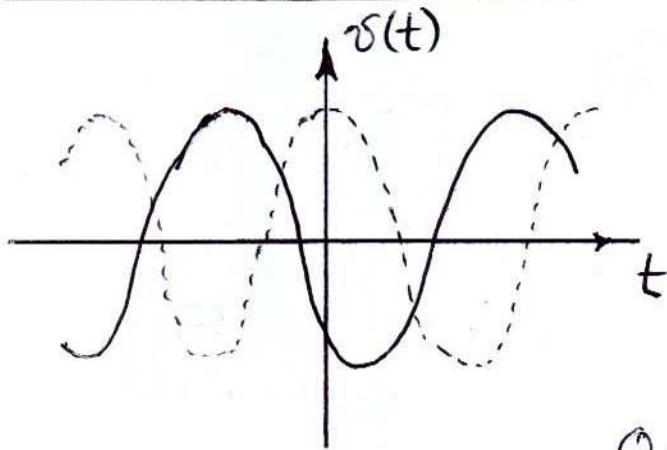


Lecture 7Linear AC circuits

$$v(t) = V_0 \cos(\omega t + \phi)$$

Q: why study AC (sine) response?

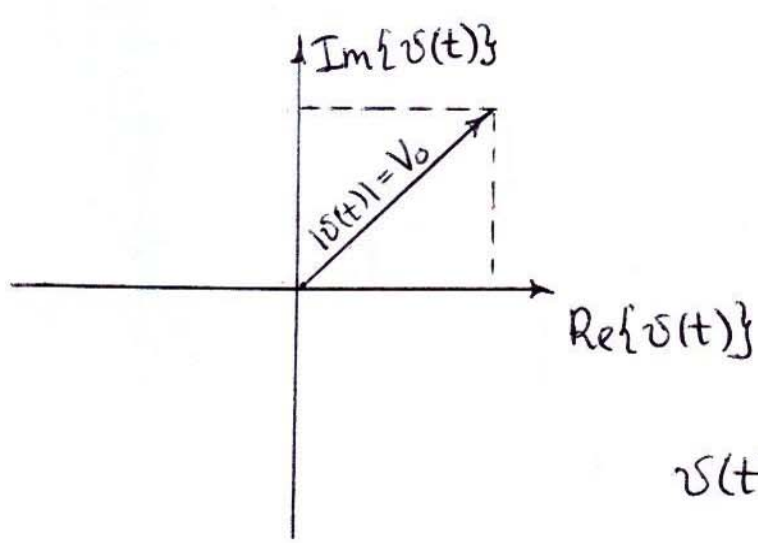
1)

Q: if you have \_\_\_\_\_ network, what shape on input remains the same on the output?

2)

3) superposition:

Complex notation



Euler formula

$$e^{j\alpha} = \cos\alpha + i\sin\alpha$$

$$j^2 = -1$$

$$v(t) =$$

Usually simply write  $v(t) =$

## Time vs. freq. domain analysis

Time domain:  $v(t), i(t)$

KVL & KCL  $\Rightarrow$

—

—

Freq. domain:  $V(\omega), I(\omega)$

$$\frac{d}{dt}(e^{j\omega t}) \rightarrow$$

$$\int e^{j\omega t} dt \rightarrow$$

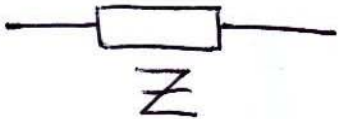
KVC & KCL  $\Rightarrow$

③



- two ways to look
- Fourier analysis :

### Complex impedance



$$V(\omega) =$$

admittance = generalized conductance

$$\text{Let } i(t) = I_0 e^{j\omega t}, \quad v(t) = V_0 e^{j(\omega t + \phi)}$$

$$Z(\omega) =$$

Impedance can change both \_\_\_\_\_ (4)

