Wavefront sensing
or why do stars twinkle?

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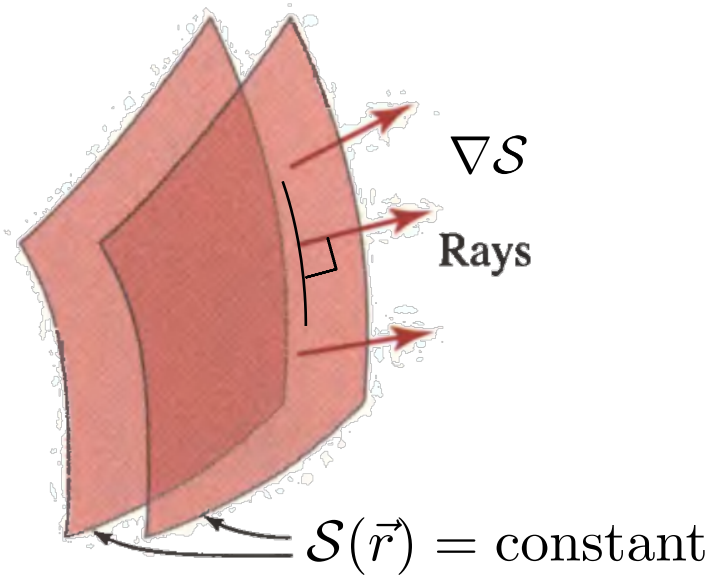
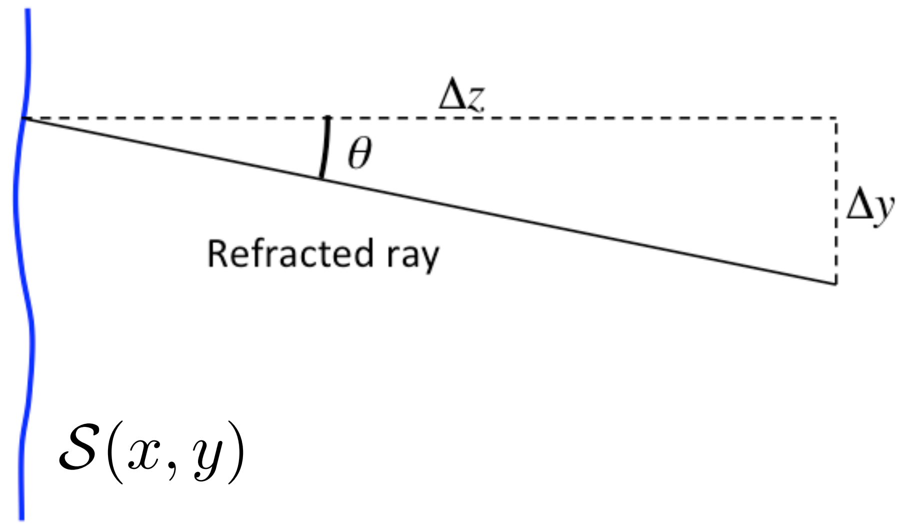
Outline

- **Connection between optical phase & ray direction**
- **Complex index of refraction**
- **Shadowgraphy**
- **Shlieren imaging**
- **Phase contrast imaging**

Phase front in geometrical optics

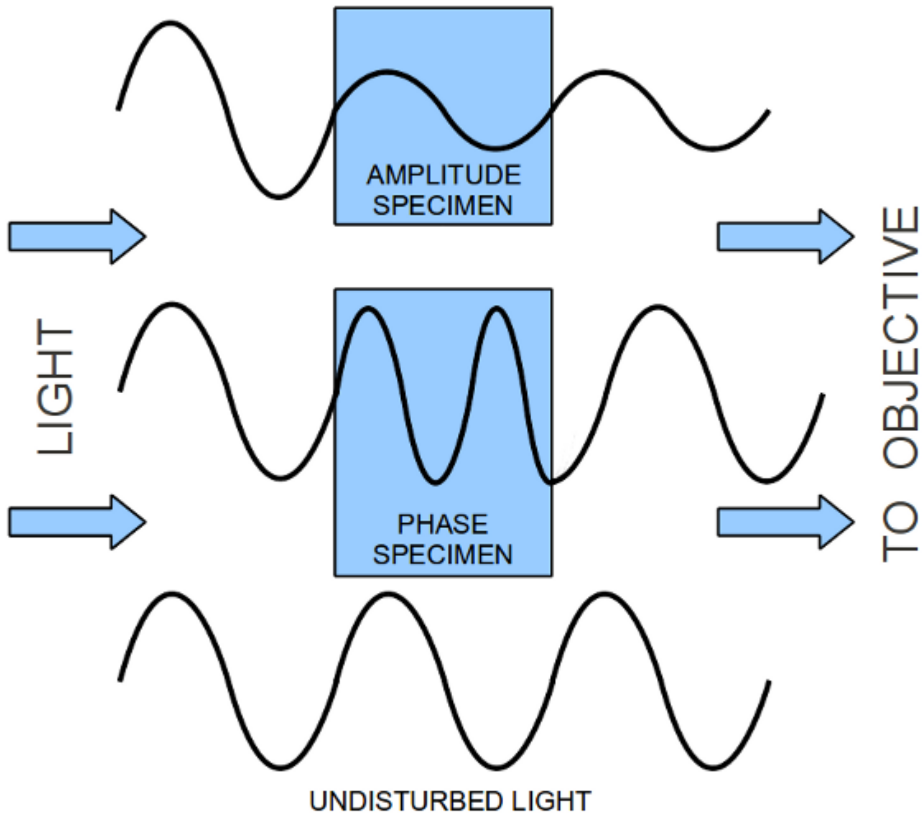
Constant phase wavefront – by the eikonal equation, the ***direction*** of the rays is given by its gradient





$$\theta_y \approx \frac{\Delta y}{\Delta z} \approx \frac{\partial \mathcal{S}}{\partial y}$$

Phase vs. amplitude changes



complex index of refraction

amplitude

$$\tilde{n} = n + i\kappa$$

phase modulation

Complex index of refraction

$$\mathbf{E}(z,t) = \mathbf{E}_0 e^{i(\tilde{k}z-\omega t)} = \underbrace{e^{-2\pi\kappa z/\lambda_0}}_{\text{amplitude}} \mathbf{E}_0 e^{\overbrace{i(kz-\omega t)}^{\text{phase}}}$$

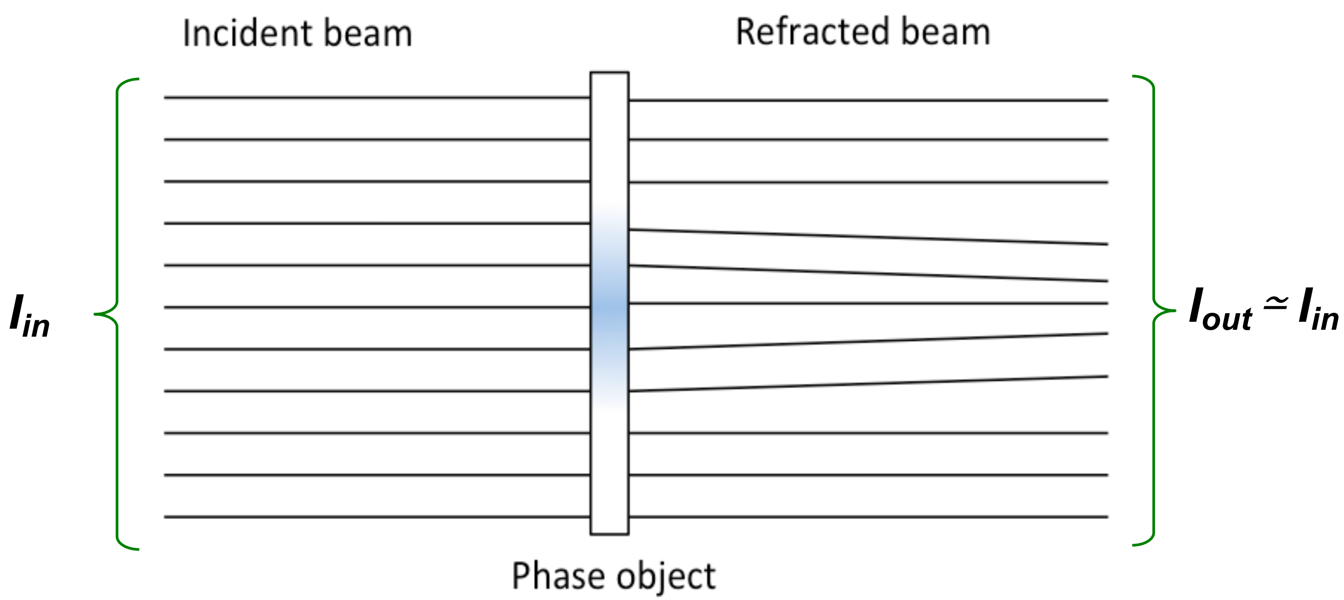
$$\tilde{k} = \frac{2\pi\tilde{n}}{\lambda_0}$$

Thus, the intensity change is given by: $\dfrac{4\pi\kappa}{\lambda_0}\Delta z$

And the phase change (in rad) given by: $\dfrac{2\pi(n-1)}{\lambda_0}\Delta z$

Fact of nature: for most transparent dielectrics (n – 1) $\gg$ κ

Phase object: $(n-1) \gg \kappa$



Intensity doesn't change, only directions of the rays do!

