Lecture 40: **05/01/09**

- Hydrogen radial wave function $R_{n\ell}(r)$
  - energy levels E_n
  - degeneracy
- Full Hydrogen wave function
- Hydrogen fine-structure
- Atoms other than Hydrogen

→ Spherical Harmonics

Recap

→ are eigenfunctions of $\hat{L}^2$:

$$\hat{L}^2 Y_\ell^m(\theta,\phi) = \ell(\ell+1)\hbar^2 Y_\ell^m(\theta,\phi)$$

→ and are eigenfunctions of $\hat{L}_z$ with $\ell = 0, 1, 2, \ldots$

$$\hat{L}_z Y_\ell^m(\theta,\phi) = m\hbar\, Y_\ell^m(\theta,\phi)$$

with $m = -\ell, -\ell+1, \ldots\ldots \ell-1, \ell$

→ Degeneracy due to spherical symmetry:

for each ℓ-value: $(2\ell+1)$ states with different m-value, but same energy

$m = -\ell, -\ell+1 \ldots\ldots +\ell$

→ Spin: angular momentum intrinsic to fundamental particles

total angl. momentum $\vec{J}$ = orb. angl. mom. $\vec{L}$ + spin $\vec{S}$

→ intrinsic:
- not associated with orbital dynamics
- particle does not rotate about its "center of mass"

⇒ total angular momentum of a particle = orbital angular moment. + spin angular momentum

$$\hat{\vec{J}} = \hat{\vec{L}} + \hat{\vec{S}}$$

→ 1925, Uhlenbeck and Goudsmith: postulated that e^- have intrinsic angular momentum (spin), described by operators similar to orbital angular momentum operators:

$$\hat{S}^2 \underbrace{|s\, m_s\rangle}_{\text{spin state}} = \hbar^2 \overset{\uparrow}{\underset{\text{spin quantum \#}}{s}}(s+1)\,|s\, m_s\rangle \qquad \hat{S}_z|s\, m_s\rangle = m_s\hbar\,|s\, m_s\rangle$$

$$[\hat{S}_x, \hat{S}_y] = i\hbar\,\hat{S}_z \ \ldots$$

for electron: $s = \frac{1}{2}$ ⇒ $m_s = -\frac{1}{2}, +\frac{1}{2}$ (not integer!)

(Cesium: $55e^-$: 54 paired such that their spin and orb. ang. momentum cancel; last unpaired e^-: $\ell = 0$, $s = \frac{1}{2}$ $m_s = -\frac{1}{2}, +\frac{1}{2}$

⇒ 2 beams! $\rightsquigarrow$

- Spin 1/2 particles (electron, proton, ...)

$s = \frac{1}{2}$ ⇒ $m_s = \pm 1/2$ allowed ⇒ <u>two</u> spin eigenstates

- "spin up" $|s\ m_s\rangle = |\tfrac{1}{2}\ \tfrac{1}{2}\rangle$ represented by $\chi_+ = \begin{pmatrix}1\\0\end{pmatrix}$
- "spin down" $|s\ m_s\rangle = |\tfrac{1}{2}\ -\tfrac{1}{2}\rangle$ represented by $\chi_- = \begin{pmatrix}0\\1\end{pmatrix}$

↑ spinor

$$\Rightarrow \hat{S}^2\begin{pmatrix}1\\0\end{pmatrix} = s(s+1)\hbar^2\begin{pmatrix}1\\0\end{pmatrix} = \frac{3}{4}\hbar^2\begin{pmatrix}1\\0\end{pmatrix}$$

$$\hat{S}^2\begin{pmatrix}0\\1\end{pmatrix} = s(s+1)\hbar^2\begin{pmatrix}0\\1\end{pmatrix} = \frac{3}{4}\hbar^2\begin{pmatrix}0\\1\end{pmatrix}$$

↑ S^2 operator → matrix! $\hat{S}^2 = \frac{3}{4}\hbar^2\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}$

⇒ similarly:

$$\left.\begin{aligned} \hat{S}_z\begin{pmatrix}1\\0\end{pmatrix} &= +\frac{\hbar}{2}\begin{pmatrix}1\\0\end{pmatrix} \\ \hat{S}_z\begin{pmatrix}0\\1\end{pmatrix} &= -\frac{\hbar}{2}\begin{pmatrix}0\\1\end{pmatrix} \end{aligned}\right\} \hat{S}_z = \frac{\hbar}{2}\underbrace{\begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix}}$$

$\equiv \sigma_z$: Pauli spin matrix

(there are two more: σ_x and σ_y ...)

VI$_6$ Hydrogen radial wave function $R_{nl}(r)$ and energy levels E_n:

→ hydrogen wave function: $\psi(r,\theta,\phi) = R(r)\, Y_l^m(\theta,\phi)$

$$= \frac{u(r)}{r}\, Y_l^m(\theta,\phi)$$

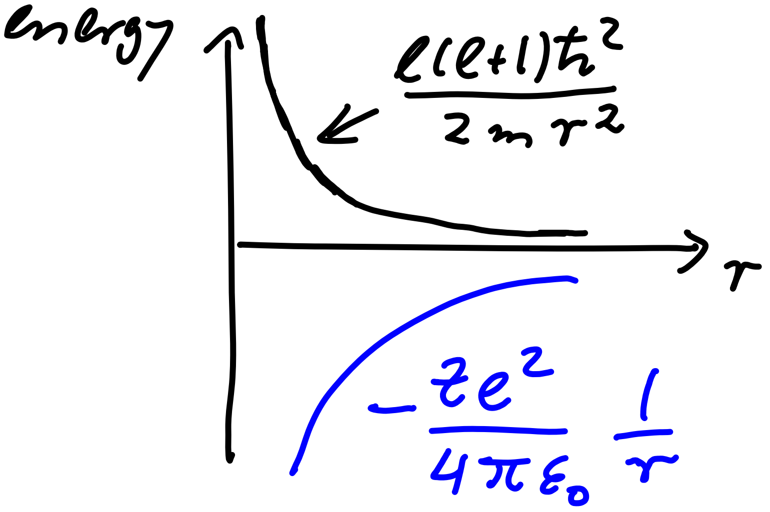
$\underline{\underline{u(r=0) \overset{!}{=} 0}}$

→ radial S.E. ②:

$$-\frac{\hbar^2}{2m}\frac{d^2u(r)}{dr^2} + \underbrace{\left\{\frac{l(l+1)\hbar^2}{2mr^2} - \underbrace{\frac{Ze^2}{4\pi\varepsilon_0}\frac{1}{r}}_{\text{coulomb potential}}\right\}}_{V_{eff}\text{: effective potential}} u(r) = E\,u(r)$$

like 1-D S.E.!

