Lecture 9

S-plane

$s \equiv \sigma + j\omega$ - generalized complex frequency

$v(t) = V_0 e^{st}$; $Re\{s\} < 0$ - exp. decay

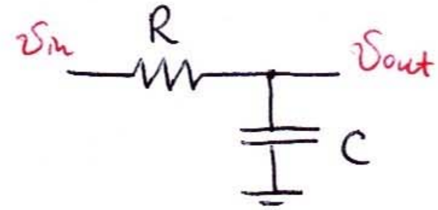
$Re\{s\} > 0$ - exp. growth

if $s = j\omega \Rightarrow$ AC sine steady case

Impedence in s-domain: $Z_C = \frac{1}{sC}$, $Z_L = sL$

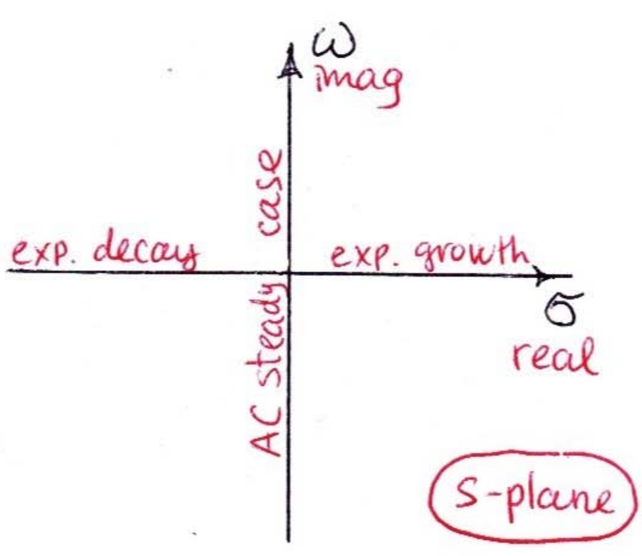
$G(s) = \frac{V_{out}(s)}{V_{in}(s)}$ - gain (transfer fcn) in s-plane

Ex.



Let: $v_{in} = V_0 e^{st}$

$$G(s) = \frac{v_{in} \cdot \frac{1}{sC}}{R + \frac{1}{sC}} \frac{1}{v_{in}} = \frac{1}{sRC + 1}$$



$G(\omega)$: $G(s \to j\omega)$

$G(s)$: $G(j\omega \to s)$

Zero-pole form of transfer fcn

Most natural when using s-plane

$$G(s) = \frac{N(s)}{D(s)} = K\frac{(s-s_{z_1})(s-s_{z_2})\ldots}{(s-s_{p_1})(s-s_{p_2})\ldots}$$

$\Rightarrow G(s)$ @ $\begin{cases} s_{z_1}\ldots & \text{polynomial roots of } N(s),\ |G|\to 0 \ \text{zeros} \\ s_{p_1}\ldots & \text{polynomial roots of } D(s),\ |G|\to\infty \ \text{poles} \end{cases}$

- convinient representation that provides insight into circuit response

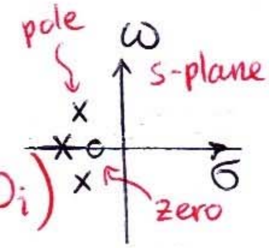
Some properties

1) # poles ≥ # zeros. Q: why?
A: otherwise $|G(\omega\to\infty)|\to\infty$ unphysical

2) circuit (filter) order:
degree of $D(s)$ polynomial (= # poles)

Filter order matches # of L, C elements

3) zeros & poles must be real, or come in complex conjugate pairs ($z_{p_i} = \sigma_i \pm j\omega_i$)



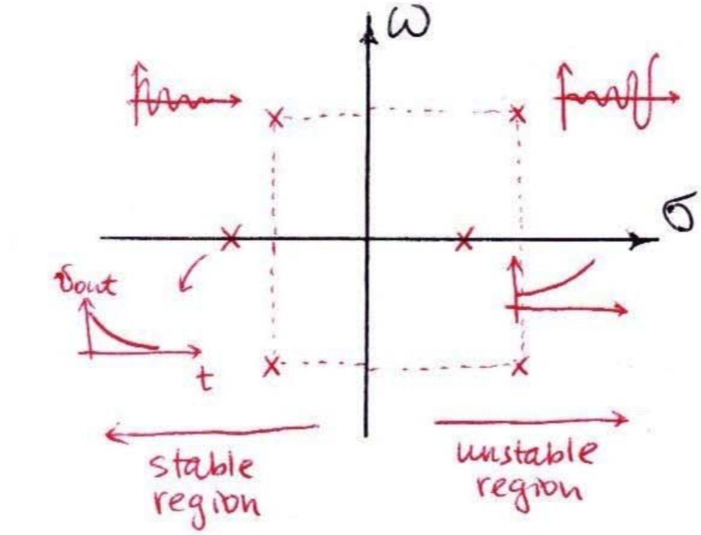
Q: why? A: real coeff. of $N(s)$, $D(s)$ poly.

