Lecture 26: **03/27/09**

- Photon Polarization States III

(2) Linear polarization analyzer loop/change of basis states:

(3) Example of Interference of Polarization States

(4) Circular polarized light/complex projection amplitudes

Recap

① $E_x = a\cos(kz-\omega t+\varphi_A) = \hat{x}\cdot\vec{E}_i$ $\boxed{\text{detect.}}$ $|x\rangle,\ \langle x|\psi\rangle$

$E_y = b\cos(kz-\omega t+\varphi_b) = \underbrace{\hat{y}\cdot\vec{E}_i}_{\text{projection!}}$ $\boxed{\text{detect.}}$ $|y\rangle,\ \langle y|\psi\rangle$

$\hat{x}$, $\hat{y}$ — x-y-analyzer

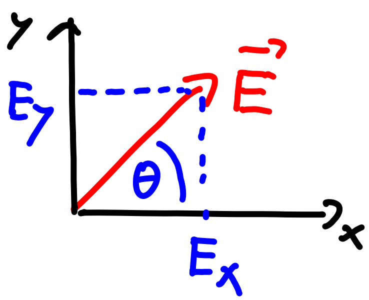
$\vec{E}_i = a\cos(kz-\omega t+\varphi_a)\hat{x} + b\cos(kz-\omega t+\varphi_b)\hat{y}$

$|\psi\rangle$

- $|x\rangle$, $|y\rangle$ linear polarization states: complete, orthonormal basis!
- Statistical behavior of individual photons

- Measurements can change the quantum state!
- Measurements give only discrete results.

- Projection amplitude for linear polarizations:



Classically

$E_x = \hat{x}\cdot\vec{E} = E\cos\theta$

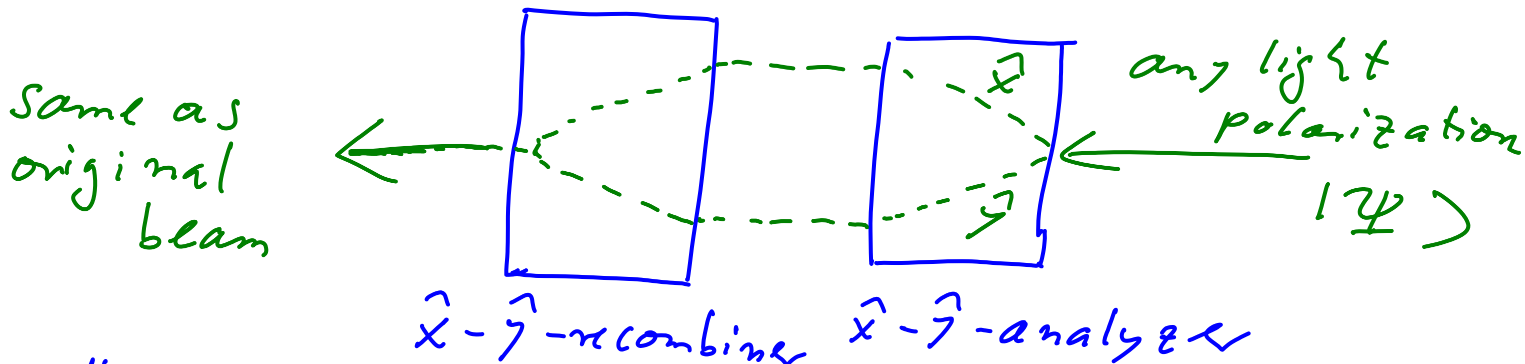
$E_y = \hat{y}\cdot\vec{E} = E\sin\theta$

QM

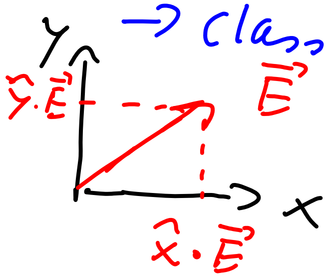
$\langle x|x'\rangle = \cos\theta$

$\langle y|x'\rangle = \sin\theta$

② Linear polarization analyzer loop / change of basis states:



→ Classically:



superposition of 2 waves

$$\vec{E} = \underbrace{(\hat{x}\cdot\vec{E})}_{\text{projection onto x-axis}}\hat{x} + \underbrace{(\hat{y}\cdot\vec{E})}_{\text{projection onto y-axis}}\hat{y}$$

→ Quantum picture:

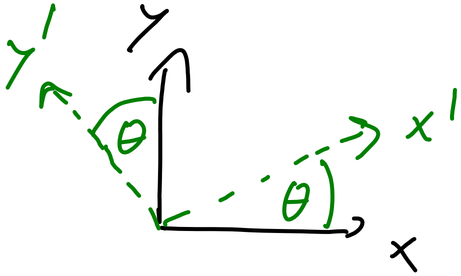
superposition of 2 states

$$|\Psi\rangle = |x\rangle\underbrace{\langle x|\Psi\rangle}_{\text{proj. of } |\Psi\rangle \text{ onto } |x\rangle} + |y\rangle\underbrace{\langle y|\Psi\rangle}_{\text{proj. of } |\Psi\rangle \text{ onto } |y\rangle}$$

$$= \{\underbrace{|x\rangle\langle x| + |y\rangle\langle y|}_{\text{unity operator}}\}|\Psi\rangle$$

recall: $|\alpha\rangle = \sum_n |e_n\rangle\langle e_n|\alpha\rangle$ $\qquad \sum_n |e_n\rangle\langle e_n| = 1$

__Now:__ Rotate analyze loop by angle θ:



$\Rightarrow$ get $x'-y'$ analyze loop

$$\Rightarrow |\Psi\rangle = |x'\rangle\langle x'|\Psi\rangle + |y'\rangle\langle y'|\Psi\rangle$$

$$= \left\{ |x'\rangle\langle x'| + |y'\rangle\langle y'| \right\} |\Psi\rangle$$

$$= \left\{ |x'\rangle\langle x'| + |y'\rangle\langle y'| \right\} \left\{ |x\rangle\langle x|\Psi\rangle + |y\rangle\langle y|\Psi\rangle \right\}$$

$$= |x'\rangle \left\{ \underline{\langle x'|x\rangle}\langle x|\Psi\rangle + \underline{\langle x'|y\rangle}\langle y|\Psi\rangle \right\}$$
$$+ |y'\rangle \left\{ \underline{\langle y'|x\rangle}\langle x|\Psi\rangle + \underline{\langle y'|y\rangle}\langle y|\Psi\rangle \right\}$$

$\Rightarrow$ can represent $|\Psi\rangle$ in terms of $|x\rangle, |y\rangle$ basis states
or $|x'\rangle, |y'\rangle$ " "

$\Rightarrow$ to change basis state, all we need to know are the projection amplitudes between the basis state
$(\langle x'|x\rangle, \langle x'|y\rangle, \ldots)$

recall: general case:

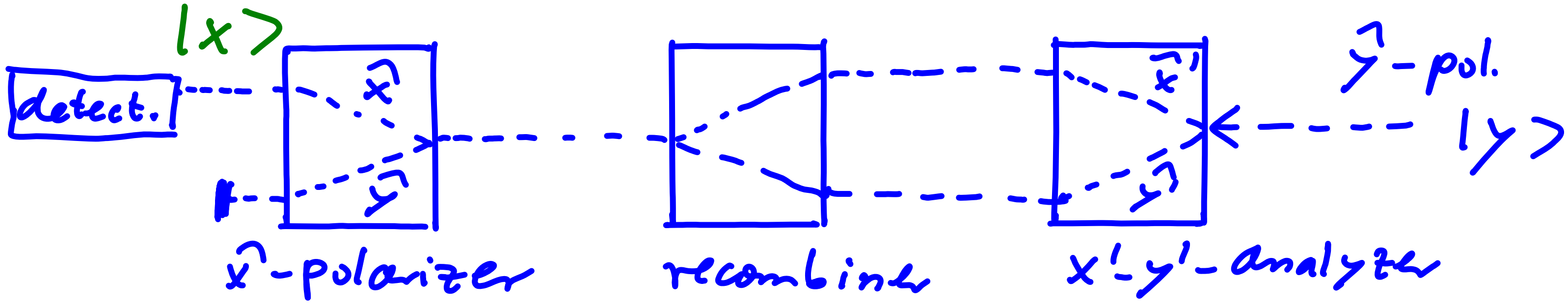
$$|\Psi\rangle = \sum_n |e_n\rangle\langle e_n|\Psi\rangle = \sum_m |e_m'\rangle\langle e_m'|\Psi\rangle$$

$$= \left\{ \sum_m |e_m'\rangle\langle e_m'| \right\} |\Psi\rangle$$

$$= \left\{ \sum_m |e_m'\rangle\langle e_m'| \right\} \left\{ \sum_n |e_n\rangle\langle e_n|\Psi\rangle \right\}$$