V_{eff} : effective potential with $l = 0, 1, 2, \dots$



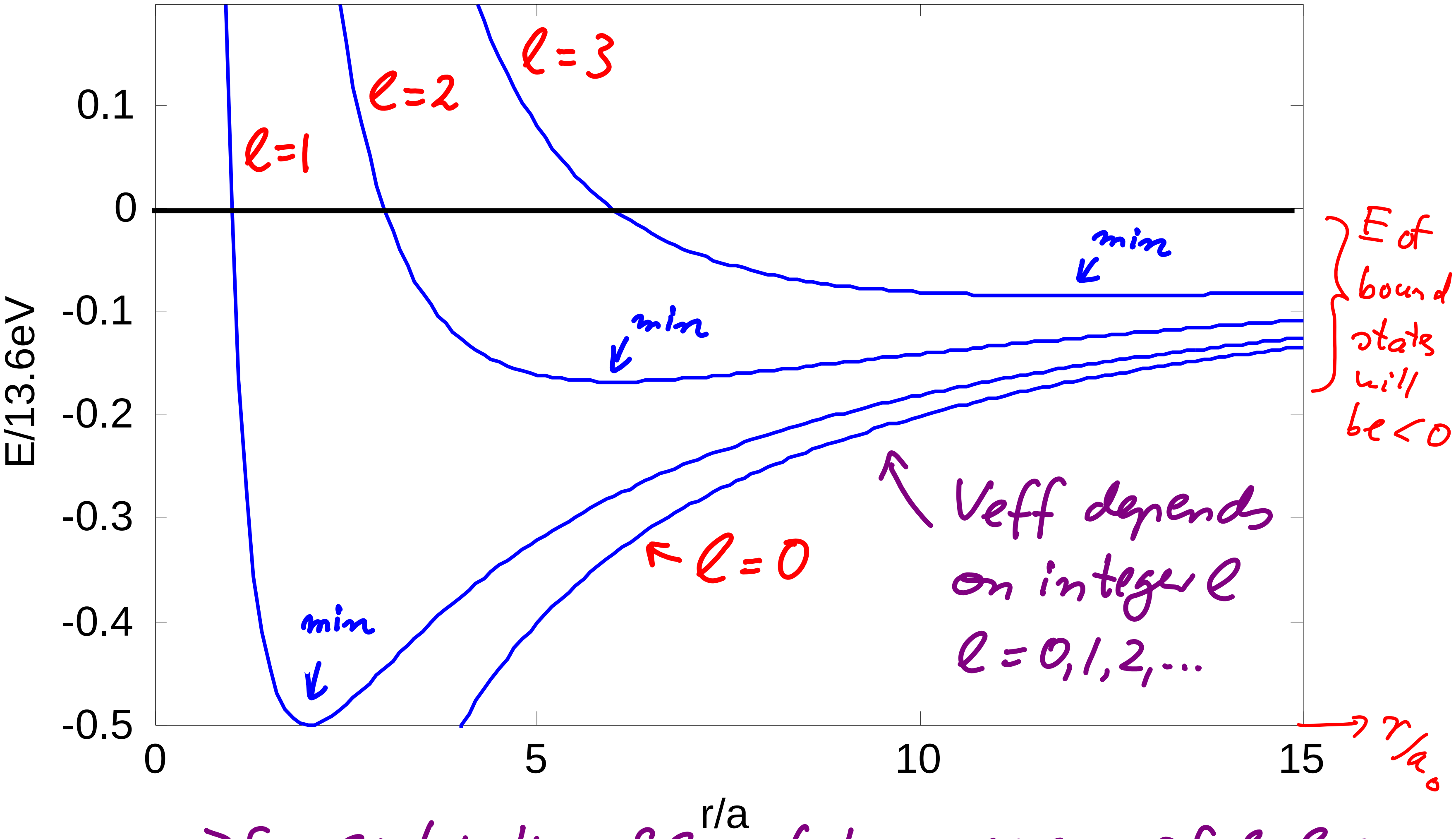
Note: as $r \to 0$:

$\frac{1}{r^2}$ diverges faster than $\frac{1}{r}$

$\Rightarrow V_{eff} \xrightarrow[r\to 0]{} \infty$ for $l > 0$

Effective Potential for Hydrogen:

$$V_{eff} = \frac{\ell(\ell+1)\hbar^2}{2mr^2} - \frac{e^2}{4\pi\varepsilon_0 r}$$



E of bound state will be < 0

Veff depends on integer ℓ

$\ell = 0, 1, 2, \dots$

$\Rightarrow$ for each value of ℓ: whole sequence of energy levels!

- Solve radial equation ② for $u(r)$:

<u>path</u>: (similar to simple harmonic oscillator)

1) study asymptotic behavior for large r and $r \to 0$

2) try: $u(r) = (\text{asymptotic form}) \cdot \begin{pmatrix}\text{power} \\ \text{series in } r\end{pmatrix}$

   ⇒ gives recursion formula for coefficients c_j of power series

3) need to terminate power series to avoid $u(r) \xrightarrow[r\to\infty]{} \infty$

   ⇒ gives E_n : allowed, quantized energies

step 1) Asymptotic behavior:

→ equation ② for large r:

as $r \to \infty$: $V_{eff}(r) \to 0$

$$\Rightarrow \frac{d^2u}{dr^2} \approx -\frac{2m}{\hbar^2} E\, u(r) \qquad \text{(for large } r)$$

← note: $E < 0$ here

$\Rightarrow$ solution: $u(r) \propto e^{-\alpha r}$ with $\alpha = \frac{\sqrt{-2mE}}{\hbar}$

$E < 0$!

[$u(r) \propto e^{+\alpha r}$ is also a solution, but not normalizable, since $e^{+\alpha r} \xrightarrow[r\to\infty]{} \infty$]

→ equation ② for $r \to 0$ (assume $\ell > 0$)

as $r \to 0$: $\frac{1}{r^2}$ - term in V_{eff} dominates over $\frac{1}{r}$ - term or E - term

$$\Rightarrow \frac{d^2 u(r)}{dr^2} \simeq \frac{\ell(\ell+1)}{r^2}\, u(r) \qquad (\text{for } r \to 0)$$

$\Rightarrow$ solution:

$$\underbrace{u(r) \propto r^{\ell+1}}_{u(r=0)=0}$$

$\Rightarrow \psi \propto \frac{u(r)}{r}$ needs to be normalizable!

[$u(r) \propto r^{-\ell}$ is also a solution, but blows up as $r \to 0$]

step 2: peel of the asymptotic behavior:

$$u(r) = r^{\ell+1} e^{-\alpha r} \underbrace{V(r)}_{\text{new function}}$$

(not the potential energy!)

$$\Rightarrow \frac{du(r)}{dr} = r^{\ell} e^{-\alpha r} \left\{ (\ell+1-\alpha r) V(r) + r \frac{dV(r)}{dr} \right\}$$

$$\Rightarrow \frac{d^2u(r)}{dr^2} = r^{\ell} e^{-\alpha r} \left\{ \left[\frac{\ell(\ell+1)}{r} - 2\alpha - 2\alpha\ell + \alpha^2 r \right] V(r) + 2(1+\ell-\alpha r) \frac{dV(r)}{dr} + r \frac{d^2V(r)}{dr^2} \right\}$$

$\Rightarrow$ insert into radial S.E. ②:

$$-\frac{\hbar^2}{2m} \left\{ r \frac{d^2V(r)}{dr^2} + 2(\ell+1-\alpha r) \frac{dV(r)}{dr} + \left[\frac{\ell(\ell+1)}{r} - 2\alpha - 2\alpha\ell + \alpha^2 r \right] \cdot V(r) \right\} + \left\{ \frac{\hbar^2 \ell(\ell+1)}{2mr^2} - \frac{Ze^2}{4\pi\varepsilon_0} \frac{1}{r} - E \right\} r V(r) = 0$$

cancel

cancel

$$\alpha^2 = \frac{-2mE}{\hbar^2}$$

Equation ③:

$$\Rightarrow 0 = r\frac{d^2V(r)}{dr^2} + 2(\ell-1-\alpha r)\frac{dV(r)}{dr} + \left[(-2\alpha)(\ell+1) + \frac{Ze^2 2m}{4\pi\varepsilon_0\hbar^2}\right]V(r)$$

<u>now:</u> express $V(r)$ as power series in r:

$$\boxed{V(r) = \sum_{j=0}^{\infty} c_j r^j}$$

$$\Rightarrow \frac{dv(r)}{dr} = \sum_{j=0}^{\infty} j c_j r^{j-1} = \sum_{j=0}^{\infty} (j+1) c_{j+1} r^j$$

$$\Rightarrow \frac{d^2v(r)}{dr^2} = \sum_{j=0}^{\infty} j(j+1) c_{j+1} r^{j-1}$$

insert into equ. ③ :

$$\sum_{j=0}^{\infty} j(j+1)\, c_{j+1}\, r^{j} + 2(\ell+1)\sum_{j=0}^{\infty} (j+1)\, c_{j+1}\, r^{j}$$

$$\underbrace{-2\alpha \sum_{j=0}^{\infty} (j+1)\, c_{j+1}\, r^{j+1}}_{-2\alpha \sum_{j=0}^{\infty} j\, c_j\, r^{j}} + \left[\frac{Ze^{2}2m}{4\pi\varepsilon_0\hbar^{2}} - 2\alpha(\ell+1)\right]\sum_{j=0}^{\infty} c_j\, r^{j} \underline{= 0}$$

↖ for all $\underline{\underline{r}}$

⇒ coefficients of <u>each</u> power of r must vanish!

$$\Rightarrow j(j+1)\, \underset{\sim}{c_{j+1}} + 2(\ell+1)(j+1)\, \underset{\sim}{c_{j+1}} - 2\alpha j\, \underset{\sim}{c_j}$$

$$+ \left[\frac{Ze^{2}2m}{4\pi\varepsilon_0\hbar^{2}} - 2\alpha(\ell+1)\right] \underset{\sim}{c_j} = 0$$

