.



$T \approx 0$

$$E_{\downarrow\uparrow} = \frac{\hbar^2k^2}{2m} - \mu_B B$$

$$E_{\uparrow\uparrow} = \frac{\hbar^2k^2}{2m} + \mu_B B$$



metal in weak $B \sim 0$ metal in $B \neq 0$

$$M = \mu_B \frac{N_{eff}}{V} \; ; \quad N_{eff} = 2\mu_B B \cdot g(E_F)$$

from lecture 13: $g(E_F)E_F = \frac{3}{2}N$

$$M = \frac{3N\mu_B^2}{2VE_F}B$$

(Landau: diamagnetic effect $= -\frac{1}{3}$ of paramagnetic in metal)

$$\Rightarrow M = \frac{N}{V}\frac{\mu_B^2}{E_F}B \text{ , or } \boxed{\chi = \frac{N}{V}\frac{\mu_B^2}{kT_F}\mu_0}$$ Pauli paramagnetism

"same" as Langevin if $kT \to kT_F$: very weak

⑤ Ferromagnetism (permanent magnet)

* need paramagnetic molecules/atoms ⇒ cooperative behaviour leads to macroscopic magnetic moment
* spontaneous alignment of magnetic moments ⇒ "domains", even when applied field $B = 0$
* only a handful of materials, most noteably Fe, Co, Ni

physics behind ferromagnetism : exchange interaction
Spin & Coulomb & Pauli principle

E.g. exchange field : $\vec{H}_E = \lambda\vec{M}$

E.g. exchange field: $\vec{H}_E = \lambda \vec{M}$

$$M = \chi_p (H_E + H)$$

external field

paramagnetic susceptibility

$$\chi_p = \frac{C}{T}$$

$$\chi = \frac{M}{H} = \frac{C}{T - \lambda C}, \text{ or } \underbrace{\chi = \frac{C}{T - T_c}, \ T_c = \lambda C}_{\text{Curie-Weiss law}}$$

singularity in χ when

$T = T_c$ Curie-temperature

a) when $T > T_c$: paramagnetic

b) when $T < T_c$: $\chi \to \infty$ ($M \neq 0$ when $B = 0$)

* magnetic saturation: all spins aligned

Fe	$T_c = 1043\,K$	$B_s = 1.7\ T$
Co	$1288\,K$	$1.4\ T$
Ni	$627\,K$	$0.5\,T$

E.g. simple estimate: $B_s \sim \mu_0 \mu_B N/V$

$$\left.\begin{array}{l} \mu_0 = 4\pi \times 10^{-7} \frac{T \cdot m}{A}, \quad \frac{N}{V} \sim 10^{29} \frac{\text{atoms}}{m^3}. \\ \mu_B = 9.3 \times 10^{-24} A \cdot m^2, \end{array}\right\} B_s \sim 1T$$

* variants of long range spin order:

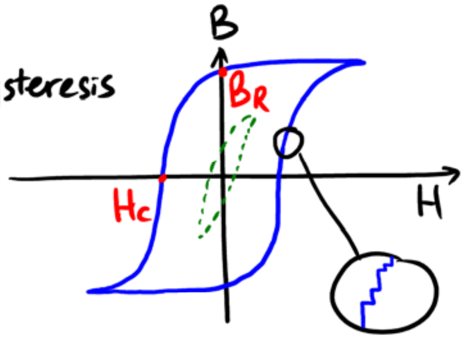
$\uparrow\uparrow\uparrow\uparrow\uparrow$	$\uparrow\downarrow\uparrow\downarrow\uparrow\downarrow$	$\uparrow\downarrow\uparrow\downarrow\uparrow\downarrow$
ferromagnet	antiferromagnet	ferrimagnet

Physics: Ising model $\hat{\mathcal{H}} \propto \sum_{ij} \hat{\vec{S}}_i \cdot \hat{\vec{S}}_j$

Domains : overall energy is minimized if spins are aligned short range (~0.1 to few mm) but allowed to anti-align at larger distances

Domains can get pinned on crystal defects, => hard (permanent) magn.

hysteresis



B_R - remanence
H_c - coercivity

Domains rearrange / change size during magnetization
"Barkhausen effect"