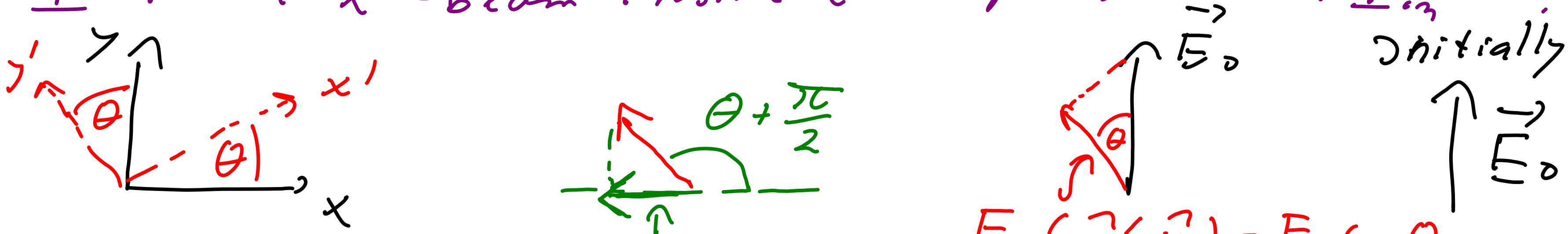
$$= \sum_{m,n} |e_m'\rangle \underbrace{\langle e_m'|e_n\rangle}_{\text{projection amplitudes between basis states!}} \langle e_n|\Psi\rangle$$

③ Example of Interference of Polarization States:



Key idea: In superposition, must add up probability amplitudes, and not probabilities!

I Block $\hat{x}'$-beam inside the loop $\Rightarrow I_{det}/I_{in} = ?$



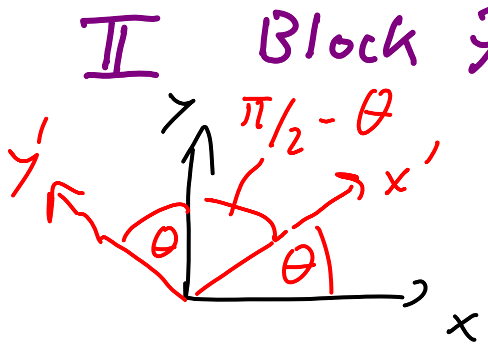
$E_0(\hat{y}'\cdot\hat{y}) = E_0\cos\theta$

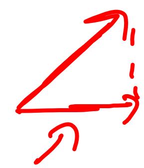
$$E_1 = E_0(\hat{y}'\cdot\hat{y})(\hat{x}\cdot\hat{y}')$$
$$= E_0\cos\theta\cos\left(\theta+\frac{\pi}{2}\right)$$

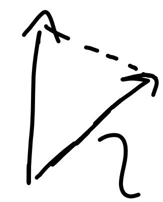
$\Rightarrow$ total transmission probability $= I_{det}/I_{in} = \cos^2\left(\theta+\frac{\pi}{2}\right)\cos^2\theta$

Example: $\theta = 30^\circ \Rightarrow$ transm. prob $= \frac{1}{4}\cdot\frac{3}{4} = \frac{3}{16} > 0$

II Block $\hat{y}'$ beam inside the analyzer loop:







Initially

$\vec{E}_0 = E_0 \hat{y}$

$E_2 = E_0(\hat{x}'\cdot\hat{y})(\hat{x}\cdot\hat{x}')$ $\quad E_0(\hat{x}'\cdot\hat{y}) = E_0\cos\left(\frac{\pi}{2}-\theta\right)$

$= E_0\cos\left(\frac{\pi}{2}-\theta\right)\cos\theta$

$= -E_0\cos\left(\theta+\frac{\pi}{2}\right)\cos\theta = -\{\text{result of case I}\}$

$= -E_1$

$\Rightarrow$ total transmission prob $= \cos^2\theta\cos^2\left(\theta+\frac{\pi}{2}\right) = \frac{I_{det}}{I_{in}}$

for $\theta = 30^\circ \Rightarrow 3/16$

III Both paths open:

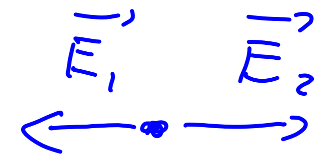
$\Rightarrow$ output of analyzer loop = input = $|y\rangle$

