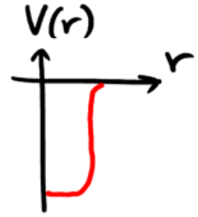
Nuclear models (contd)

Recap: a) Fermi model b) Liquid drop model

c) Shell model

similar to atomic model



$$\left[\frac{\hat{p}^2}{2m_n} + V(\vec{r})\right]\psi(\vec{r}) = E\psi(\vec{r})$$

spherical symmetry $\psi(\vec{r}) = \frac{u(r)}{r} Y_{\ell m}(\theta,\varphi)$

$$\left[\frac{d^2}{dr^2} - \left(\frac{\ell(\ell+1)}{r^2} - \frac{2m_n V(r)}{\hbar^2}\right)\right] u(r) = -\frac{2m_n E}{\hbar^2} u(r)$$

$\left.\begin{array}{l} n_r = 1,2,3,\dots \\ \ell = 0,1,\dots \end{array}\right\}$ eigenvalues $E_{n_r\ell}$

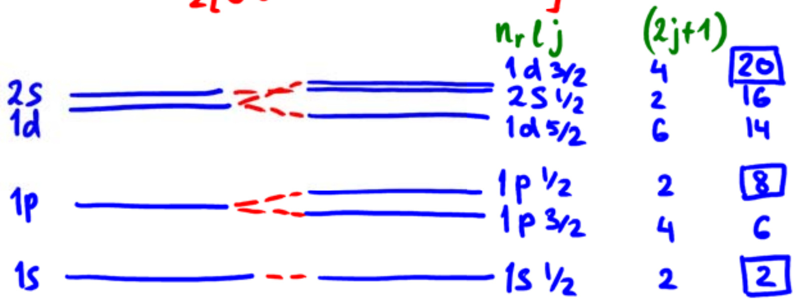
n_r is # of nodes of $u(r)$; different from n in Coulomb potential, e.g. no same restriction for ℓ at a given n_r

Mayer and Jensen (Nobel 1963)

add $V_{so} = -K\frac{\hat{\ell}\cdot\hat{s}}{r^3}$ spin-orbit nuclear interaction to explain magic numbers

$\Rightarrow |n_r j \ell s m_j\rangle$, $E_{n_r \ell j}$

recall $\langle\hat{\ell}\cdot\hat{s}\rangle = \frac{1}{2}[j(j+1) - \ell(\ell+1) - s(s+1)]$



* explains magic numbers (excitation gaps)
* explains spin e.g. magic or double-magic nuclei ⇒ spin 0

  ^{17}O spin : 5/2 (from 1d 5/2 neutron)

  ^{15}N spin : 1/2 (proton hole in 1p 1/2)

d) collective model : potential $V(\vec{r},t)$ is allowed to be non-spherical, allow collective oscillations, ⇒ electric quad moment, etc.
agrees with measurements

Radioactivity

emission of $\left\{ \begin{array}{l} \alpha\text{-part. } ({}^4He^{2+}) \\ \beta\text{-part. } (e^\pm) \\ \gamma\text{-radiation} \end{array} \right\}$ from unstable nuclei

$$\frac{dN}{dt} = -\lambda N \qquad \lambda: \text{decay const}$$

$$\Rightarrow \quad N(t) = N_0 e^{-\lambda t}$$

Decay rate or activity: $R = \left|\frac{dN}{dt}\right| = \lambda N$

Half-life: $N(t_{1/2}) = \frac{1}{2}N_0$, $\quad t_{1/2} = \frac{\ln 2}{\lambda}$

Units: 1 Bq (becquerel) ≡ 1 decay/s (SI)
1 Ci (curie) ≡ 3.7×10^{10} decays/s (1 g of ${}^{226}Ra$)
radium

Ex. ${}^{204}_{82}Pb$: half-life > 10^{17} years (α-decay)
≫ age of the universe

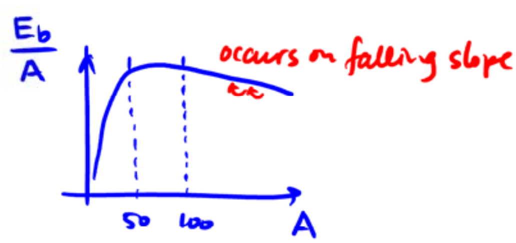
${}^{60}_{27}Co$: β-decay to ${}^{60}_{28}Ni$, then γ-decay @ 1.17 and 1.33 MeV
half-life 5.27 years (technologically important)

Half-life times vary between 0 and ∞
↳ proton?