4) poles ⇒ homogeneous response of the system
(some initial cond. but $v_{in}\to 0$)

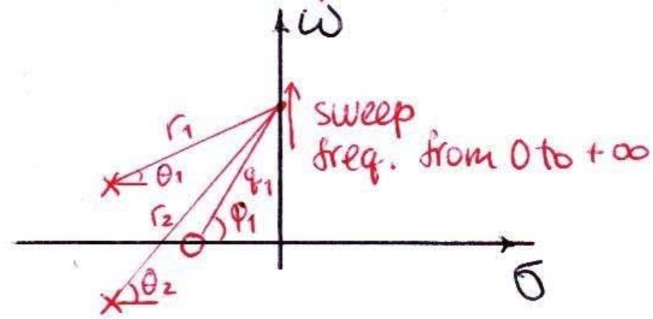
N(s) does not matter for homog. response

$$v_{out} = \sum_i c_i e^{s_{p_i} t}$$

c_i ← initial cond.



Q: How to get $G(\omega)$ from zero-pole form?



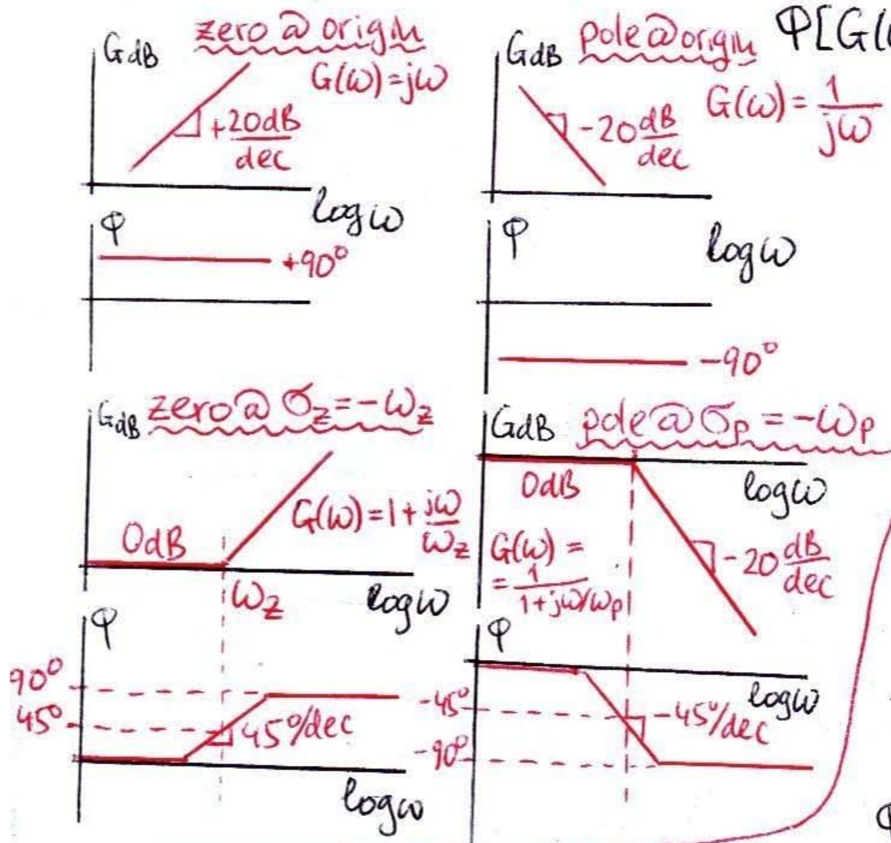
$$|G(\omega)| = K \frac{q_1 q_2 \dots}{r_1 r_2 \dots}$$

$$\varphi[G(\omega)] = (\varphi_1 + \varphi_2 + \dots) - (\theta_1 + \theta_2 + \dots)$$