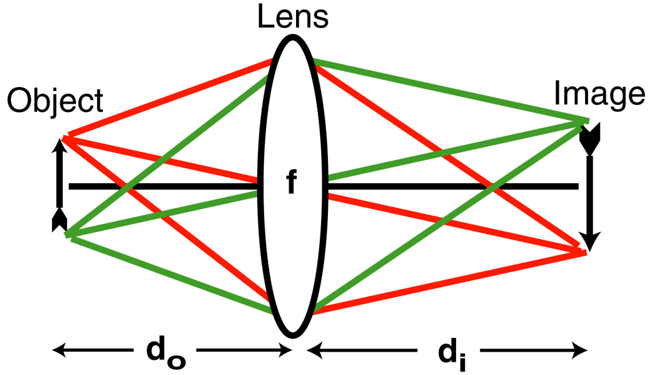
Temperature induced variations in gasses, liquids, thin biological objects, transparent films – all phase rather than amplitude objects

Problem with imaging of phase objects

When $B = 0$, all rays from a point x_{in} arrive at a point x_{out}, independent of angle.

$$\begin{bmatrix} x_{out} \\ \theta_{out} \end{bmatrix} = \begin{bmatrix} A & 0 \\ C & D \end{bmatrix} \begin{bmatrix} x_{in} \\ \theta_{in} \end{bmatrix} = \begin{bmatrix} A\,x_{in} \\ C\,x_{in} + D\,\theta_{in} \end{bmatrix}$$

$$\boxed{x_{out} = A\,x_{in}}$$

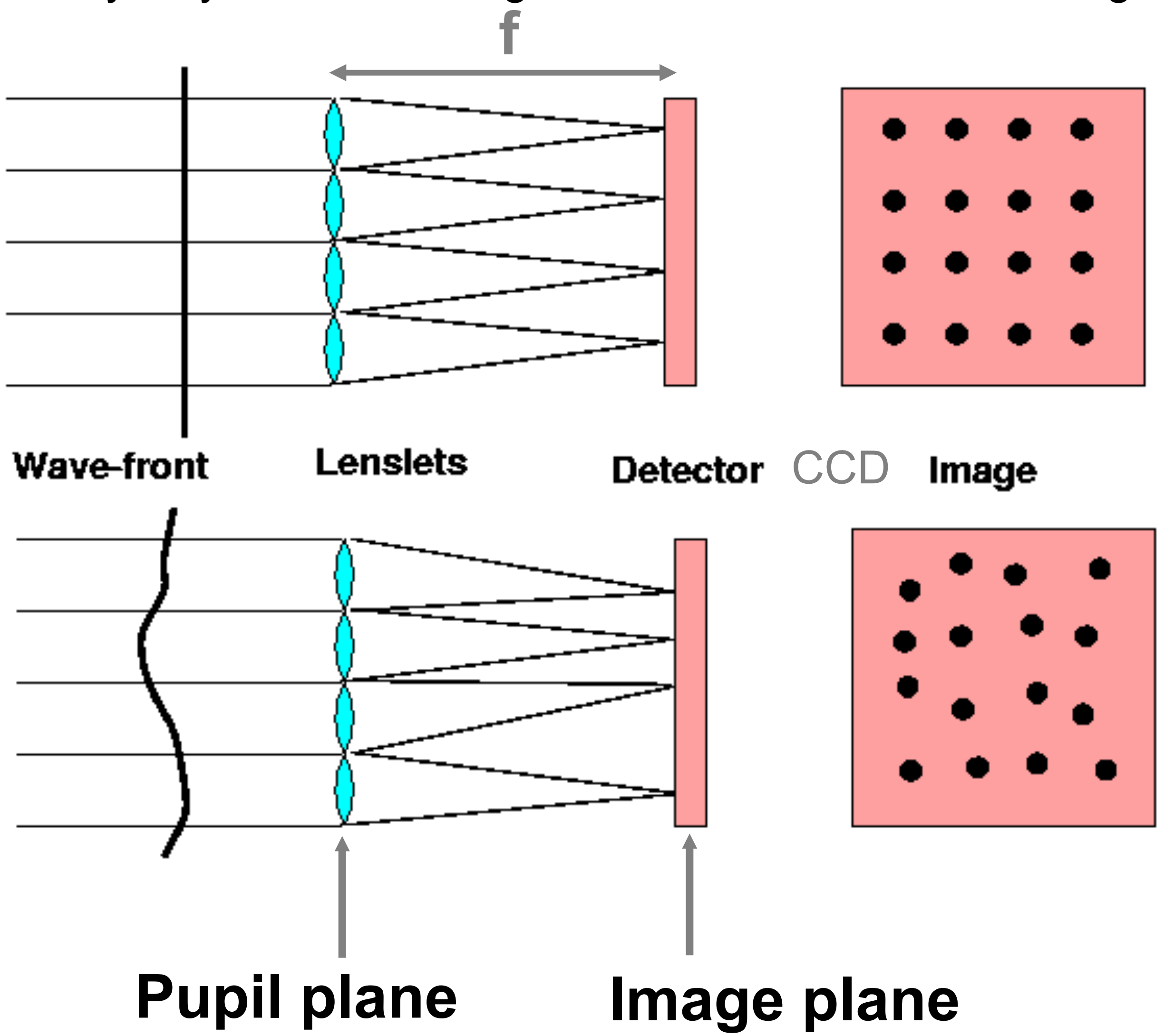


The *imaging condition* for ABCD matrix is B = 0, i.e. ray position in the image plane is *independent from* the angle (or phase) in the object plane.

Thus, phase objects are ***invisible*** when imaged directly!

Direct wavefront sensing: Hartmann-Shack

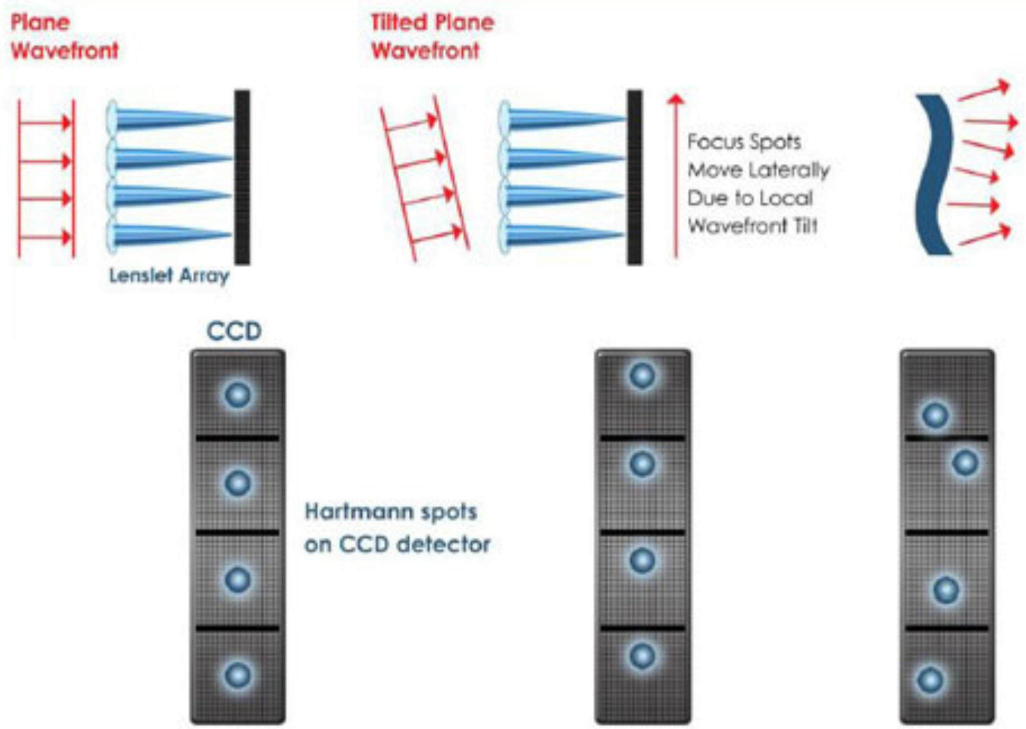
- Many ways exist to "image" or sense the wavefront, e.g. Shack-Hartmann