$$Z = R + jX = Z_0 e^{j\varphi}, \Rightarrow$$

### Capacitance

$$\frac{d\psi}{dt} = \frac{i}{C}$$

$$Z_C = \frac{v(t)}{i(t)} =$$

voltage \_\_\_\_\_ current by  $90^\circ$

### Inductance

$$v = L \frac{di}{dt}$$

$$Z_L = \frac{v(t)}{i(t)} =$$

voltage \_\_\_\_\_ current by  $90^\circ$

→ i

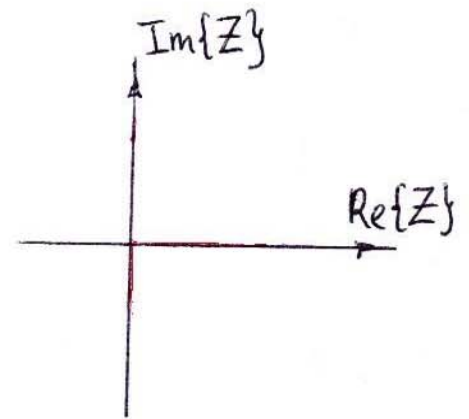
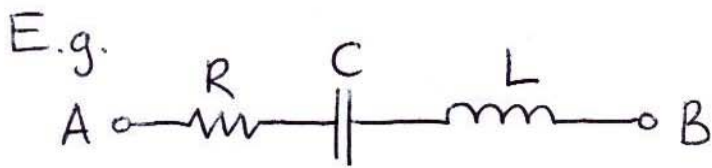
	$\omega \rightarrow 0$	$\omega \rightarrow \infty$
$Z_R$		
$Z_L$		
$Z_C$		

Lecture 8

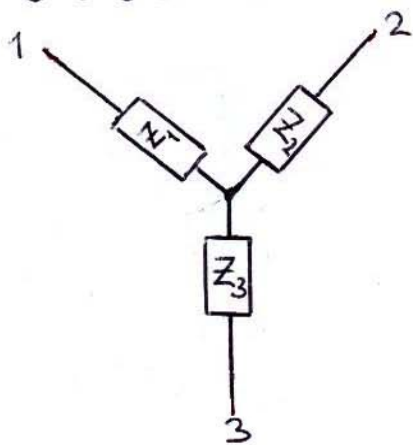
Combining R, L, C

series:

parallel:



Y- $\Delta$  transformation



same as before, but now complex

$\Delta \rightarrow Y$ :

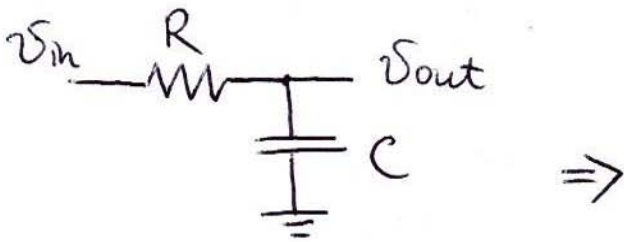
$Y \rightarrow \Delta$ :

# AC circuit analysis

②

- \* superposition
- \* Thevenin & Norton
- \* KCL & KVL

## Lo-pass filter



Transfer fcn (a.k.a. complex gain)

$$G(\omega) \equiv$$

$$\text{L.P. : } G(\omega) =$$

magn.  $|G(\omega)| =$

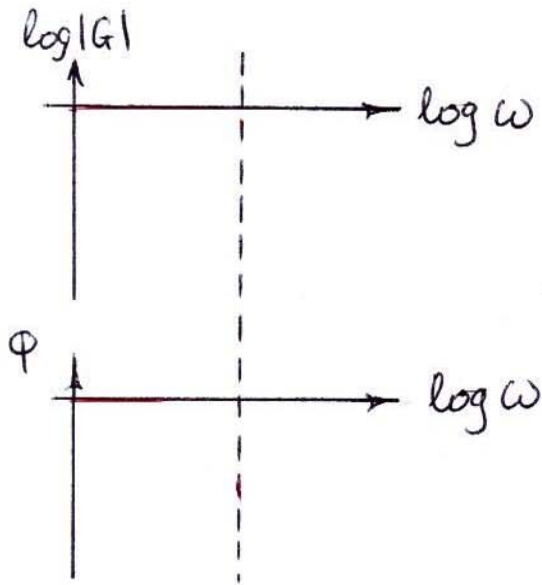
phase  $\Phi[G] =$

# Bode plots

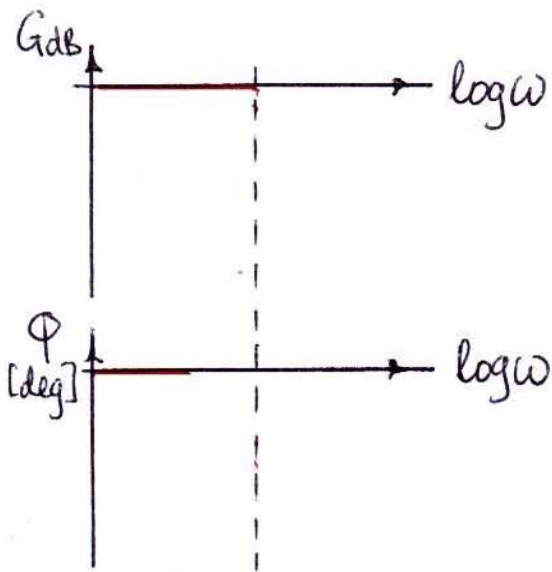
(3)