① α-decay

$${}^{A}_{Z}X \rightarrow {}^{A-4}_{Z-2}Y + {}^{4}_{2}He$$

parent   daughter



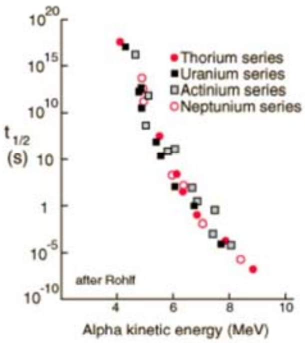
Energy released (Q-value): $Q = (m_X - m_Y - m_\alpha)c^2$

Q: where does the energy go?

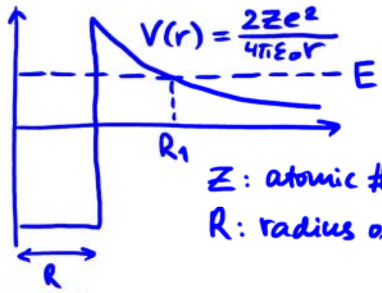
$$\vec{p}_\alpha + \vec{p}_Y = 0, \quad Q = KE_\alpha + KE_Y = \frac{p_\alpha^2}{2m_\alpha} + \frac{p_Y^2}{2m_Y} = \frac{p_\alpha^2}{2}\left(\frac{1}{m_\alpha} + \frac{1}{m_Y}\right)$$

into KE of $\alpha \rightarrow \approx p_\alpha^2/2m_\alpha$

* KE of α particles from various emitters ~ 4-9 MeV
* lifetimes vary wildly : from 10^{-7}s to $\gg 10^{10}$ yrs (age of the univ.)



* correlation b/w α's KE & half-life
* well understood in terms of tunneling



Z: atomic # of daughter nucl.
R: radius of daughter (~10 fm)

$$T \approx e^{-2\int_R^{R_1}\sqrt{\frac{2m_\alpha}{\hbar^2}(V-E)}\,dR'}$$

decay rate = T × collision freq

$\approx \frac{v}{2R}$ with $v = \frac{p}{m_\alpha}$, and $p \sim \frac{\hbar}{R}$

$\Rightarrow \frac{\hbar/m_\alpha}{2R^2} \sim 10^{21}\,s^{-1}$

Gamow, Gurney, Condon (1928) – gives good order of magn.

② β-decay

* emission of e^- (β^-)
* emission of e^+ (β^+) or atomic e^- capture (K shell)

if $^A_ZX \rightarrow {}^A_{Z\pm1}Y \pm e^{\mp}$, then problems with

* spin not being conserved
* recoil energy of heavy daughter ~0
  but KE of β is known to vary

$\Rightarrow$ must have another particle for conservation laws to work

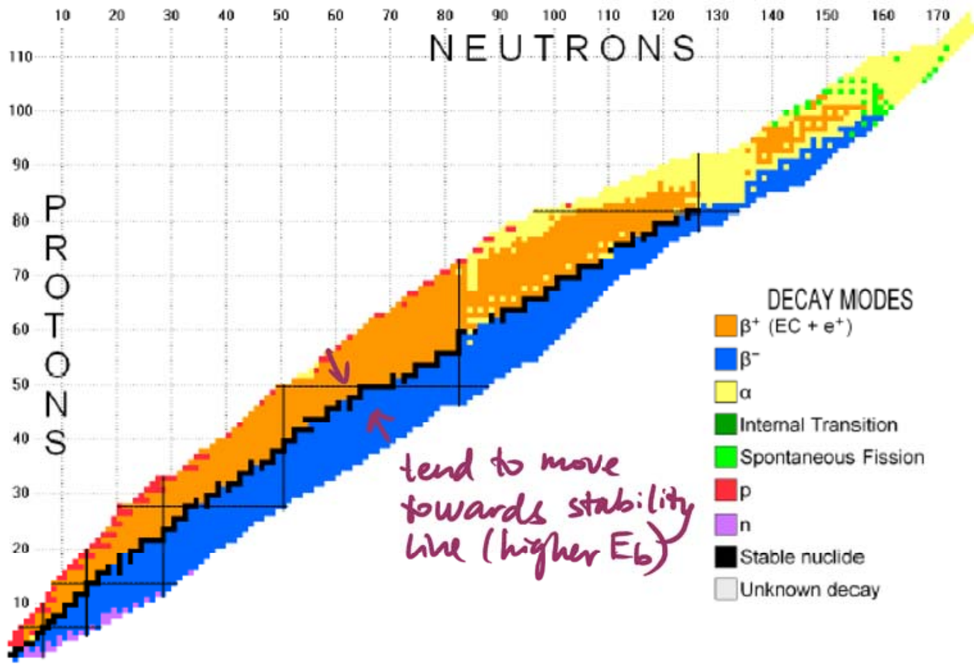
$\left.\begin{array}{l}\text{* spin } 1/2 \\ \text{* neutral} \\ \text{* weakly interacting} \\ \text{* little mass } (<0.2\,eV)\end{array}\right\}$ neutrino and antineutrino
small neutral one (it)