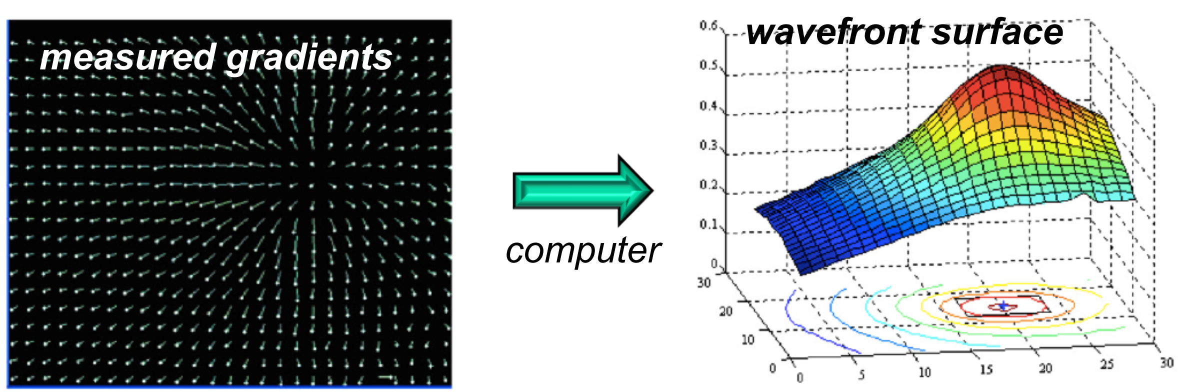


Microlens arrays in nature

Shack-Hartmann

- If a plane wave incident: all spots from the micro-lens array form a perfect grid.
- When the wavefront is distorted, the spots shift measuring directly the wavefront angle.
- Can reconstruct the wavefront surface from the directly measured gradients

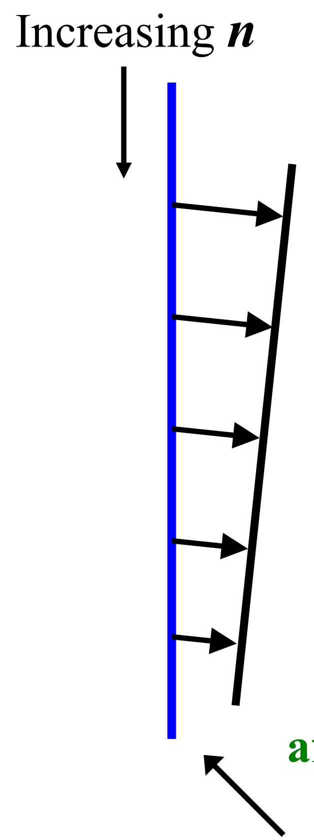




Angle change due to flat phase object is given by the derivative of the index of refraction

Refraction (angle change) will be due to the phase gradient (=***derivative***), with the phase itself related to the optical path: $n \times$ thickness

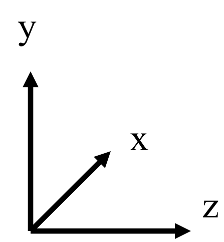
Rays bend towards larger indexes of refraction (e.g. due to Fermat's principle).



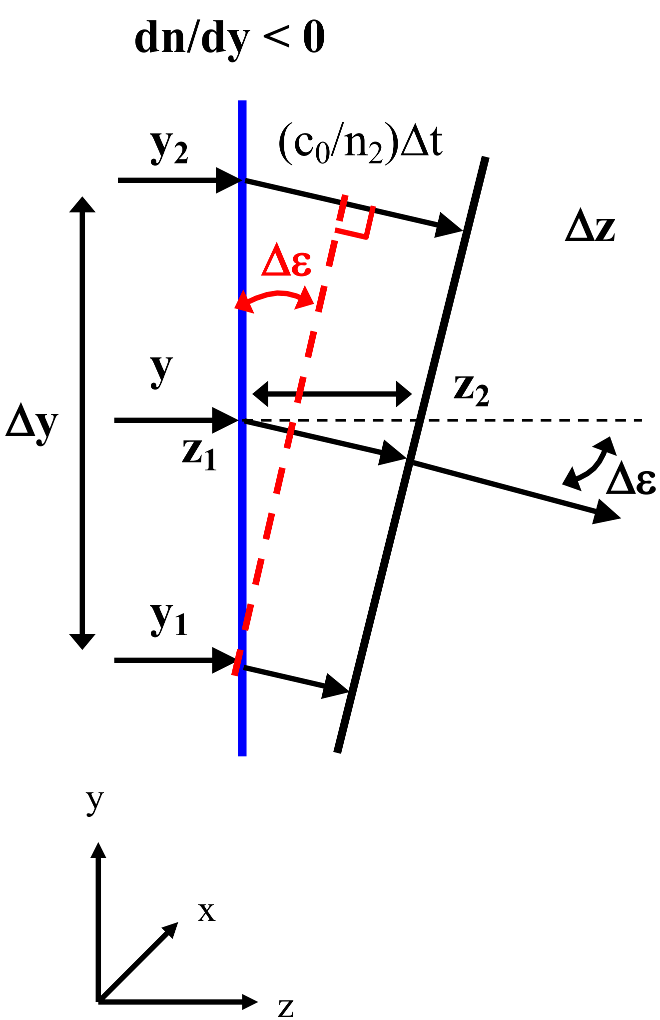
*It's derivative, not **n** itself that determines the refraction angle.*

after time = Δt

Negative vertical refractive-index gradient *dn/dy* < 0



Mathematical model



Distance wave front moves in time Δt:

$$\Delta z = c\Delta t = \frac{c_0}{n}\Delta t$$

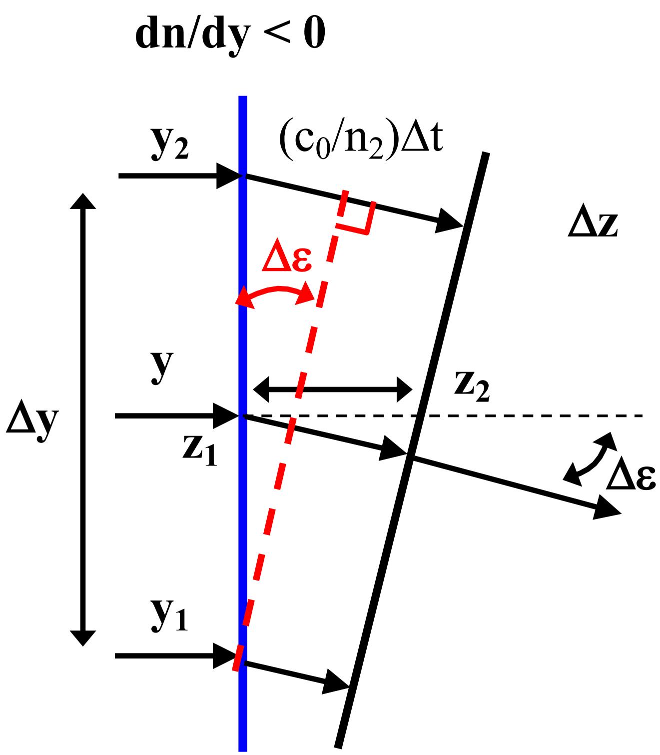
Refraction angle:

$$\tan(\Delta\varepsilon) \approx \Delta\varepsilon = \frac{\left(\frac{c_0}{n_2}\right)\Delta t - \left(\frac{c_0}{n_1}\right)\Delta t}{\Delta y}$$

