Lecture 7: 02/02/09

- More on Bohr's Model of the Atom
- Wave Properties of Particles
  - De Broglie's Hypothesis

Louis de Broglie (1892 – 1987):
Nobel Prize for Physics (1929)

Recap:

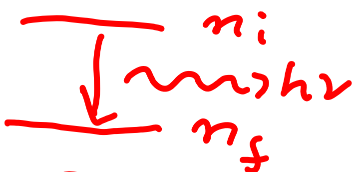
I$_{2,1}$ Evidence for quantized energy levels in atoms:

Franck-Hertz experiment, x-ray spectra, e^- scattering on He...

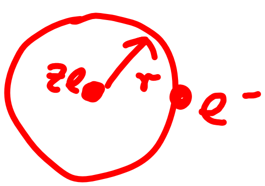
$\Rightarrow$ Particle confined into small volume $\Rightarrow$ Quantized Energy

I$_{2,2}$ The Bohr Atom (1913)

e^- in circular orbit; does not radiate!



$E_{photon} = h\nu = E_i - E_f$



Quantized Energy Levels: $E_n = -\frac{1}{8\pi\varepsilon_0}\frac{Ze^2}{a_0 n^2}$

Quantized Radii: $r_n = a_0 n^2$

"correspondence principle": $\Rightarrow$ $a_0 = \dfrac{h^2(4\pi\varepsilon_0)}{(2\pi)^2 m Z e^2}$

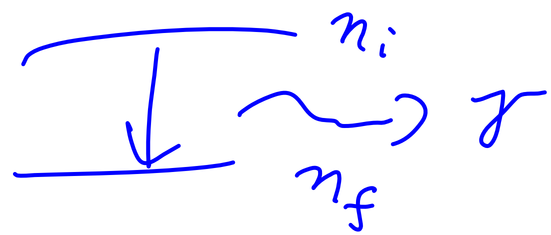
In the Bohr atom, an electron radiates …

~~A.~~ **when accelerating in its orbit around the nucleus**

B. during transition between orbits

C. both of the above

D. neither of the above



- **Bottom Line: Bohr Atom**

Quantized Energy: $E_n = -\frac{1}{8\pi\varepsilon_0}\frac{Ze^2}{a_0 n^2} = -13.6\,eV \cdot \frac{Z^2}{n^2}$

$n = 1, 2, 3, \ldots$

Quantized Radii: $r_n = a_0 n^2$; $a_0 = \frac{4\pi\varepsilon_0 \hbar^2}{4\pi^2 m Z e^2}$

- good prediction for spectra of 1-electron atoms and ions (H, He^+, Li^{2+} ....)
- But: Does not explain detaileds in spectrum
- But: Arbitrary quantization to fit data
- But: Bohr model can not be expanded to atoms with more than one e^-

$\Rightarrow$ Why quantized values?

- $\underline{4^{th}}$: Consider angular momentum of electron in Bohr atom:

$L = mvr \Rightarrow$ for quantized orbits:

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$$L_n^2 = (m v r_n)^2 = m(mv^2) r_n^2 = m\left(\frac{1}{4\pi\varepsilon_0} \frac{Ze^2}{r_n}\right) r_n^2$$

$$= m \frac{1}{4\pi\varepsilon_0} Z e^2 r_n \leftarrow r_n = a_0 n^2 = \frac{4\pi\varepsilon_0 h^2}{(2\pi)^2 m Z e^2} n^2$$

$$\Rightarrow L_n^2 = \cancel{m} \frac{1}{\cancel{4\pi\varepsilon_0}} \cancel{Z e^2} \frac{\cancel{4\pi\varepsilon_0} h^2}{(2\pi)^2 \cancel{m} \cancel{Z e^2}} n^2 = \frac{h^2}{(2\pi)^2} n^2$$

angular momentum: $L_n = \frac{h}{2\pi} n = \hbar n \qquad n = 1, 2, 3, \ldots$

with $\hbar = h/2\pi$

$\Rightarrow$ Angular momentum is quantized in Bohr atom!