$\Rightarrow$ recursion formula:

$$\boxed{c_{j+1} = \left\{ \frac{2\alpha(j+\ell+1) - \frac{Ze^2 2m}{4\pi\varepsilon_0 \hbar^2}}{(j+1)(j+2\ell+2)} \right\} c_j}$$

$\Rightarrow$ start with c_0 (to be fixed later by normalization) $\Rightarrow c_1 \Rightarrow c_2 \Rightarrow \ldots$

step3: As for the simple harmonic oscillator: need to terminate the power series at some $j = j_{max}$ to avoid that the wave function blows up ($u(r) \xrightarrow[r\to\infty]{} \infty$)

$\Rightarrow$ require: $\boxed{c_{j_{max}+1} = 0}$

Proof that this is required:

- for large j (these terms dominate at large r)

$$c_{j+1} \simeq \frac{2j\alpha}{j(j+1)} c_j = \frac{2\alpha}{j+1} c_j$$

- suppose this were exact:

$$\Rightarrow c_j = \frac{(2\alpha)^j}{j!} \underline{c_0}$$

$$\Rightarrow v(r) = \sum_{j=0}^{\infty} c_0 \frac{(2\alpha)^j}{j!} r^j = c_0\, e^{2\alpha r}$$

$$\Rightarrow u(r) = c_0\, r^{\ell+1} e^{-\alpha r} e^{2\alpha r} = c_0\, r^{\ell+1} \underline{\underline{e^{\alpha r}}} \text{ for large } r$$

which blows up at large r $\Rightarrow$ not normalizable

$\Rightarrow$ power series must terminate!

$$\Rightarrow \quad C_{j_{max}+1} = 0$$

$$\Rightarrow \quad 2\alpha\left(j_{max} + \ell + 1\right) - \frac{Ze^2 2m}{4\pi\epsilon_0\hbar^2} = 0$$

define "principle quantum number" n:

$$\boxed{n \equiv j_{max} + \ell + 1} \qquad n = \underline{\underline{1}}, 2, 3, \dots$$

$$\Rightarrow \quad 2\alpha n = \frac{Ze^2 2m}{4\pi\epsilon_0\hbar^2} = 2\underbrace{\frac{\sqrt{-2mE}}{\hbar}}_{\alpha} n$$

mass of e^-

$$\Rightarrow \left\| \; E_n = -\left(\frac{Ze^2}{4\pi\varepsilon_0}\right)^2 \frac{\overset{\swarrow\,\text{mass!}}{m}}{2\hbar^2}\,\frac{1}{n^2} \; \right\|$$

} allowed energy values for stationary states

$n = 1, 2, 3, \ldots$

⇒ quantized!

with Bohr radius $a_0 = \dfrac{4\pi\varepsilon_0\hbar^2}{mZe^2}$

$$\Rightarrow \quad E_n = -\frac{Ze^2}{4\pi\varepsilon_0\, 2a_0}\,\frac{1}{n^2} = -13.6\,\text{eV}\,\frac{1}{n^2} \quad \text{for } Z=1$$

⇒ same as in Bohr model!

⇒ recursion formula:

$$c_{j+1} = \frac{2\alpha(j+\ell+1-n)}{(j+1)(j+2\ell+2)}\, c_j$$

} depends on n and ℓ
⇒ so will the radial function $R_{n\ell}(r) = u_{n\ell}(r)/r$

- Energy-level chart and energy degeneracy:

$$E_n = -\left(\frac{Ze^2}{4\pi\varepsilon_0}\right)^2 \frac{m}{2\hbar^2}\,\frac{1}{n^2} = -\left(\frac{Ze^2}{4\pi\varepsilon_0}\right)^2 \frac{m}{2\hbar^2}\,\frac{1}{(j_{max}+\ell+1)^2}$$

(n, ℓ, m) representation | (j_{max}, ℓ, m) representation

Principle quantum number:

$$n \equiv j_{max} + \ell + 1$$

$\Rightarrow$ $n \geq \ell + 1$

$\Rightarrow$ $\boxed{\ell \leq n-1}$

$\Rightarrow$ same E_n for all states with different $\ell \leq n-1$ for given n

$\Rightarrow$
$n = 1, 2, 3, \ldots$
$\ell = 0, 1, 2, \ldots\, n-1$
$m = -\ell, -\ell+1, \ldots, +\ell$

quantum numbers:

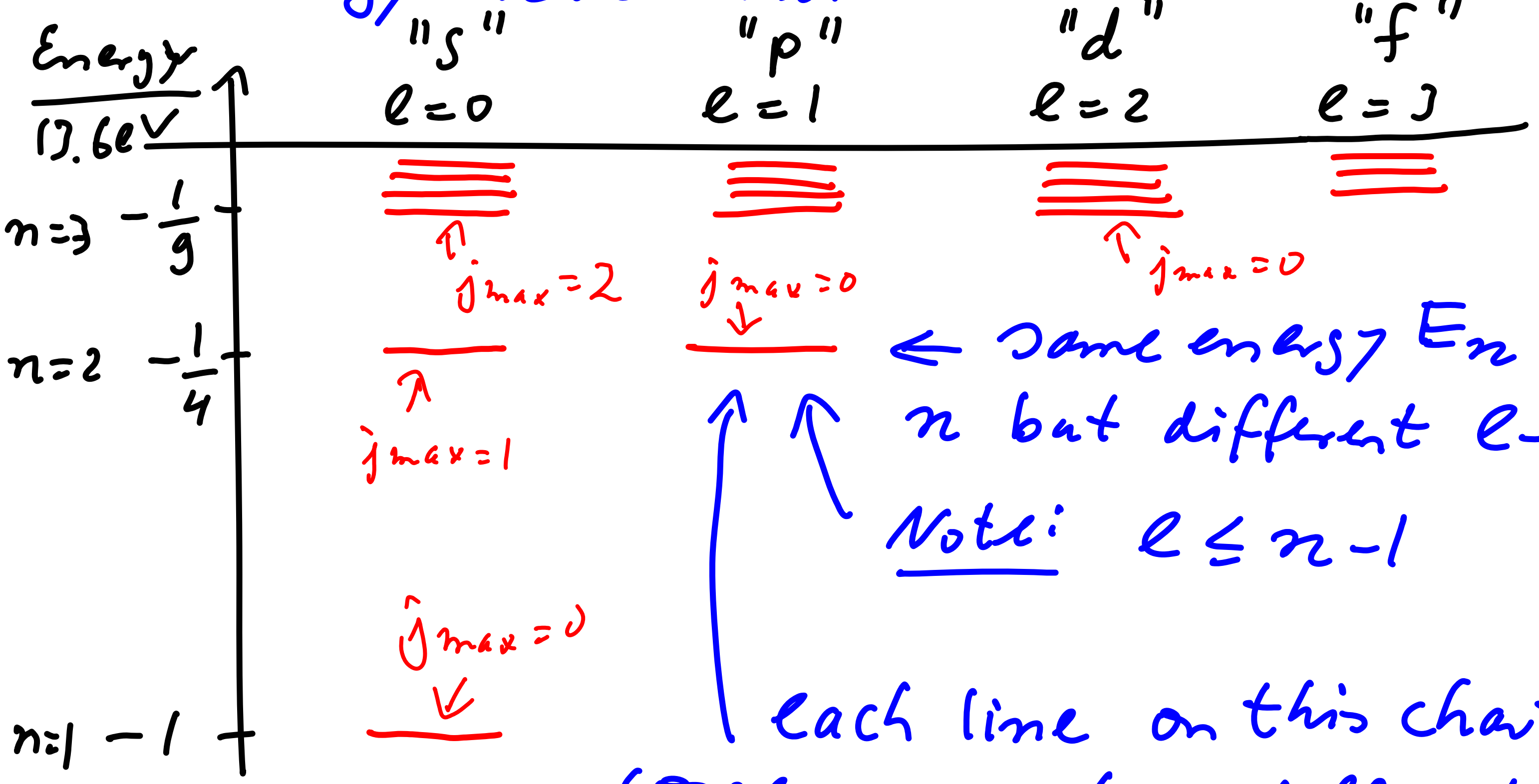
$j_{max} = 0, 1, 2, \ldots$
$\ell = 0, 1, 2, \ldots$
$m = -\ell, -\ell+1, \ldots +\ell$

$\Rightarrow$ whole series of energy values for each ℓ-value

$\Rightarrow$ degeneracy: lowest energy state with $\ell = N$ has same energy as $(N+1)^{st}$ state with $\ell = 0, \ldots$

$\Rightarrow$ this is a special feature of the $1/r$ Coulomb potential!