Also: $\Delta t = \Delta z \dfrac{n}{c_0}$

$$\Rightarrow \quad \Delta\varepsilon = \frac{n(n_1 - n_2)}{n_1 n_2}\frac{\Delta z}{\Delta y}$$

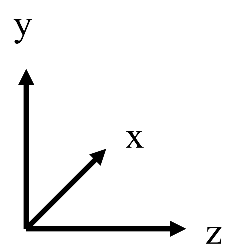
Angle of refraction



$$\Delta y \to 0, \Delta z \to 0$$

$$\boxed{\frac{d\varepsilon}{dz} = \frac{1}{n}\frac{dn}{dy}}$$

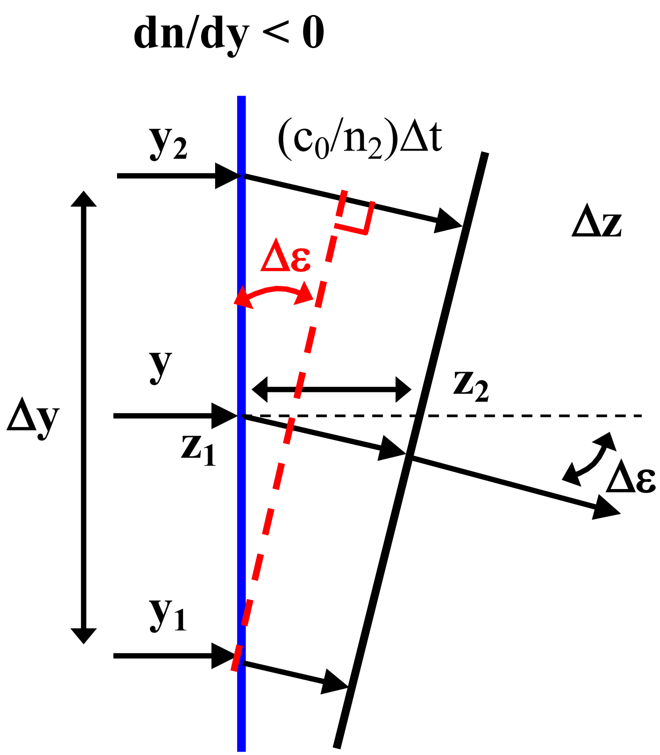
Because ε is a very small angle, it is approximately equivalent to dy/dz, the slope of the refracted ray.



$$\boxed{\frac{\partial^2 y}{\partial z^2} = \frac{1}{n}\frac{dn}{dy} \quad \text{and} \quad \frac{\partial^2 x}{\partial z^2} = \frac{1}{n}\frac{dn}{dx}}$$

Curvature of refracted ray

Angle of refraction (contd.)



So the angular ray deflection in the x and y directions are:

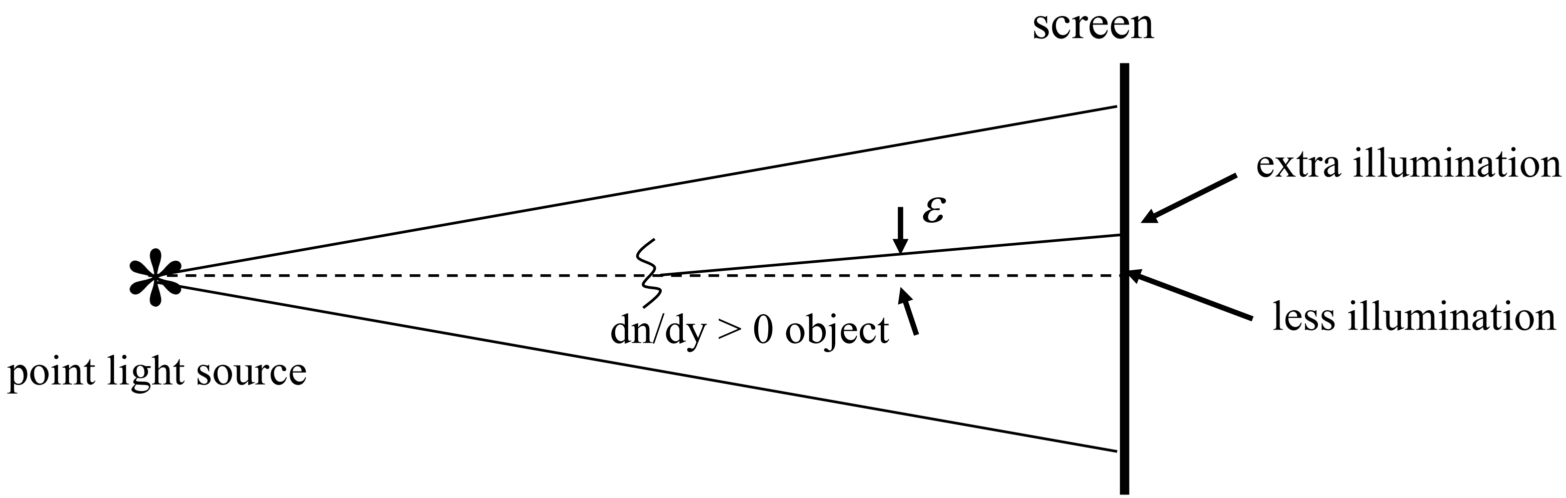
$$\epsilon_y = \int \frac{1}{n}\frac{\partial n}{\partial y} dz \quad \text{and} \quad \epsilon_x = \int \frac{1}{n}\frac{\partial n}{\partial x} dz$$

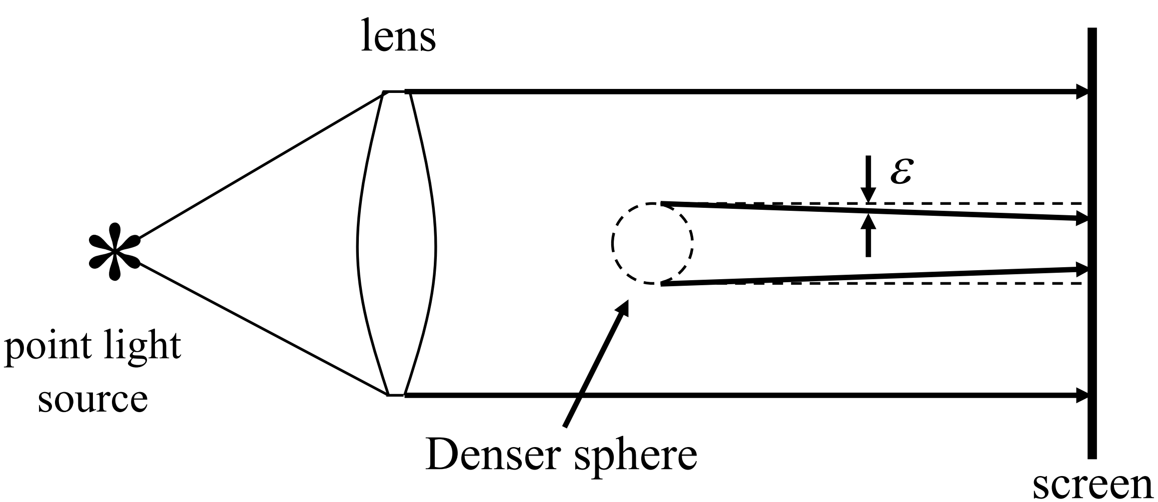
Thus, two things that matter:

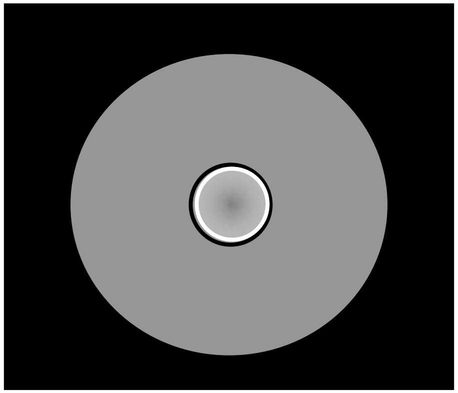
- ***Derivative of the index of refraction***
- And/or ***object thickness***