$\Rightarrow L_1 = \hbar$, but actually $L_1 = 0 \ldots$

$\Rightarrow$ Conclusion:

- Particles confined to small volume (like atom, nucleus, molecule...)

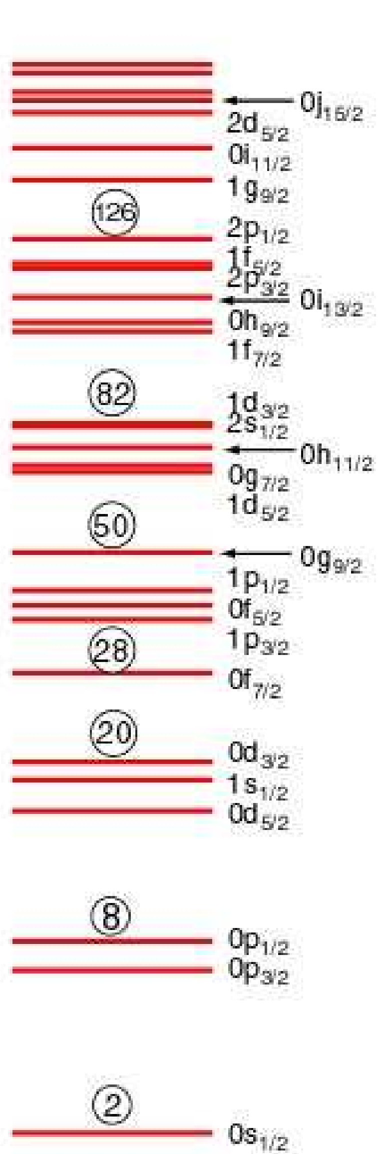
  $\Rightarrow$ quantized energy levels!

- Bohr model of atom:

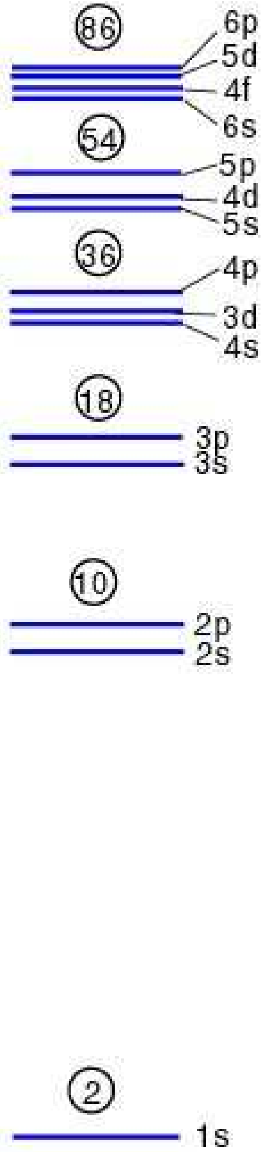
  quantized angular momentum

$\Rightarrow$ Why?

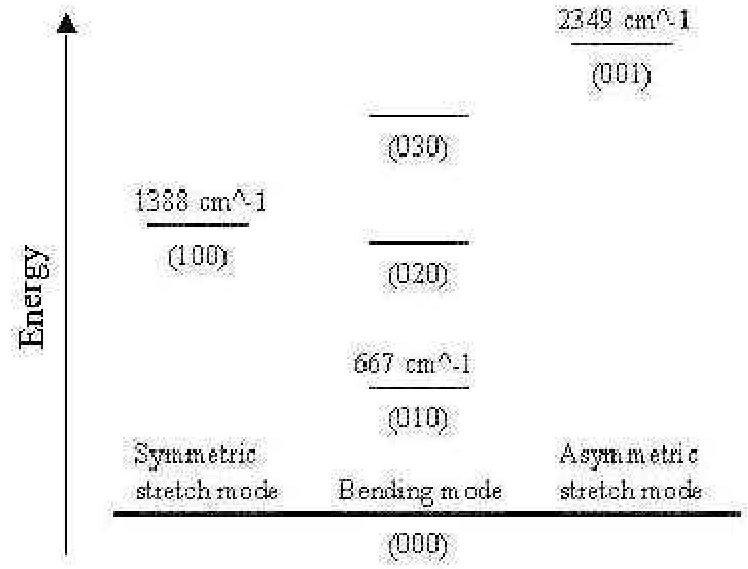
More Quantized Energy Level Examples: Atomic Nuclei and Molecule Vibration