Energy-level chart:



← same energy E_n for given n but different ℓ-values

Note: $\ell \leq n-1$

each line on this chart corresponds to a different solution of the radial S.E.

$\Rightarrow$ different radial wave functions:

$$R_{n\ell}(r) = \frac{u_{n\ell}(r)}{r}$$

What is the total degeneracy of each energy level E_n in Hydrogen?

for given n:

allowed ℓ's	orbital degeneracy of each ℓ: $2\ell+1$ m-values! $(-\ell \dots +\ell)$
0	1
1	3
2	5
⋮	
$n-1$	$2(n-1)+1$

total: $\displaystyle\sum_{\ell=0}^{n-1}(2\ell+1) = \underline{\underline{n^2}}$

in addition: electron can have spin up $(m_s = +\frac{1}{2})$ or spin down $(m_s = -\frac{1}{2})$

$\Rightarrow$ total degeneracy for given E_n: $\underline{\underline{E_n : 2n^2}}$

- radial wave function

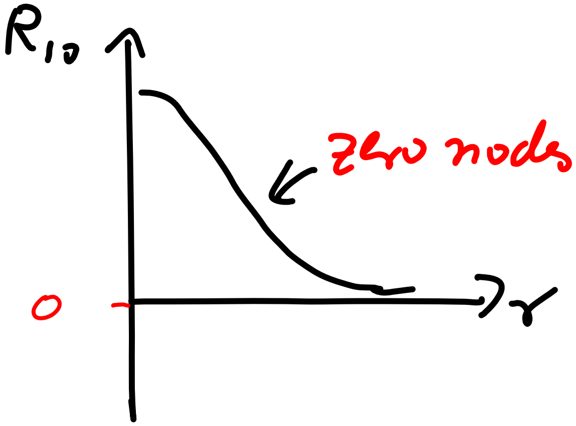
$$R_{n\ell}(r) = \frac{u_{n\ell}(r)}{r} = r^{\ell} e^{-\alpha r} \sum_{j=0}^{\infty} c_j r^j$$

with c_j from recursion formula

Examples:

→ $n=1,\ \ell=0$:

$$R_{10}(r) = 2\, a_0^{-3/2}\, e^{-r/a_0}$$



Ground state!

$\langle r\rangle_{n=1,\ell=0}$

$R_{n\ell}(r)\cdot Y_\ell^m(\theta,\phi)$

↓

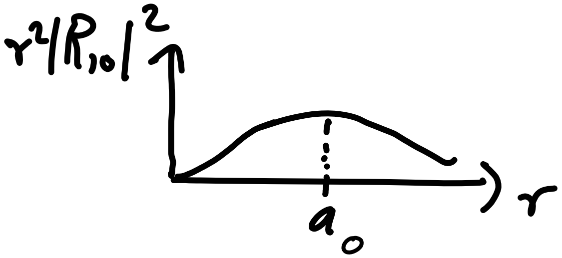
$$= \int_{\text{all space}} d\theta\, d\phi\, dr\, \sin\theta\, r^2\, \psi_{100}^{*}\, r\, \psi_{100}$$

$$= 1 \cdot \int_0^{\infty} dr\, r^2\, r\, |R_{10}(r)|^2$$

$$= \frac{3}{2} a_0 \neq \text{radius in Bohr model}$$

(a_0 for $n=1$)

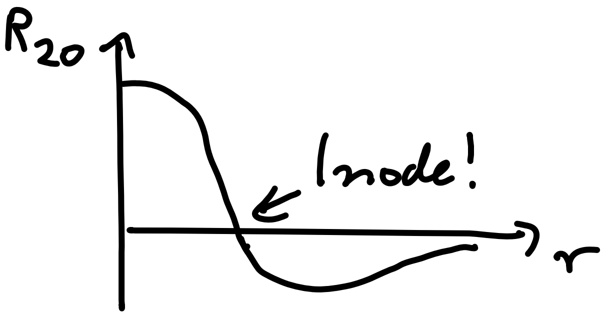
Can think about $r^2|R_{n\ell}(r)|^2 = |u(r)|^2$ as radial probability density. (= probability of measuring electron between r and $r+dr$)



$r^2|R_{10}|^2$ has maximum at $r = a_0$ ...

→ $n=2,\ \ell=0$:

$$R_{20} = \frac{1}{\sqrt{2}}\, a^{-3/2}\left(1-\frac{1}{2}\frac{r}{a_0}\right)e^{-r/2a_0}$$

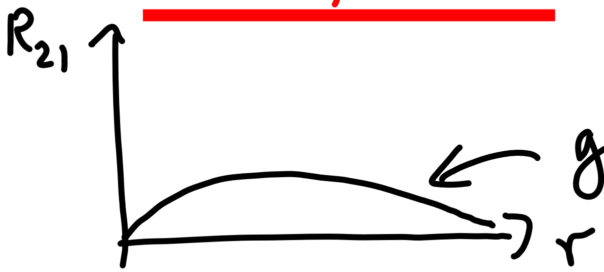


$\langle r \rangle_{20} = 6a_0 \neq 4a_0$ as in Bohr model

$r^2|R_{20}|^2$ has maximum at $(3+\sqrt{5})a_0$

→ $n=2,\ \ell=1$

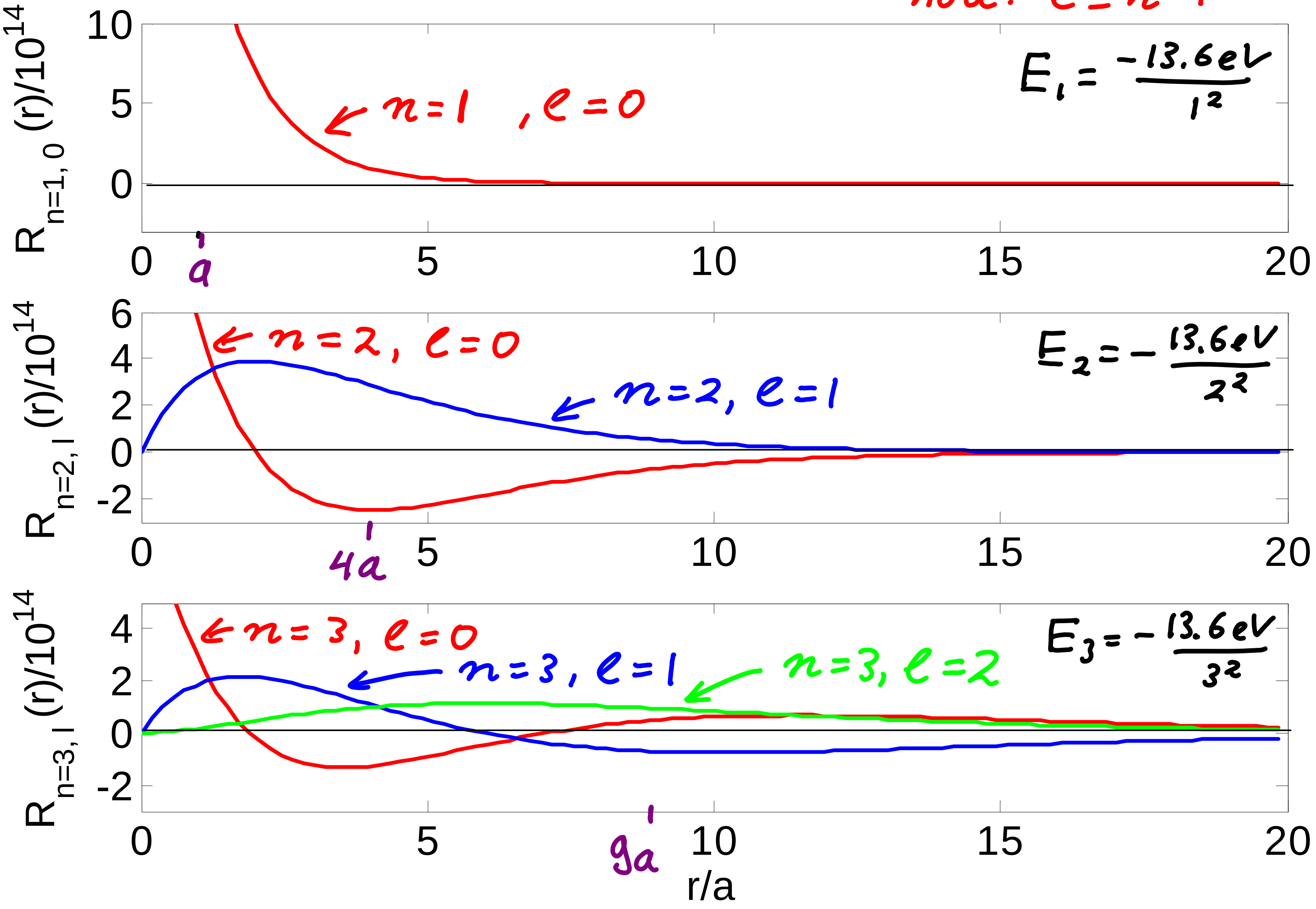
$$R_{21} = \frac{1}{\sqrt{24}}\, a_0^{-3/2}\,\frac{r}{a_0}\, e^{-r/2a_0}$$



ground state for $\ell=1$ effective potential ⇒ zero nodes!