Shadowgraphy

Simplest way to visualize derivative of the index of refraction:

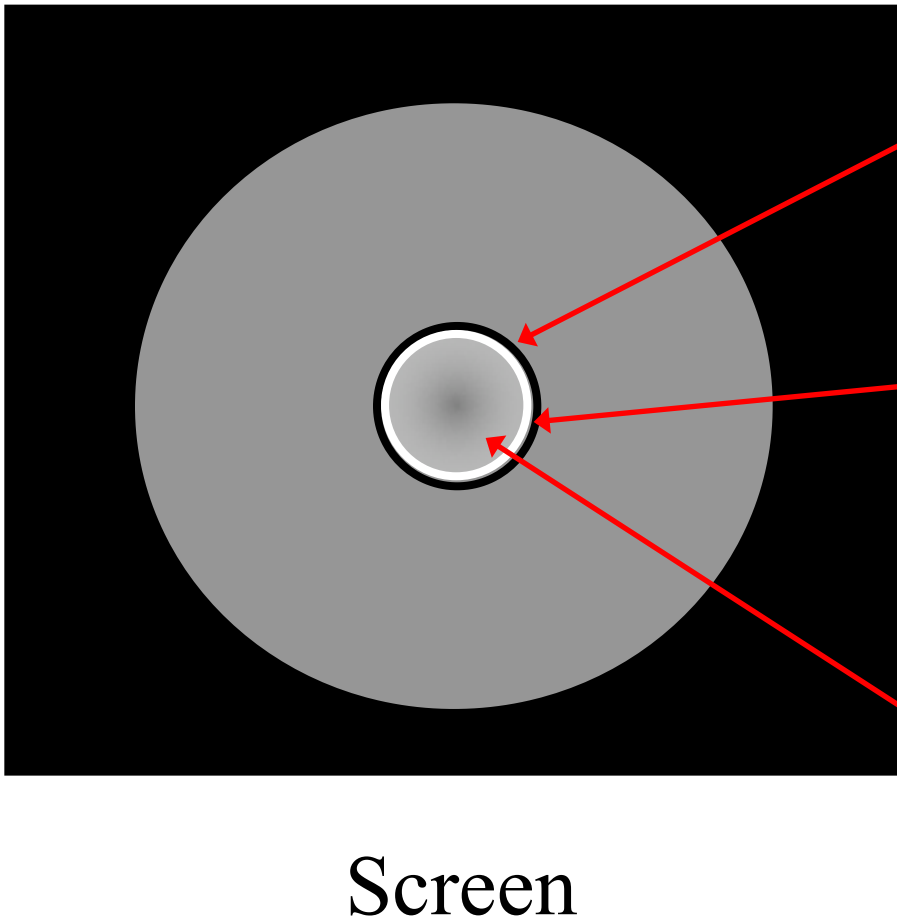






Screen

Shadowgraphy



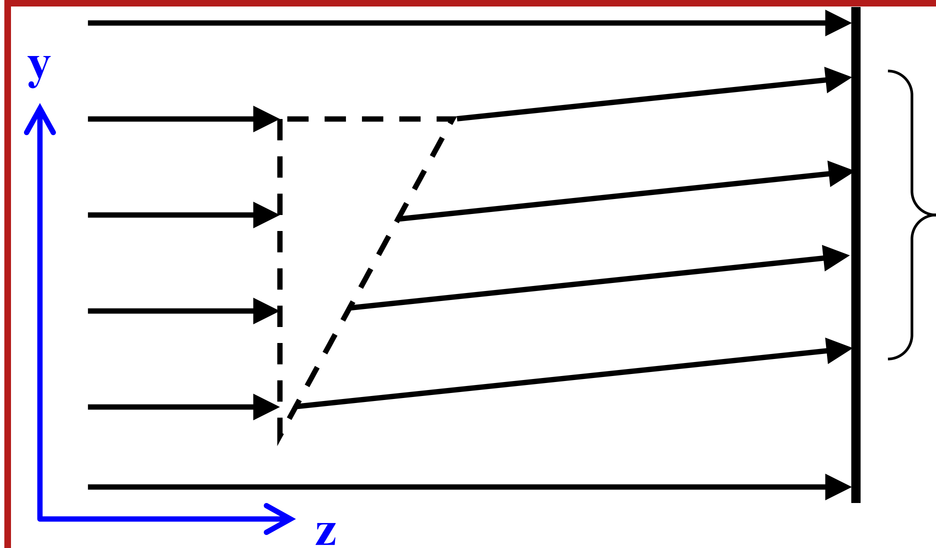
Dark circle due to light refracted from outline of sphere

Light circle due to refracted light from the outline illuminating this part of the screen

Gradient back to background illumination due to non-uniform refraction of rays as the light wave travels down the optical axis

Quality of the source & distance to the screen defines sharpness of features

Variations of gradients are critical!

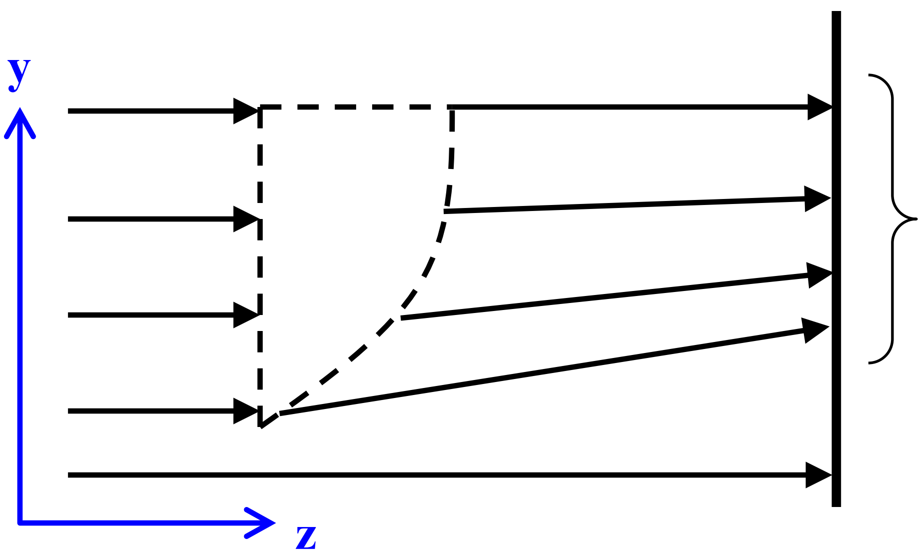


Uniform shift of illumination

- see some shadow, but don't get outline of the phase object
- as move down optical path (z-direction),

$$\partial n / \partial y = \text{constant}$$

so all rays shift the same!



Nonuniform illumination

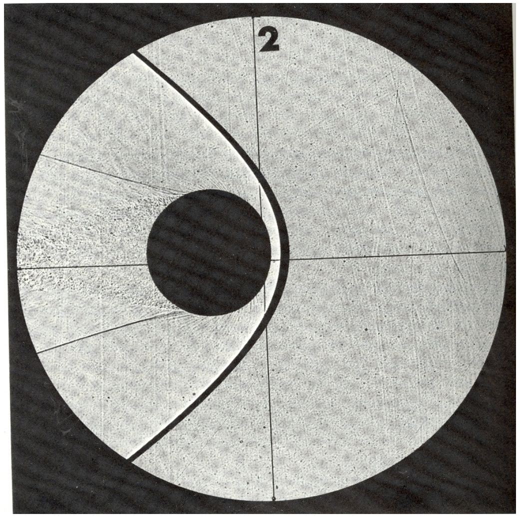
- as move down optical path,

$$\partial^2 n / \partial y^2 = \text{constant}$$

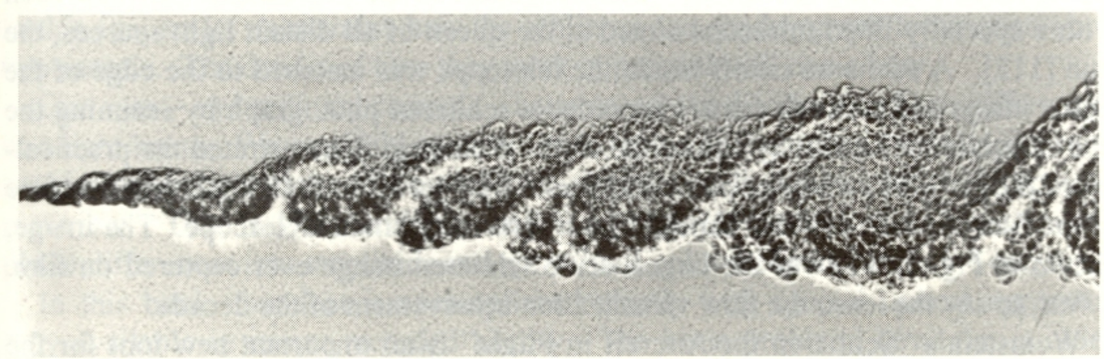
so rays shift non-uniformly.