Pauli 1930, seen 1956

* emission of e^- : ${}^{A}_{Z}X \rightarrow {}^{A}_{Z+1}Y + e^- + \bar{\nu}_e$

* emission of e^+ : ${}^{A}_{Z}X \rightarrow {}^{A}_{Z-1}Y + e^+ + \nu_e$

${}^{A}_{Z}X + e^- \rightarrow {}^{A}_{Z-1}Y + \nu_e$ (e^- capture)



on a more fundamental level :

$n \rightarrow p + e^- + \bar{\nu}_e$ (~11 min for free n)

$p \rightarrow n + e^+ + \nu_e$ ($>6.6\times10^{33}$ yr for free p?)

$p + e^- \rightarrow n + \nu_e$

on more fundamental level :

six flavors of quarks :

up, down, strange, charmed, bottom, top

p : uud

n : ddu

e.g. $n \rightarrow p + e^- + \bar{\nu}_e$

$d \rightarrow u + e^- + \bar{\nu}_e$ quark flavor change (more later)

β-decay is due to "weak interaction"

③ γ-decay

if nucleus is left in excited state (e.g. after α/β decay)

$${}^{A}_{Z}X^{*} \to {}^{A}_{Z}X + \gamma$$

Ex. ${}^{12}_{5}B \to {}^{12}_{6}C^{*} + e^{-} + \overline{\nu_e}$

$${}^{12}_{6}C^{*} \to {}^{12}_{6}C + \gamma$$

photon energy ~ MeV

Radiation - matter interaction

important in dosimetry, instrument design, etc.

simple picture: linear energy loss with path of charged particles in matter

ionization
inelastic collision with atoms / e^-'s
bremsstrahlung
e^+/e^- pair creation

energy change of radiation particle

$\frac{dE}{dx}$ - varies for many materials, density, etc.

Better quantity $\left(\frac{1}{\rho}\frac{dE}{dx}\right)$ - nearly const for a wide range of parameters

"mass stopping power"

From NIST web-site

α-rad : $(7 \div 5) \times 10^2$ MeV $\frac{cm^2}{g}$ for $5 \div 10$ MeV α's

β^--rad : ~ 2 MeV $\frac{cm^2}{g}$ for $0.3 \div 30$ MeV e^-'s

γ-rad : different behaviour, exp-attenuation
"mass attenuation coefficient"
$(6 \div 2) \times 10^{-2}$ $\frac{cm^2}{g}$ for 1-10 MeV γ's

Ex. to shield 5 MeV radiation using Al ($\rho = 2.7\,\frac{g}{cm^3}$)

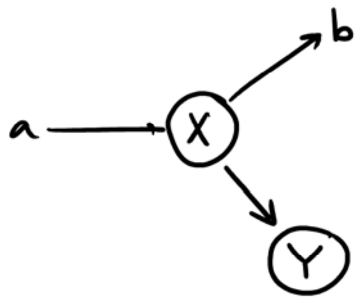
α-rad: $\dfrac{5\,\text{MeV}}{\text{mass stopping power}}\,\dfrac{1}{\rho} \sim 0.0026\,\text{cm}$

β-rad: $\dfrac{5\,\text{MeV}}{\text{mass stopping power}}\,\dfrac{1}{\rho} \sim 0.92\,\text{cm}$

stopped complet. (except for γ-creation)

γ-rad: $\dfrac{1}{\text{mass attenuation}}\,\dfrac{1}{\rho} \sim 10\,\text{cm}$ for $1/e$ attenuation

