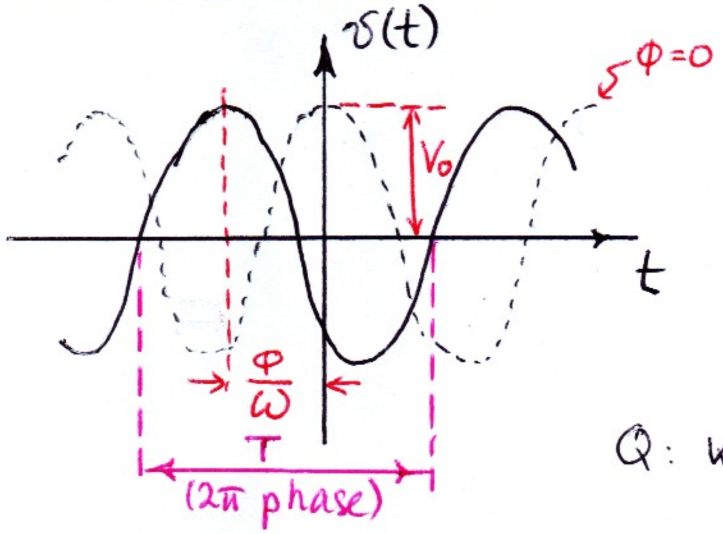
P3360/AEP3630

Lecture 7

Linear AC circuits



$\omega = 2\pi f$ ang. freq.

$$v(t) = V_0 \cos(\omega t + \varphi)$$

leading phase φ ("+"-ve)

Q: why study AC (sine) response?

1) natural waveform of electr. utilities, easily produced (rotating generators), transported & manipulated

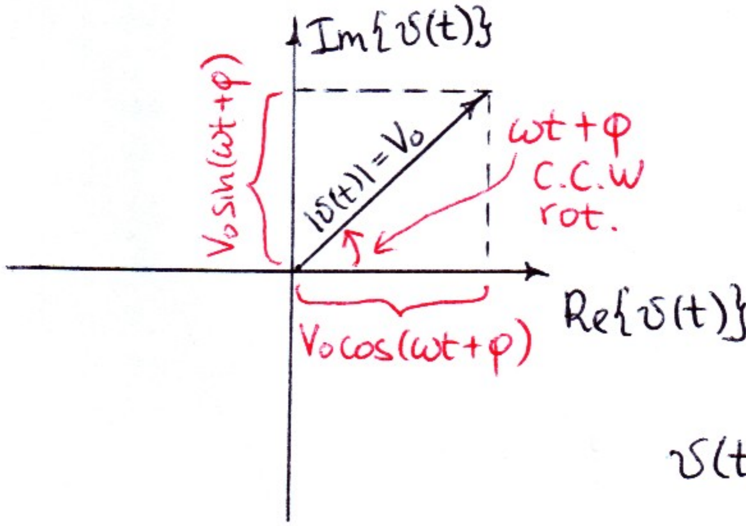
Q: if you have R, L, C network, what shape on input remains the same on the output?

2) sine input wave into a linear network remains a sine (+ phase shift) on the output

3) superposition: if response of a circuit is known @ all frequencies => can understand response to arbitrary waveform (through Fourier analysis)

Complex notation

– a way to avoid dealing with trig. identities



"phasor diagram"

Euler formula

$$e^{j\alpha} = \cos\alpha + i \sin\alpha$$

$j^2 \equiv -1$ (can't use $i^2 = -1$ b/c i = curr.)

$$v(t) = V_0 \cos(\omega t + \varphi)$$
$$= Re\{V_0 e^{j(\omega t + \varphi)}\}$$

Usually simply write $v(t) = \underbrace{V_0}_{\text{magn.}} e^{j\overbrace{(\omega t + \varphi)}^{\text{phase}}}$

with Re{ } assumed

(no Im scope probe!)

Time vs. freq. domain analysis

Time domain: $v(t), i(t)$ fcn. of time

KVL & KCL $\Rightarrow$ differential eqn's

- Solve them. completely general
- but need initial conditions, driving terms, ...

☹ difficult (except for a computer)

Freq. domain: $V(\omega), I(\omega)$ fcn of ω

$$\frac{d}{dt}(e^{j\omega t}) \rightarrow \times j\omega$$

$$\int e^{j\omega t} dt \rightarrow \times \frac{1}{j\omega}$$

KVC & KCL $\Rightarrow$ algebraic eqn's

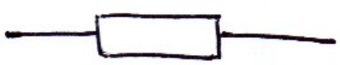
much simpler to solve ☺

Time domain	$\xleftrightarrow[\text{(relates the two)}]{\text{Fourier analysis}}$	Freq. domain

- two ways to look at the same situation
- Fourier analysis : superposition allows to decompose / combine periodic (arbitrary) wave into sum (integral) of harmonics (freq.)
  $t \leftrightarrow \omega$

Complex impedance

extends Ohm's law to AC circuits



Z

$$V(\omega) = Z(\omega)\, I(\omega)$$

$$Z(\omega) = \frac{V(\omega)}{I(\omega)} = R + jX$$

reactance (Ω) no power diss.

resistance (Ω), power dissipation

admittance = generalized conductance

$$Y \equiv Z^{-1} = G + jB$$

susceptance

conductance

Let $i(t) = I_0 e^{j\omega t}$, $v(t) = V_0 e^{j(\omega t + \varphi)}$

$$Z(\omega) = \frac{v(t)}{i(t)} = \frac{V_0 e^{j(\omega t+\varphi)}}{I_0 e^{j\omega t}} = \underbrace{\frac{V_0}{I_0}}_{\text{mag.}} \underbrace{e^{j\varphi}}_{\text{phase}} = Z_0 e^{j\varphi}$$

Impedance can change both volt. magnitude & phase

$$Z = R + jX = Z_0 e^{j\varphi}, \Rightarrow Z_0 = \sqrt{R^2 + X^2}$$

$$\varphi = \tan^{-1}\left(\frac{X}{R}\right)$$

Capacitance

$\frac{dv}{dt} = \frac{i}{C}$. For steady state AC: $v(t) = V_0 e^{j\omega t}$

$$\frac{dv}{dt} = j\omega \underbrace{V_0 e^{j\omega t}}_{v(t)} = \frac{i(t)}{C}, \Rightarrow i(t) = j\omega C \cdot v(t)$$

$$Z_C = \frac{v(t)}{i(t)} = \frac{1}{j\omega C}, \quad |Z_C| = \frac{1}{\omega C} = \begin{cases} \text{large for low freq.} \\ \text{small for high freq.} \end{cases}$$

$$\varphi_C = -\frac{\pi}{2}$$

voltage lags current by 90°

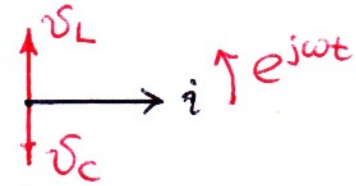
Inductance

$v = L\frac{di}{dt}$ Let $i(t) = I_0 e^{j\omega t}$, $\Rightarrow v(t) = Lj\omega \underbrace{I_0 e^{j\omega t}}_{i(t)}$

$$Z_L = \frac{v(t)}{i(t)} = j\omega L, \quad |Z_L| = \omega L = \begin{cases} \text{small for low } f \\ \text{large for hi } f. \end{cases}$$

$$\varphi_L = \frac{\pi}{2}$$

voltage leads current by 90°



"ELI the ICE man"

	$\omega \to 0$ (dc)	$\omega \to \infty$ (e.g. sharp step)
Z_R	R	R
Z_L	short	open
Z_C	open	short