<u>Variation</u> of gradients critical for shadowgraphy!

Example shadowgraphs



Sphere flying at M=1.7 (Merzkirch 1987)

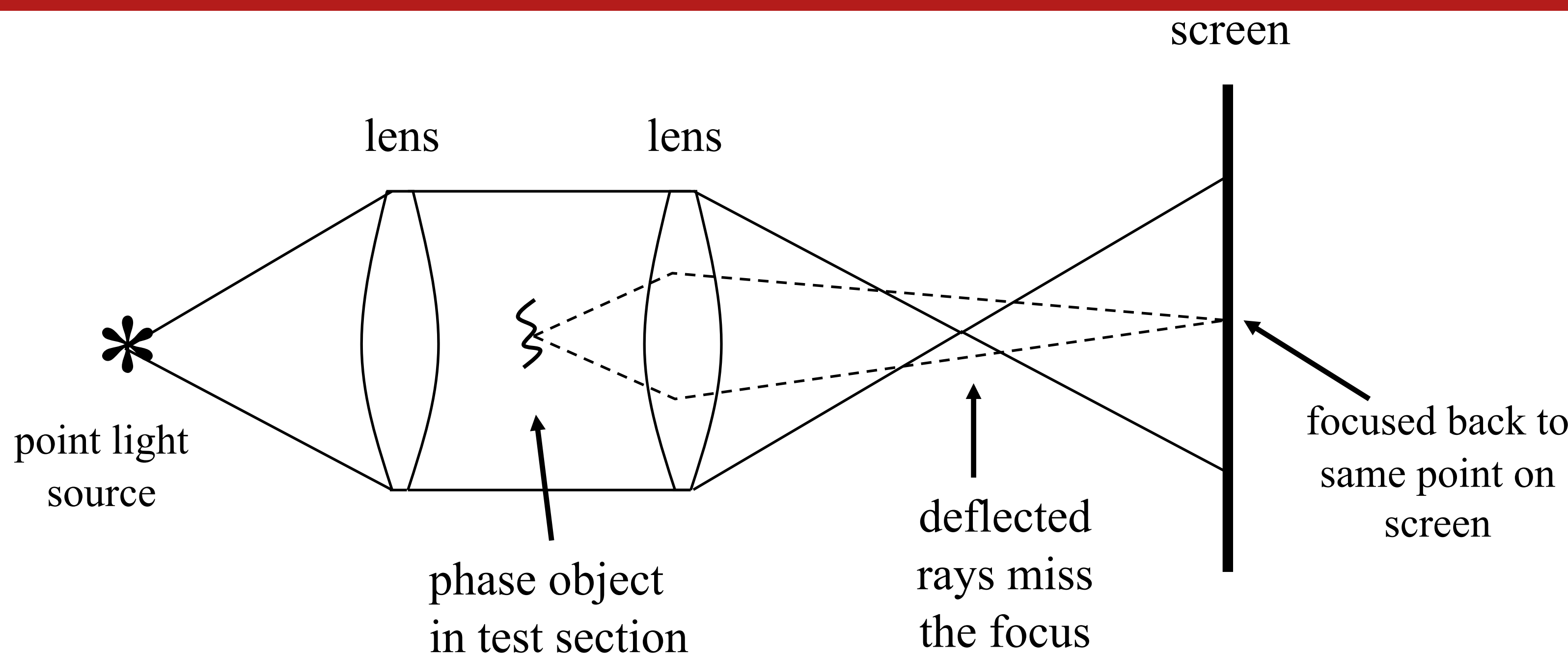


He/N_2 mixing layer (Settles 2001)

Schlieren (ger. *"streak"*) Imaging

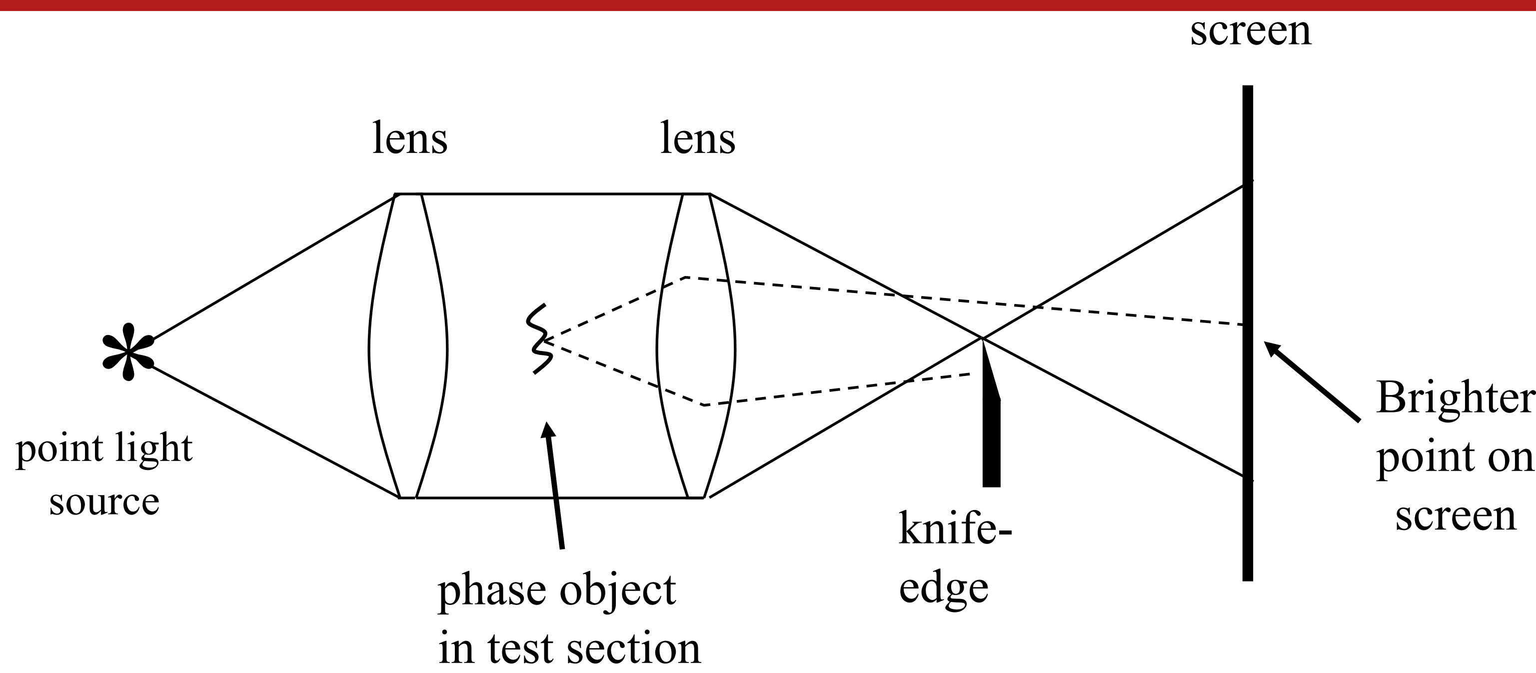
Schlieren Imaging	Shadowgraphy
Focused optical image formed by a lens	Not an image but a shadow
Requires cutoff of the refracted light	No cutoff of refracted light
Illumination level responds to $\partial n/\partial x$ and $\partial n/\partial y$	Responds to second spatial derivative, $\partial^2 n/\partial x^2$ and $\partial^2 n/\partial y^2$
Schlieren image displays the deflection angle ε	Shadowgraph displays ray displacement
More sensitive in general	Less sensitive except for special cases (e.g. shock waves)
More difficult to set up – uses lamps, mirrors, lenses	Extremely easy to setup, occurs naturally