$$G(\omega) = |G(\omega)| e^{j\phi[G(\omega)]}$$

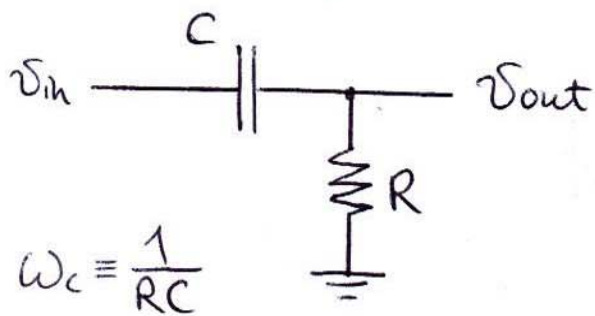
Bode plots =



Usually draw uncorrected Bode plots.

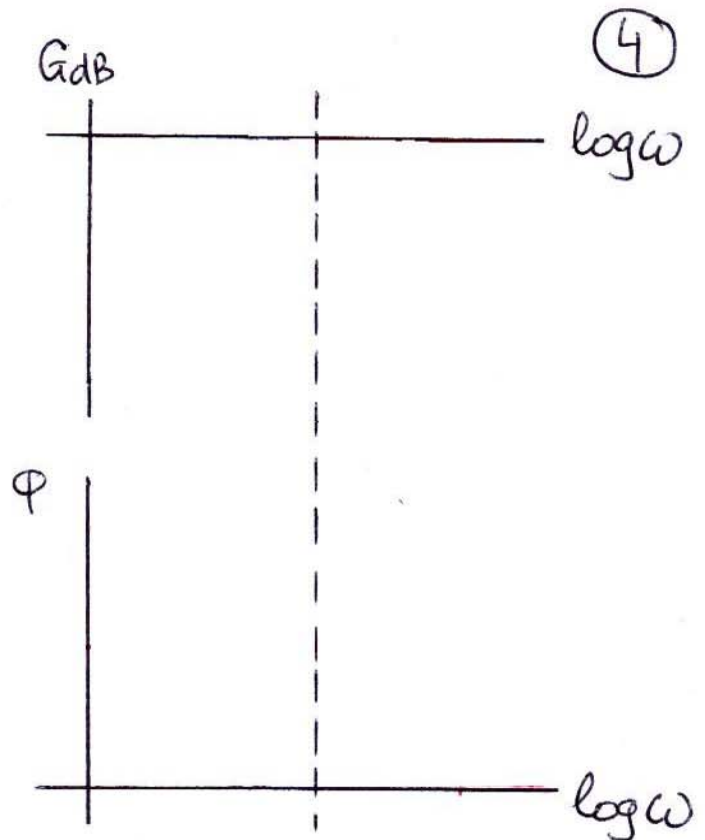


## Hi-pass filter



$$\omega_c \equiv \frac{1}{RC}$$

$$G(\omega) =$$



Note: Bode plots  $\Rightarrow$  always two  
 $G(\omega) = |G(\omega)| e^{j\phi[G(\omega)]}$