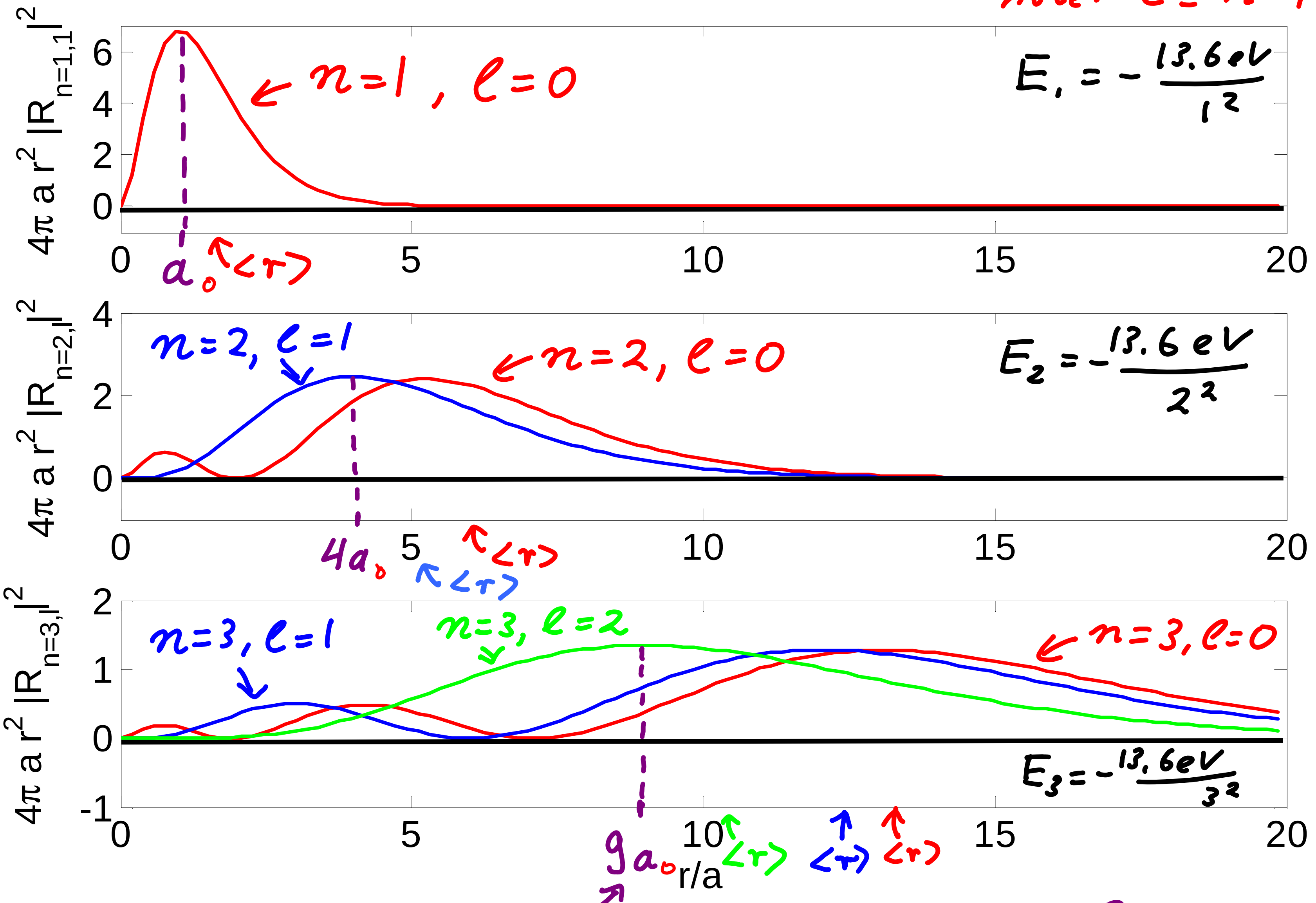
Hydrogen radial wave functions: $R_{n\ell}(r)$

note: $\ell \le n-1$



Hydrogen radial probability distributions: $4\pi r^2 |R_{n\ell}(r)|^2$

note: $\ell \le n-1$



Bohr model radius for n=3

- Full wave function for the electron in the Hydrogen atom:

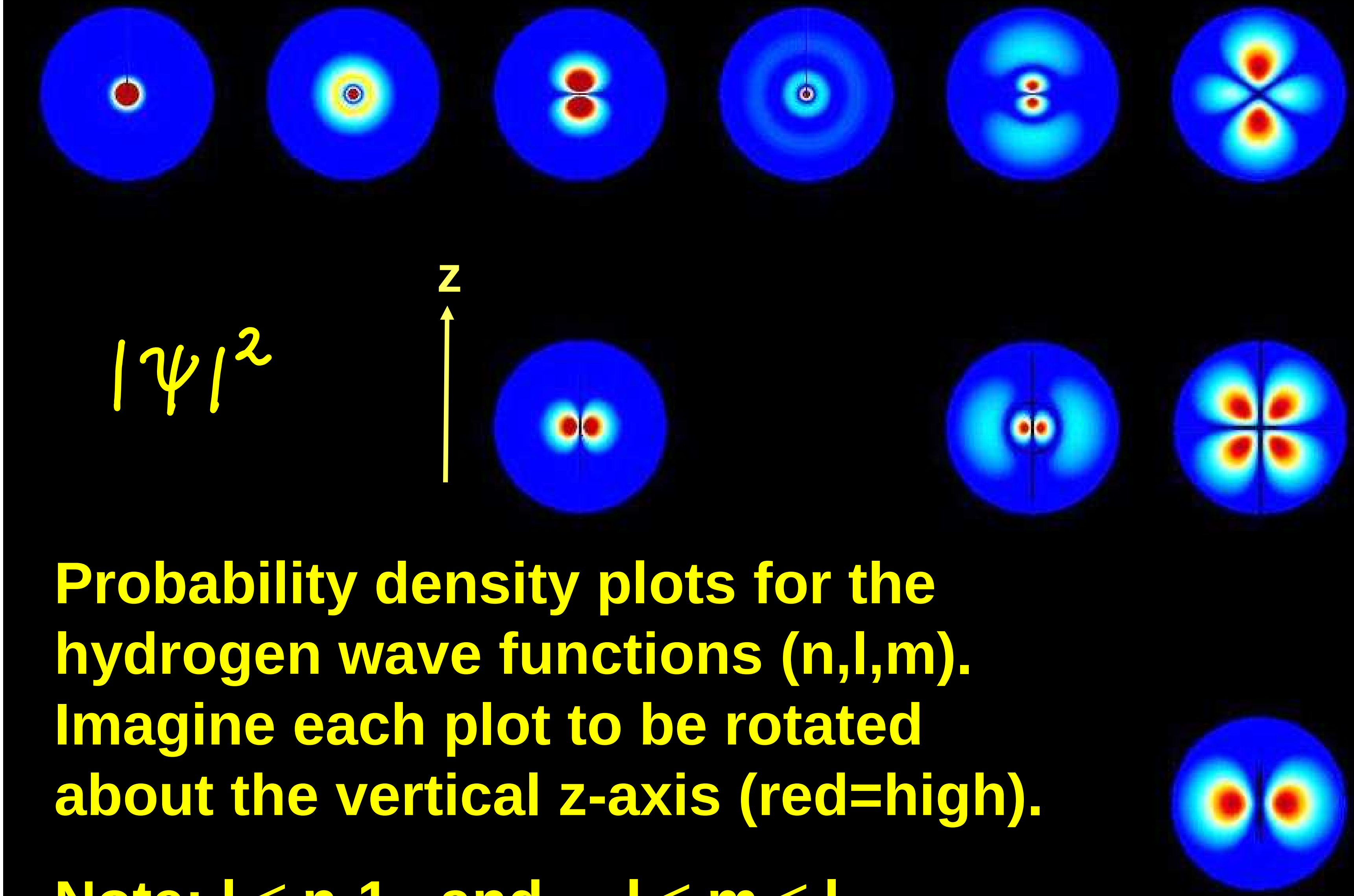
stationary states: $\psi_{n\ell m}(r,\theta,\phi) = R_{n\ell}(r)\, Y_\ell^m(\theta,\phi)$