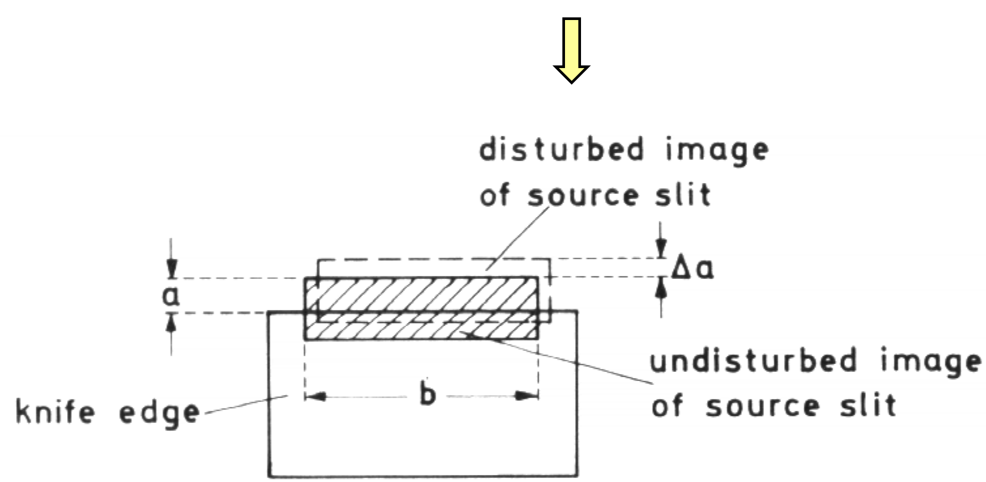
Schlieren system – point light source



- merely a projector, imaging opaque objects in the test section

Schlieren system – knife block





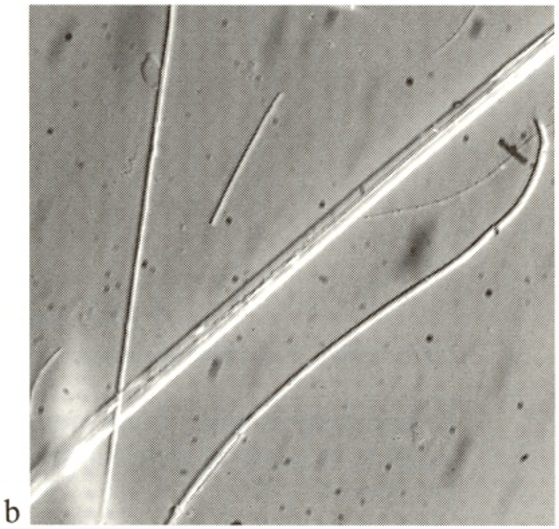
Detecting 1st derivative

$\partial n/\partial y$

Some Schlieren images

a

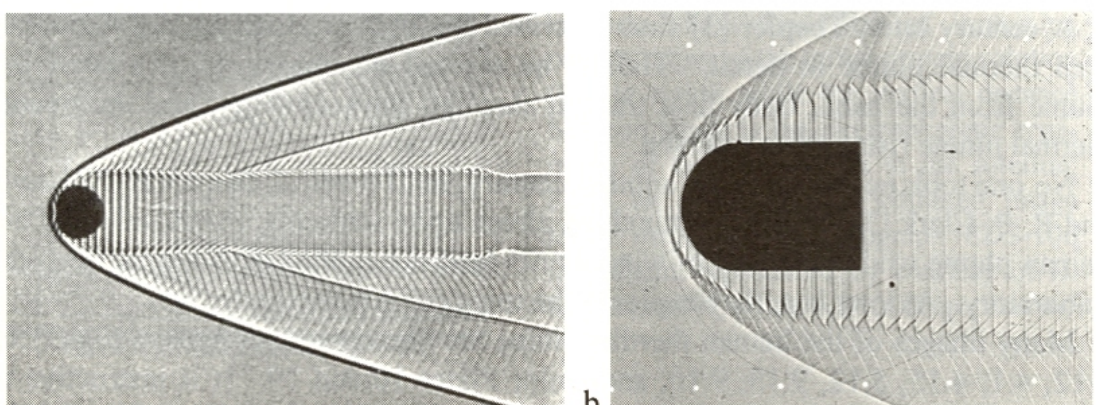
Bullet and candle flame (Settles 2001)



b

Glass fibers (Settles 2001)

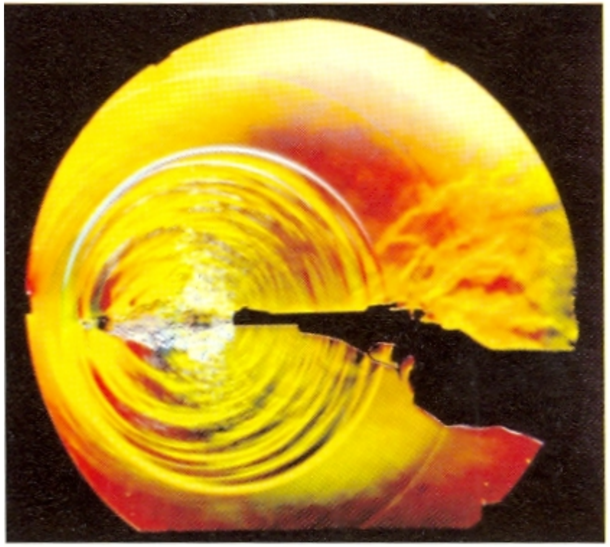
Projectile fired at Mach 4.75 in reactive H_2/air mixture – cyclic detonation behind the shock (Settles 2001)



a b

More Schlieren images

Removing frozen pizza from case (Settles 2001)

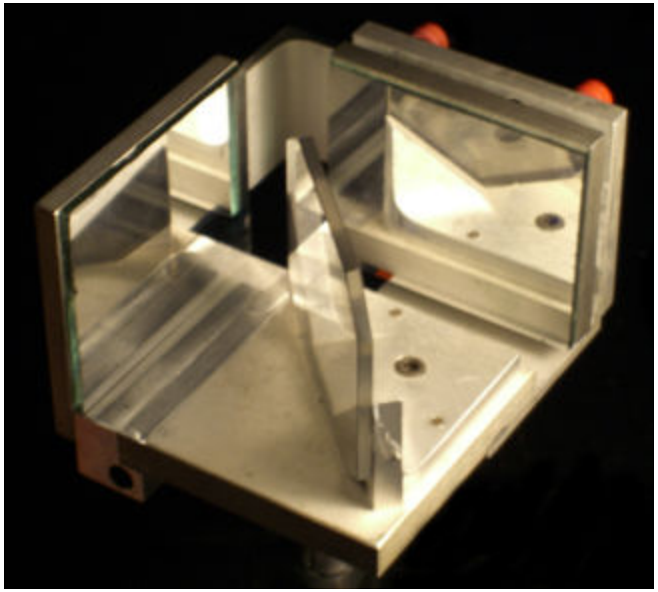


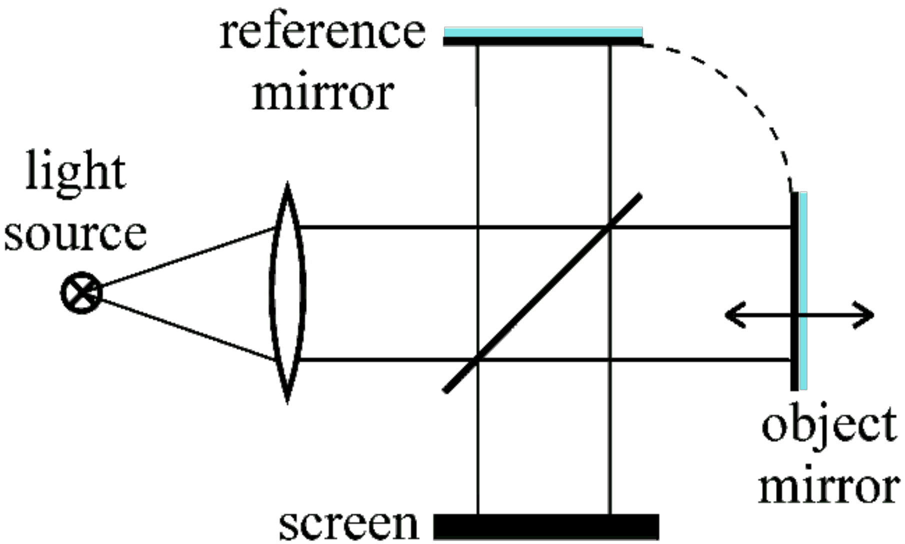
A gun firing 0.22 caliber bullet (Settles 2001)

Heat from grill (Settles 2001)

Phase sensitivity

In upcoming lectures: ***interference*** is the most sensitive way of detecting small phase changes.