⇒ on Bode plots: add zeros & subtract poles

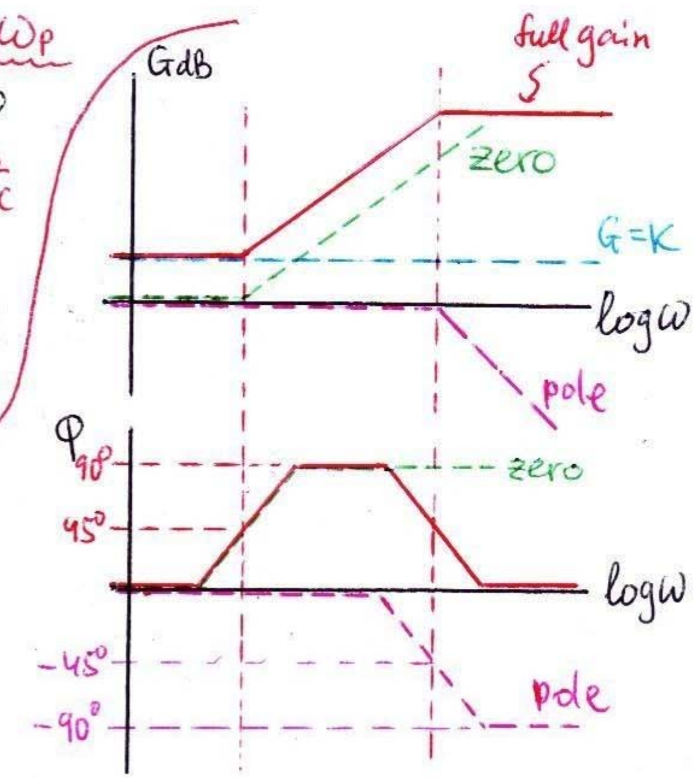
Bode Plots:

E.g. consider basic cases when zeros & poles are real & nonpositive

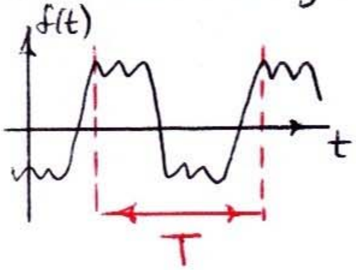


Ex. $G = K \dfrac{1 + j\omega/\omega_z}{1 + j\omega/\omega_p}$ ⇒

let $\omega_z \ll \omega_p$



Fourier Analysis (connect time & freq. domains)



Basic idea: decompose periodic $f(t)$ into harmonics

$$f(t) = \sum_{n=-\infty}^{\infty} c_n e^{jn\omega t}$$

$\omega = \frac{2\pi}{T}$ - fund. freq. ; c_n - generally complex, corresponds to harm. n

Definition: $g(t)$ and $h(t)$ orthogonal if

$$\int_a^b g(t)\, h^*(t)\, dt = 0$$

$\{e^{jn\omega t}\}$ forms orthogonal set : $\int_{t_0}^{t_0+T} e^{jn\omega t} (e^{jm\omega t})^* dt = \begin{cases} 0, & m \neq n \\ T, & m = n \end{cases}$

$$\int_{t_0}^{t_0+T} e^{-jm\omega t} f(t)\, dt = \sum_{n=-\infty}^{\infty} c_n \underbrace{\int_{t_0}^{t_0+T} e^{-jm\omega t} e^{jn\omega t} dt}_{\text{orthogonal if } m \neq n} \Rightarrow \text{only } c_m \text{ survives}$$

$$\Rightarrow \boxed{c_m = \frac{1}{T} \int_{t_0}^{t_0+T} e^{-jm\omega t} f(t)\, dt}$$

If $f(t)$ is real, the sum sym. with n

$$f(t) = \mathrm{Re}\left\{ \sum_{n=0}^{\infty} \tilde{c}_n e^{jn\omega t} \right\}$$ with $\tilde{c}_0 = c_0$ and $\tilde{c}_n = 2c_n$, $n > 0$

Strategy

1) source decomp. (superp.) $\underbrace{v_{in}}_{\text{arbitrary periodic wave}} = \sum_{n=0}^{\infty} \hat{V}_{in,n} e^{jn\omega t}$

2) find response @ each harmonic $n\omega$ $\hat{V}_{out,n} = \hat{V}_{in,n} \underbrace{G(n\omega)}_{\text{network gain}}$

3) find $v_{out} = \mathrm{Re}\left\{ \sum_{n=0}^{\infty} \hat{V}_{out,n} e^{jn\omega t} \right\}$