$\Rightarrow$ projection of $\hat{y}$ onto $\hat{x}$ is zero: $\hat{x}\cdot\hat{y} = 0$

$\Rightarrow$ total transmission probability $= 0$

$\Rightarrow$ transmission with both paths open $<$ transmission with one path open only $\Rightarrow$ destructive interference

→ classically:



$$\vec{E}_{total} = \vec{E}_1 + \vec{E}_2 = 0$$

$$= E_0 \left\{ (\hat{x}\cdot\hat{y}')(\hat{y}'\cdot y) + (\hat{x}\cdot\hat{x}')(\hat{x}'\cdot\hat{y}) \right\}$$

⇒ Intensity $\propto |E_{total}|^2 = |\vec{E}_1 + \vec{E}_2|^2 = 0$

← add up vectors and not intensities!

and not $E_1^2 + E_2^2 > 0$

→ Quantum mechanics:

total projection ampl. $= \langle x|y'\rangle\langle y'|y\rangle + \langle x|x'\rangle\langle x'|y\rangle$

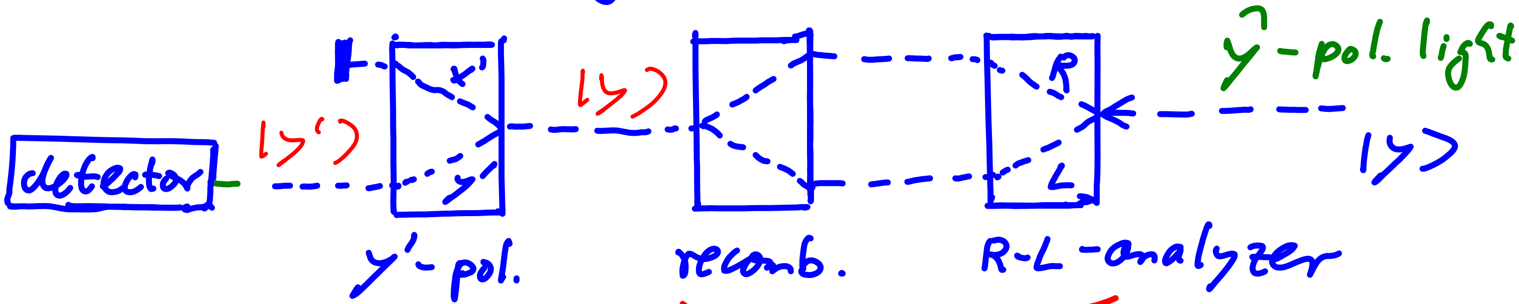
$$= \langle x| \{ \underbrace{|y'\rangle\langle y'| + |x'\rangle\langle x'|}_{\text{unity operator}} \} |y\rangle$$

$$= \langle x|y\rangle = 0$$

⇒ total prob. of measuring a photon at the detector

$$= |\langle x|y\rangle|^2 = 0$$

④ Circular polarized light / complex projection amplitudes



RL analyzer loop

- no intensity lost ⇒ R, L polarization states are a complete set
- R and L circular pol. states are orthogonal:

$$\langle L|R\rangle = \langle R|L\rangle = 0$$

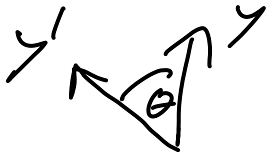
$$\langle R|R\rangle = \langle L|L\rangle = 1$$

⇒ can represent any pol. state in terms of $|R\rangle$ and $|L\rangle$

$$|\Psi\rangle = c_R|R\rangle + c_L|L\rangle = |R\rangle\langle R|\Psi\rangle + |L\rangle\langle L|\Psi\rangle$$

$$= \underbrace{\{|R\rangle\langle R| + |L\rangle\langle L|\}}_{\text{RL-analyzer loop} = 1}|\Psi\rangle$$

here:



$$\langle y'|y\rangle = \cos\theta = \langle y'|\{|R\rangle\langle R| + |L\rangle\langle L|\}\cdot|y\rangle$$

insert RL analyzer loop (unity operator)

$$= \underbrace{\langle y'|R\rangle\langle R|y\rangle}_{\text{path I}} + \underbrace{\langle y'|L\rangle\langle L|y\rangle}_{\text{path II}}$$

$\Rightarrow$ what are the projection amplitudes

$\langle R|y\rangle, \langle L, y\rangle \ldots$?