- magnitude & phase \_\_\_\_\_
- e.g. change in magn. \_\_\_\_\_

Standard (zero-pole) form of  $G$

$$G(\omega) =$$



Lecture 9s-plane

$$s \equiv \sigma + j\omega$$

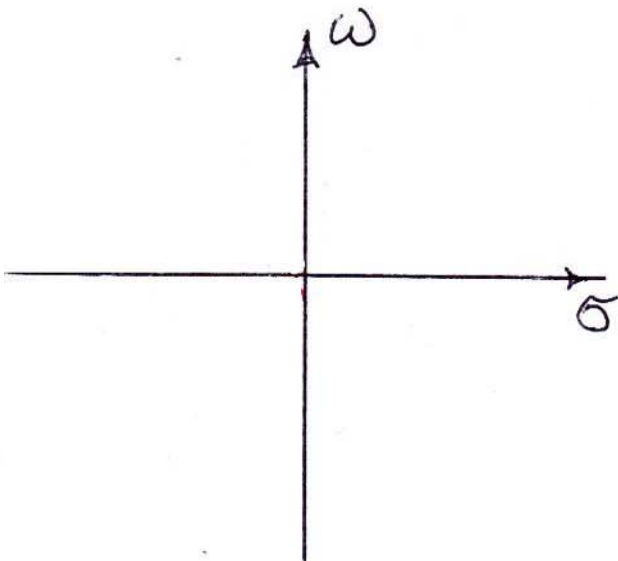
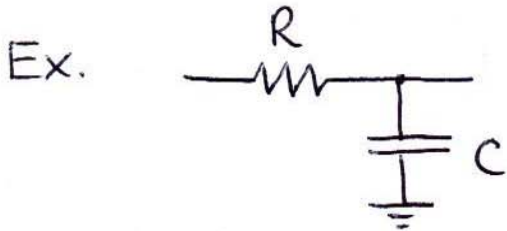
$$v(t) = V_0 e^{st} \quad ; \quad \operatorname{Re}\{s\} < 0$$

$$\operatorname{Re}\{s\} > 0$$

$$\text{if } s = j\omega \Rightarrow$$

$$\text{Impedance in } s\text{-domain: } Z_C = \frac{1}{sC} \quad , \quad Z_L = sL$$

$$G(s) = \frac{V_{\text{out}}(s)}{V_{\text{in}}(s)} \quad - \quad \text{gain (transfer fcn) in } s\text{-plane}$$



$$G(\omega) :$$

$$G(s) :$$

## Zero-pole form of transfer fcn

(2)

Most natural when using s-plane

$$G(s) = \frac{N(s)}{D(s)} =$$

$$\Rightarrow G(s) \text{ @}$$

-

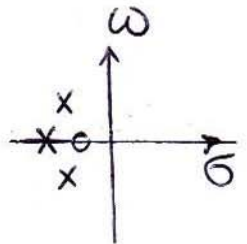
### Some properties

1)

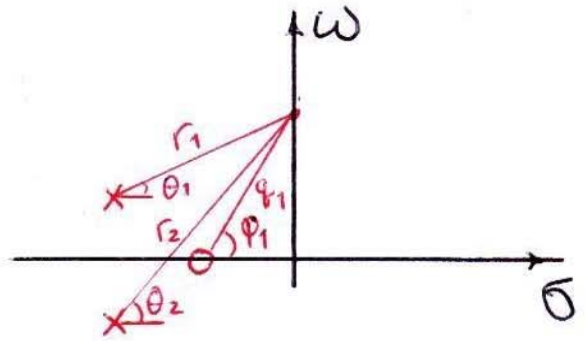
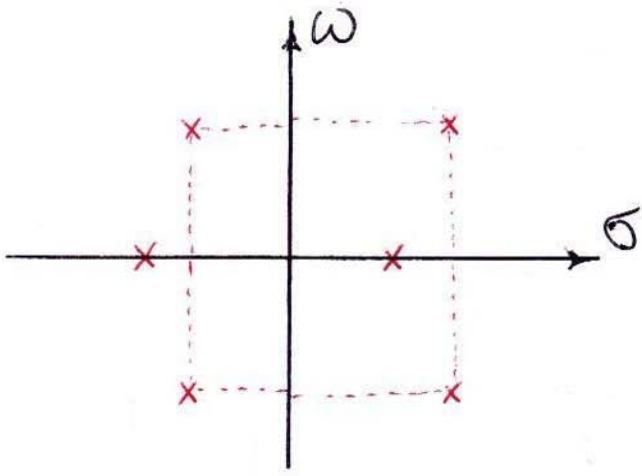
2)

3)

4)