with $n = 1, 2, 3, \ldots$ $\qquad \ell \leq n-1$

$m = -\ell, -\ell+1, \ldots +\ell$ and $E_n = -\frac{13.6\,\text{eV}}{n^2}$

Note: full quantum mechanical theory solves problems of the Bohr model:

- no patchwork between classical physics and some postulates
- electron does not orbit around the nucleus: stationary probability density! no need for the electron to radiate in the ground state as it should in the Bohr model (due to acceleration!)
- correct angular momentum: $L = 0$ in ground state ($n=1, \ell=0, m=0$) not $L = \hbar$ as in Bohr model



n=1 l=0 | n=2 l=0 | n=2 l=1 | n=3 l=0 | n=3 l=1 | n=3 l=2

m=0

z

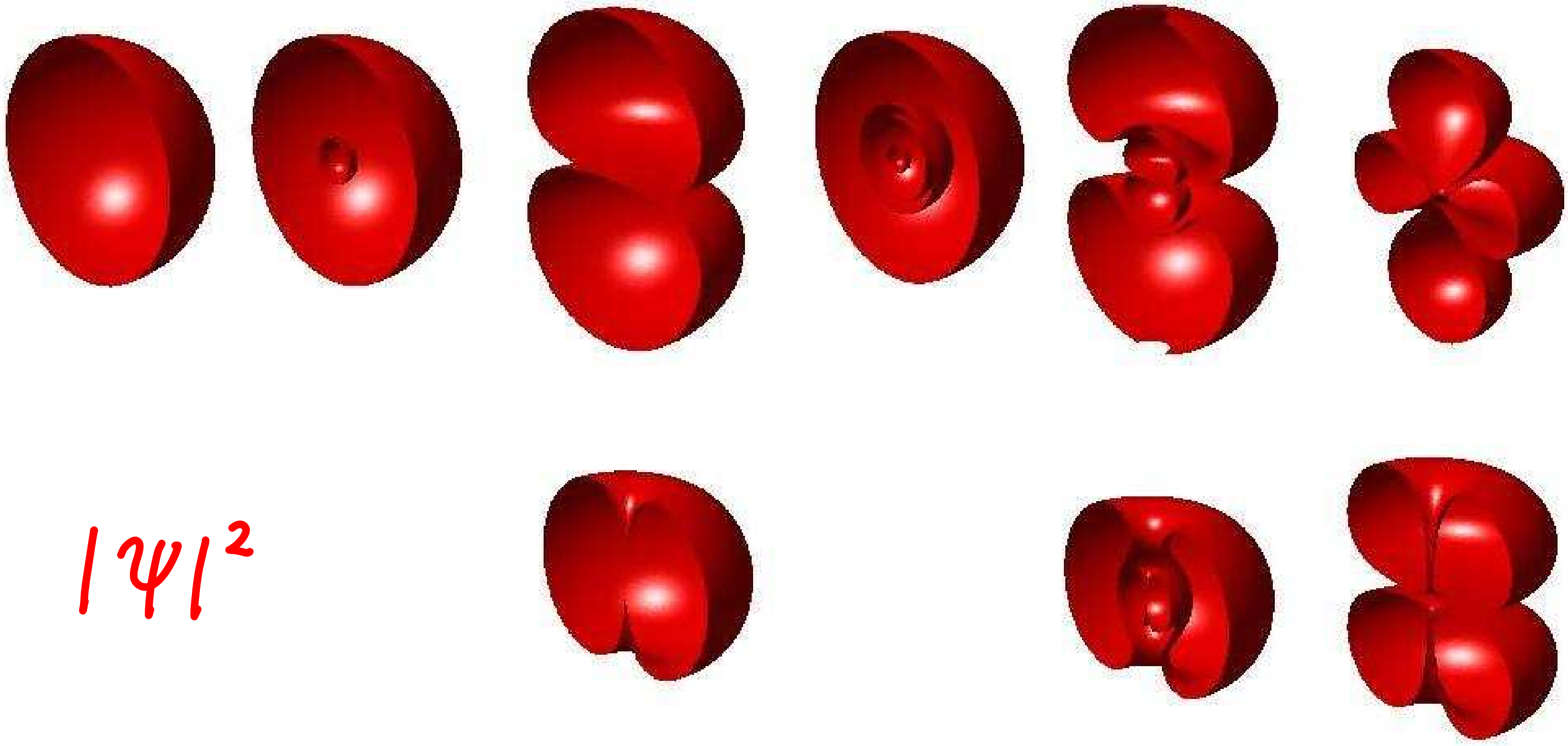
$|\psi|^2$

m=±1

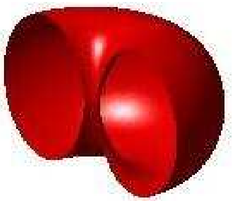
m=±2

Probability density plots for the hydrogen wave functions (n,l,m). Imagine each plot to be rotated about the vertical z-axis (red=high).