Nuclear reactions



$a + {}^{A}_{Z}X \rightarrow b + {}^{A'}_{Z'}Y$

a, b – typically particles

Another notation: ${}^{A}X(a,b)\,{}^{A'}Y$

Q-value : $Q = \underbrace{KE_b + KE_Y - KE_a}_{\text{kin. energy difference}}$ ($KE_X = 0$)

conservation of energy:

$$(m_a c^2 + KE_a) + m_X c^2 = (m_b c^2 + KE_b) + (m_Y c^2 + KE_Y)$$

$$\Rightarrow \quad Q = (m_a + m_X - m_b - m_Y)\,c^2$$

$Q > 0$ exothermic; $Q < 0$ endothermic

$KE_a \geqslant E_{th} = -Q\left(1 + \dfrac{m_a}{m_X}\right)$

if non-rel. particles for reaction to occur

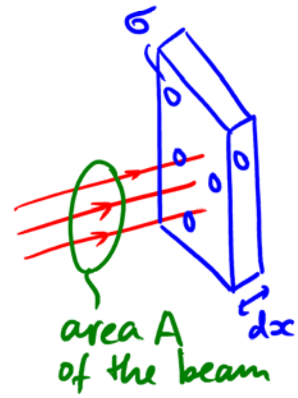
Ex. $^{2}\mathrm{H} \searrow d(n,\gamma)t \swarrow {}^{3}\mathrm{H}$ $\quad Q = 6.26$ MeV

$^{6}\mathrm{Li}(p,\alpha)t$ $\quad$ 4.02 MeV

$^{7}\mathrm{Li}(p,\alpha)^{4}\mathrm{He}$ $\quad$ 17.34 MeV

$^{13}\mathrm{C}(p,n)^{13}\mathrm{N}$ $\quad$ −3.0 MeV

Cross-section



$dN_{tgt} = nA\,dx$ – number of nuclei in target exposed to the beam

n: target density

$dA_{tgt} = \sigma\, dN_{tgt} = \sigma n A\, dx$

→ total effective area seen by each projectile

$dp = \dfrac{dA_{tgt}}{A} = \sigma n\, dx$ – probability to interact with one proj.

$dN_{int} = \dot{N}\, dp$, $\Rightarrow$ $d\dot{N}(x) = -dN_{int} = -\dot{N}(x)\, dp$

$\dot{N}$: beam flux; $d\dot{N}$: projectiles taken out of the beam

$\dot{N}(x) = \dot{N}(0)\, e^{-\frac{x}{\ell}}$, $\ell = \dfrac{1}{\sigma n}$ "interaction length"

units for σ: m^2 or 1 barn $= 10^{-28} m^2$

σ: can be ≪ or ≫ than physical size of atom/nuclei, depends on process, energy, etc.

Examples:

a) pp scattering : $p + p \to p + p + X$ ← anything

"strong"

say $KE_p = 50\,GeV$, $\sigma \sim 0.03$ barn

close to "geometric" cross-section, e.g. $r_p \sim 0.8\,fm$

$\Rightarrow \pi r_p^2 \sim 0.02$ barn

characteristic of strong interaction

b) electromagnetic photoelectric effect

$\gamma + Pb \to e^- + Pb^*$ ← here excited atom, not nucleus

For $E_\gamma \sim 100\,keV$ (K-edge of lead),

$\sigma \sim 10^{-25} m^2 \sim 1000$ barn

K-shell size $\sim \dfrac{a_0}{Z} = \dfrac{0.53\,\text{Å}}{82} \sim 650\,fm$

$\pi R_K^2 \sim 13{,}000$ barn (σ is smaller by ×10)

c) <u>weak interaction</u> inverse β-decay

$$\bar{\nu}_e + p \rightarrow e^+ + n$$ (used to detect antineutrinos)

cross-section $\sigma \sim 10^{-19}$ barn for $E_{\bar{\nu}_e}$ of few MeV
(10^{18} times smaller than physical area of p)

E.g. estimate interaction length:

11 g/cm³ for Pb

$$\ell = \frac{1}{n\sigma}, \quad n = Z \times \frac{6\times10^{23}}{A} \times \rho = 2.6\times10^{30}\ \text{cm}^{-3}$$

$\ell = 3.8\times10^{16}$ m!

1 light-year ~ 10^{16} m, => 4 light-years of Pb!!