Michelson interferometer

Amplitude vs. phase sensing in microscopy

Amplitude Objects:
Cells with stains, pigments, absorb, scatter light, change its amplitude

Most biological objects do not absorb white light
But scatter & phase shift: phase objects

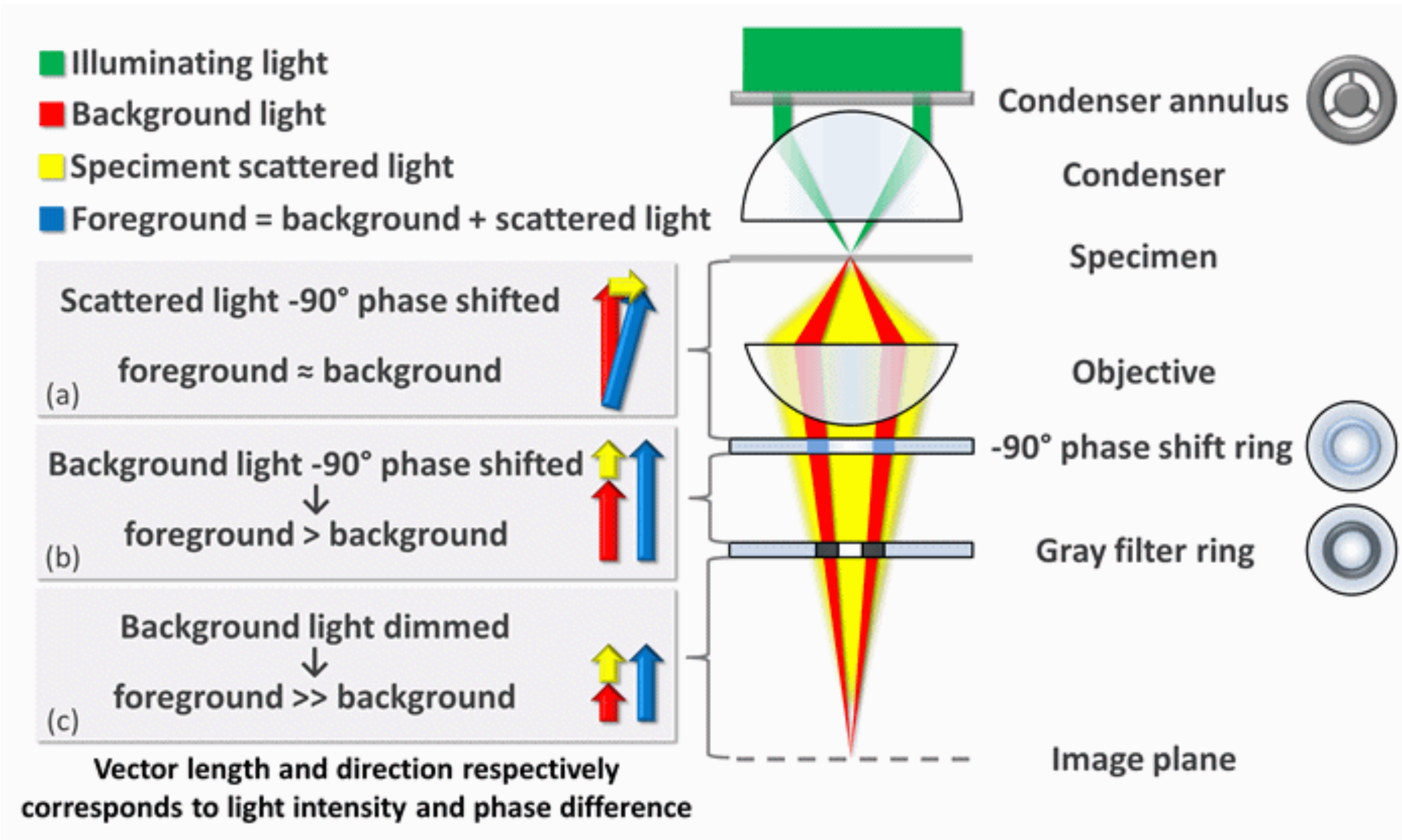
$$n \equiv \frac{c}{v_{\mathrm{phase}}}$$

But may be hard to see with contrast by bright field

Use phase differences to visualize differences in refractive index:
Phase contrast microscopy
Nobel Prize in 1953: Zernike

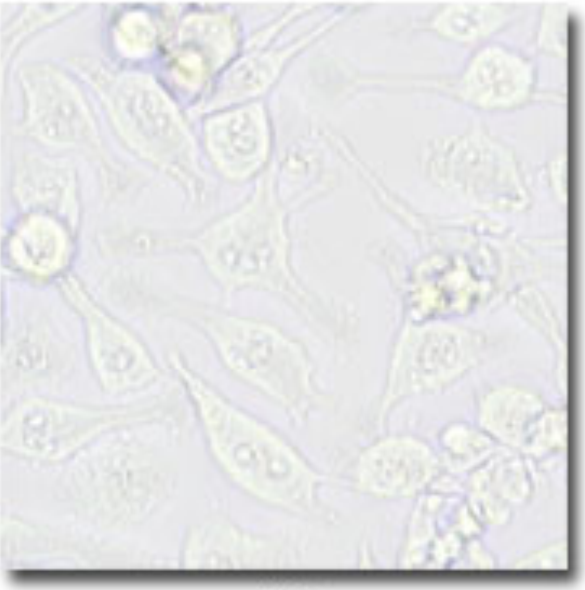
Working principle of phase contrast microscopy

Use the facts that scattered light: a) goes in all directions, b) is phase-shifted relative to the incoming light.

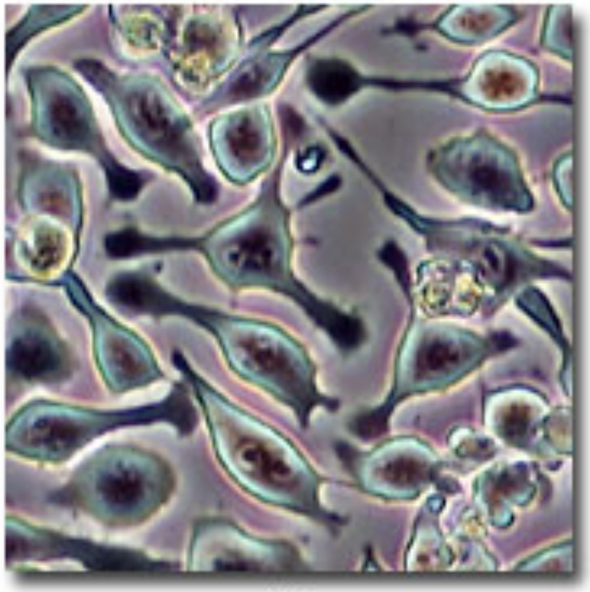


Amplitude vs. phase contrast images of living cells

http://www.microscopyu.com/articles/phasecontrast/phasemicroscopy.html



amplitude



phase

Links/References

http://www.ucolick.org/~max/289/Lectures%20Final%20Version/Lecture%207%202013/Lecture7_2013_v1.ppt

http://galcit.caltech.edu/~sallym/schlieren_presentation.ppt

http://www.ccam.uchc.edu/yu/class/Lectures/2.ppt

Microlens array insect eye http://www.newshonk.com/wp-content/uploads/2015/05/insecy-eye-e1269258069203.jpg

Phase contrast microscope principle figure is taken from Wikipedia