Note: l ≤ n-1   and   –l ≤ m ≤ l

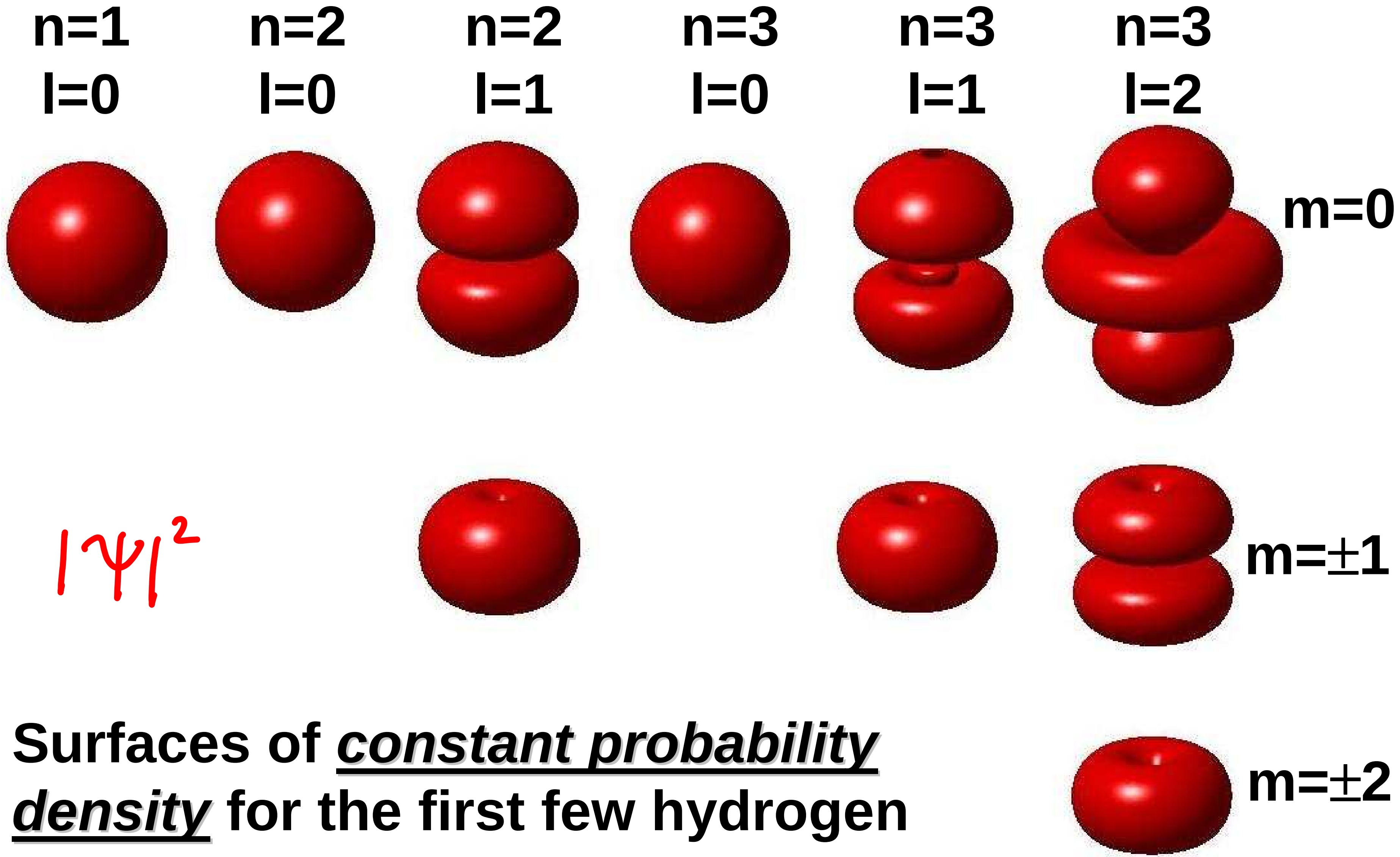


$|\psi|^2$



Surfaces of *constant probability density* for the first few hydrogen wave functions (cutaway view)

Note: $l \leq n-1$ and $-l \leq m \leq l$



$|\psi|^2$

Surfaces of *constant probability density* for the first few hydrogen wave functions

Note: l ≤ n-1   and   –l ≤ m ≤ l

VI$_7$ Outlook

- Are these states of the Hydrogen atom exactly right?

$\Rightarrow$ No! (but quite close...)

$\rightarrow$ There are several small corrections:

1) there is a weak spin-orbit interaction in addition to the Coulomb force:

$$E_{S.O} \propto \hat{L}\cdot\hat{S} = \hat{L}_x\hat{S}_x + \hat{L}_y\hat{S}_y + \hat{L}_z\hat{S}_z$$

$\Rightarrow$ energy operator:

$$\hat{E} = \frac{\hat{p}_r^2}{2m} + \frac{\hat{L}^2}{2mr^2} - \frac{Ze^2}{4\pi\varepsilon_0 r} + A\hat{L}\cdot\hat{S}$$

$\Rightarrow$ $\hat{E}$ and $\hat{L}^2$ still commute, but $\hat{E}$ and $\hat{L}_z$ or $\hat{S}_z$ no longer commute

but: total angular momentum in z-direction

$J_z = L_z + S_z$ and $\hat{J}^2$ both still commute with $\hat{E}$

$\Rightarrow$ label stationary states by E, L^2, J^2 and J_z-value!

2) in addition: weak interaction between the angular momentum of the electron and the spin of the nucleus ⇒ hyper-fine structure

3) relativistic corrections (Schrödinger-equ. is non-relativistic!)

• Atoms other than Hydrogen:

⇒ more complicated (see Phys 317...)

- 2 or more electrons ⇒ $\psi = \psi(\underset{\overset{\uparrow}{\text{electron 1}}}{\vec{r}_1}, \underset{\nwarrow \text{electron 2}}{\vec{r}_2}, \dots)$
- need to include electron-electron interaction
  ⇒ dynamics of different electrons are correlated
- electrons are indistinguishable particles!
  ⇒ wave function has to have certain symmetry properties:

$$|\psi(\vec{r}_1, \vec{r}_2)|^2 = |\psi(\vec{r}_2, \vec{r}_1)|^2$$

prob. density can not depend on which electron is labeled 1, and which is 2

⇒ for fermions, like electrons: $\psi(\vec{r}_1, \vec{r}_2) = -\psi(\vec{r}_2, \vec{r}_1)$

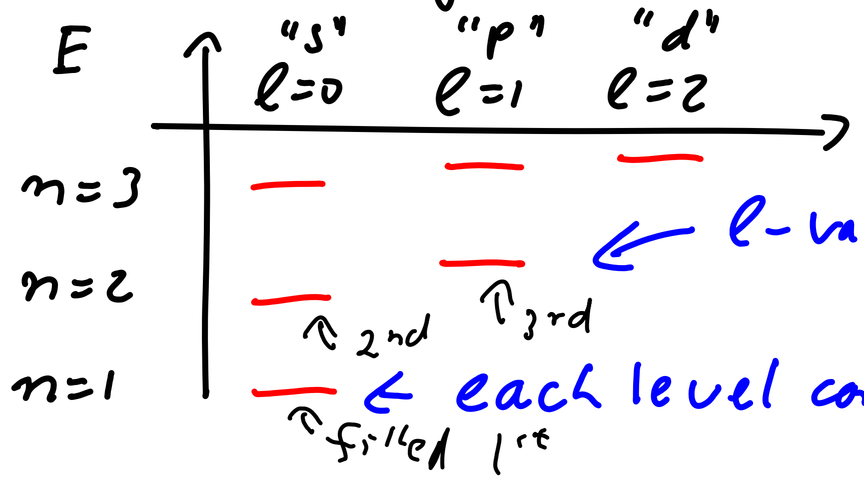
→ can make some simple approximations to get a rough estimate:

(1) compute energy levels of <u>one</u> electron assuming an average time-independent interaction with all other electrons in the atom ⇒ effective potential for 1-electron: $V(r) \propto \begin{cases} e^2/r & \text{far from nucleus} \\ Ze^2/r & \text{very close to nucleus} \end{cases}$

(2) indistinguishable particles ⇒

Pauli exclusion principle: at most one electron in any (n, ℓ, m, m_s) level

⇒ can draw approximate energy-level diagram for each atom:



← ℓ-value degeneracy broken!

← each level can be filled with up to $\underset{\text{spin}}{2}\,\underset{m\text{-values}}{(2\ell+1)}\ e^-$

Alkali Atom Energy Levels:

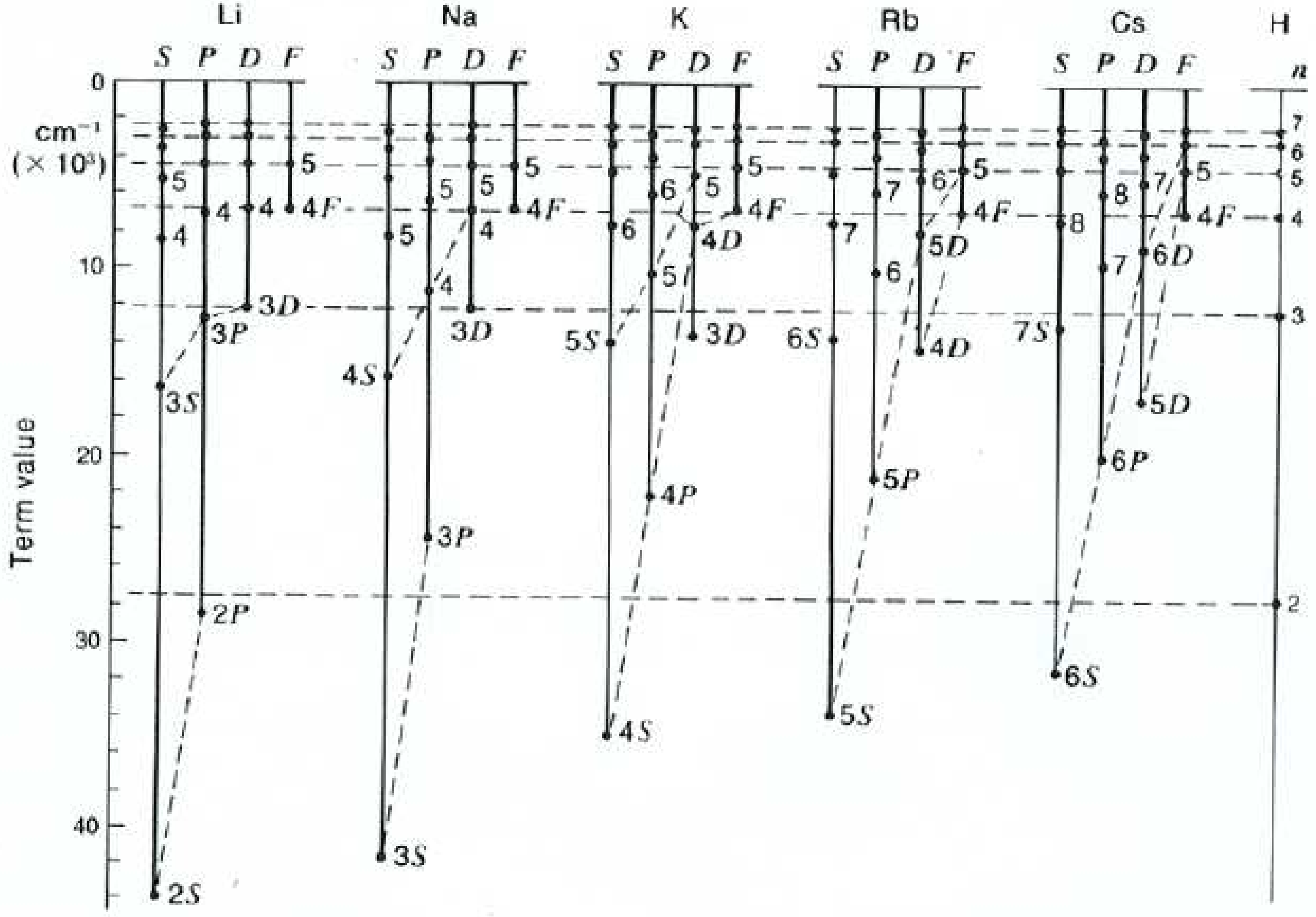


Fig. 12-12 Comparison of alkali atom energy levels with energy levels for hydrogen (right-hand column). Notice that for higher angular momentum states the energies approach those of the hydrogen atom for the same principal quantum number. (After F. K. Richtmeyer, E. H.

→ Fill pattern of outermost electrons:

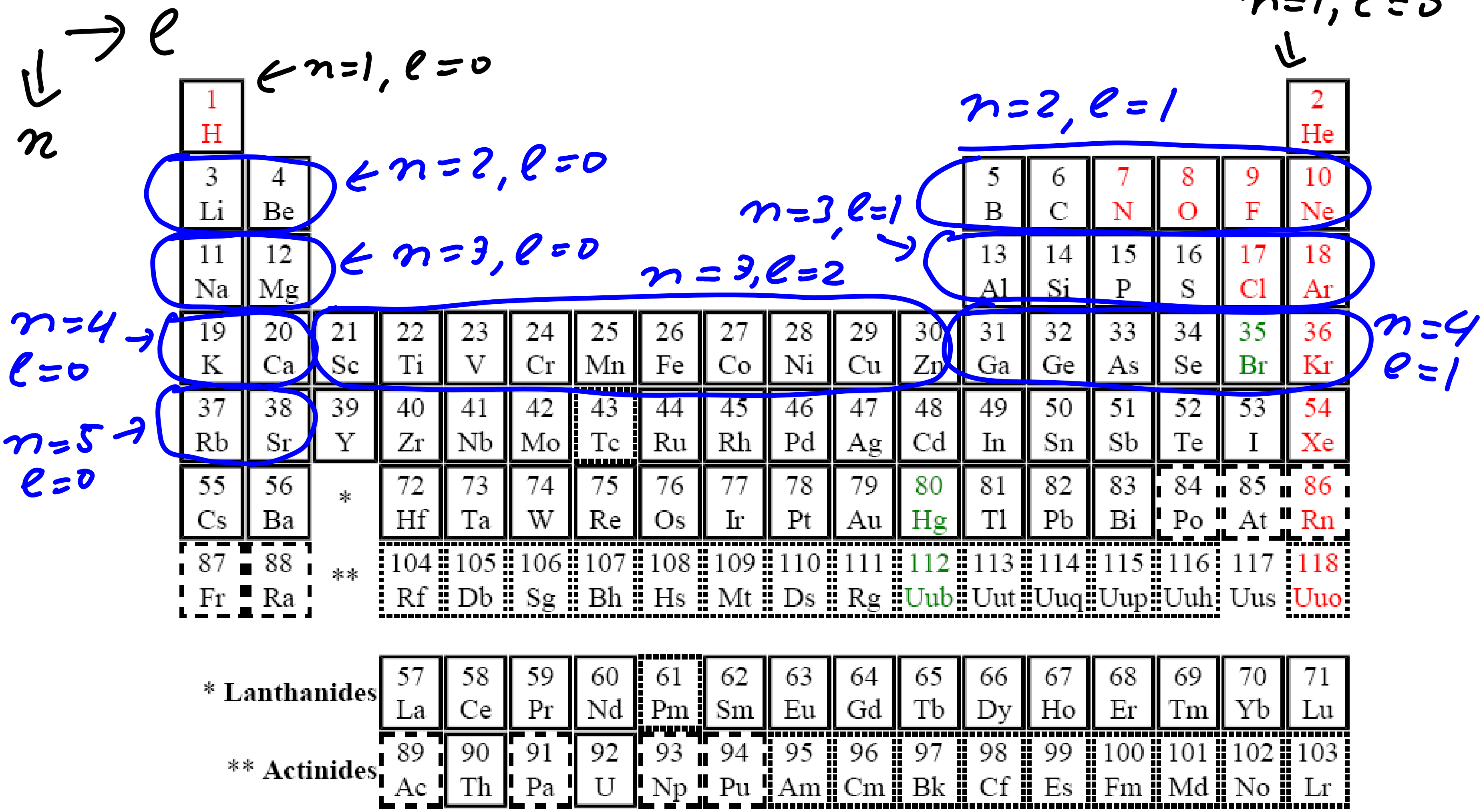
fill ↓ first	$n=1$, $\ell=0$	up to $2e^-$	H, He
	$n=2$, $\ell=0$	" " $2e^-$	Li, Be
	$n=2$, $\ell=1$	" " $6e^-$	B .... Ne
	$n=3$, $\ell=0$	" " $2e^-$	Na, Mg
	$n=3$, $\ell=1$	" " $6e^-$	Al .... Ar
next:	$n=4$, $\ell=0$	" " $2e^-$	K, Ca

(has lower energy generally than $n=3, \ell=2$)

↓	$n=3$, $\ell=2$	" " $10e^-$	Sc ... Zn
	$n=4$, $\ell=1$	" " $6e^-$	
	$n=5$, $\ell=0$	" " $2e^-$	
	$n=4$, $\ell=2$	" " $10e^-$	
	$n=5$, $\ell=1$	" " $6e^-$	

↖ periodic table reflects this...

Periodic table and n,l values of “outermost” e$^-$



1 H																	2 He
3 Li	4 Be											5 B	6 C	7 N	8 O	9 F	10 Ne
11 Na	12 Mg											13 Al	14 Si	15 P	16 S	17 Cl	18 Ar
19 K	20 Ca	21 Sc	22 Ti	23 V	24 Cr	25 Mn	26 Fe	27 Co	28 Ni	29 Cu	30 Zn	31 Ga	32 Ge	33 As	34 Se	35 Br	36 Kr
37 Rb	38 Sr	39 Y	40 Zr	41 Nb	42 Mo	43 Tc	44 Ru	45 Rh	46 Pd	47 Ag	48 Cd	49 In	50 Sn	51 Sb	52 Te	53 I	54 Xe
55 Cs	56 Ba	*	72 Hf	73 Ta	74 W	75 Re	76 Os	77 Ir	78 Pt	79 Au	80 Hg	81 Tl	82 Pb	83 Bi	84 Po	85 At	86 Rn
87 Fr	88 Ra	**	104 Rf	105 Db	106 Sg	107 Bh	108 Hs	109 Mt	110 Ds	111 Rg	112 Uub	113 Uut	114 Uuq	115 Uup	116 Uuh	117 Uus	118 Uuo

*** Lanthanides**	57 La	58 Ce	59 Pr	60 Nd	61 Pm	62 Sm	63 Eu	64 Gd	65 Tb	66 Dy	67 Ho	68 Er	69 Tm	70 Yb	71 Lu
**** Actinides**	89 Ac	90 Th	91 Pa	92 U	93 Np	94 Pu	95 Am	96 Cm	97 Bk	98 Cf	99 Es	100 Fm	101 Md	102 No	103 Lr

The End