③

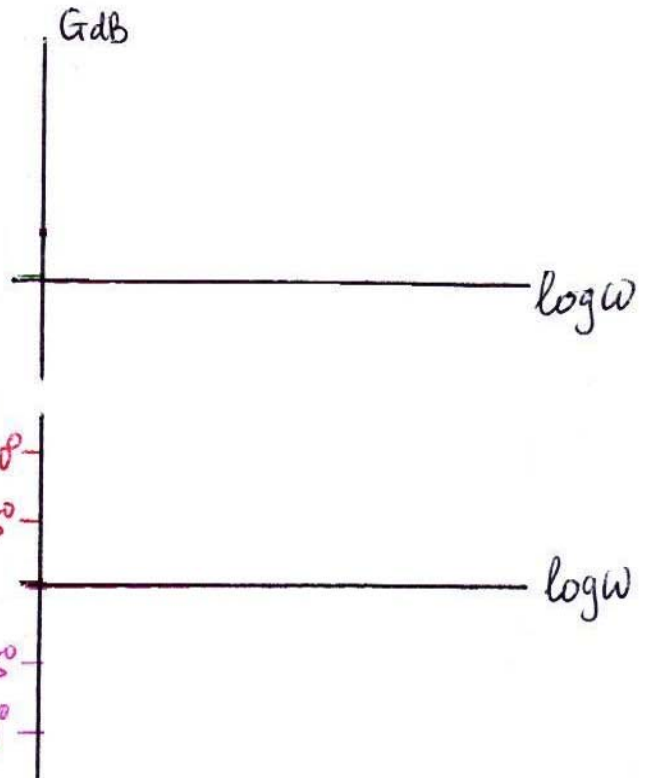
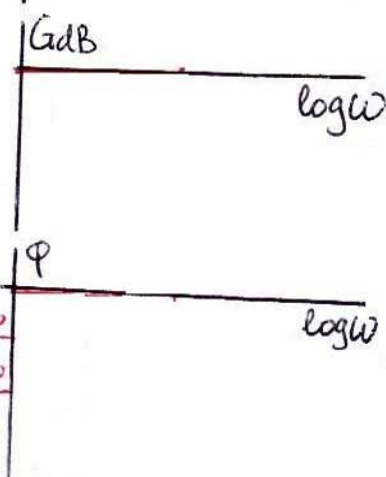
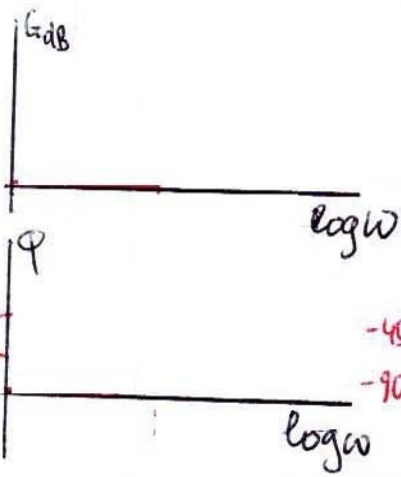
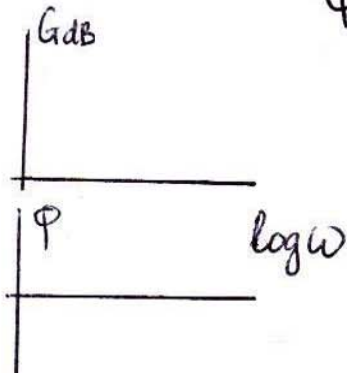
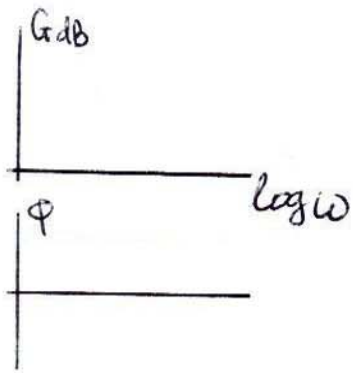


Bode Plots:

$|G(\omega)| =$

$\Phi[G(\omega)] =$

$\Rightarrow$  on Bode plots:



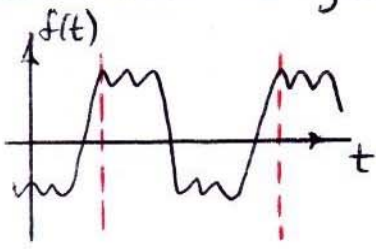
Ex.  $G = K \frac{1+j\omega/\omega_z}{1+j\omega/\omega_p}$

let  $\omega_z \ll \omega_p$

$\Rightarrow$

# Fourier Analysis

(4)



Basic idea:

$$f(t) = \sum_{n=-\infty}^{\infty} c_n e^{jn\omega t}$$

Definition:  $g(t)$  and  $h(t)$  orthogonal if

$\{e^{jn\omega t}\}$  forms orthogonal set:

$$f(t) = \sum_{n=-\infty}^{\infty} c_n e^{jn\omega t}$$

Strategy

- 1) source decomp. (superp.)
- 2) find response @ each harmonic  $n\omega$
- 3) find  $v_{out}$