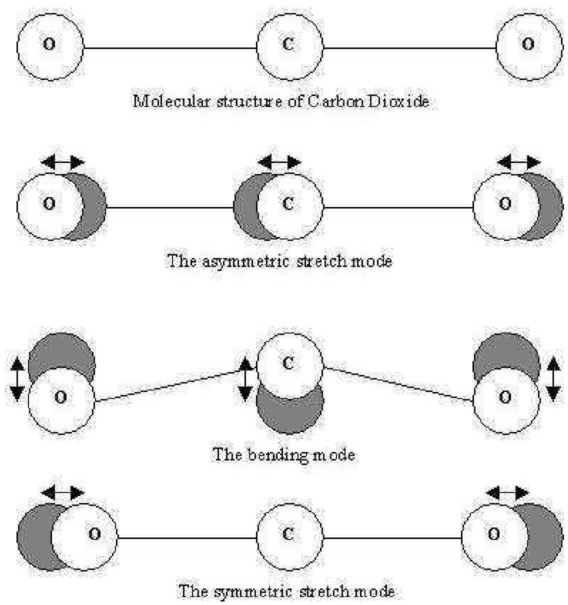
Shell Model of Nuclei



Shell Model of Atoms



The first few vibrational energy levels of the CO2 molecule



I_3 Particle Waves

$I_{3,1}$ De Broglie Hypothesis:

- Louis de Broglie (1924)

1. All particles have wave-like and particle like properties, not only photons.
2. Particle with momentum p has a "particle wave" associated with its motion with wavelength:

wavelength ⟷ wave like property

momentum ⟷ particle like

$$\lambda = h/p \qquad (p = h/\lambda = \hbar k)$$

3. and frequency:

total energy of particle

$$\nu = E/h \qquad (E = h\nu)$$

De Broglie's "particle waves"…

A. describes the shape / spatial distribution of the particle / object

B. governs the motion of the particle / object

C. Something else

The De Broglie wavelength of a particle…

A. can be *smaller* than the linear dimensions of the particle

B. can be *larger* than the linear dimensions of the particle

C. Both of the above

D. Neither of the above

$\lambda = h/p$, depends on the momentum, but not related to particle size!

- for photons ($m=0$)

$$\left.\begin{array}{ll} E = h\nu & \text{(Planck-Einstein)} \\ E = pc & \text{(E\&M, relativity)} \end{array}\right\} \underline{\underline{p}} = \frac{h\nu}{c} = \frac{h}{\underline{\underline{\lambda}}} \quad \checkmark$$

- for $m \neq 0$ particle: How did he got $\lambda = h/p$?

de Broglie assumed:

$E = mc^2 = h\nu_0 \leftarrow$ relates frequency ν_0 to rest energy for particle with zero velocity (in rest frame)

$\Rightarrow$ consider particle at rest in rest frame and in motion in Lab frame

- Special Relativity:

– total relativistic energy: $E = \sqrt{p^2c^2 + m_0^2c^4} = \gamma m_0 c^2$

– relativistic momentum: $p = \gamma m_0 v$ $\qquad \gamma = \dfrac{1}{\sqrt{1 - v^2/c^2}}$

($v \ll c \Rightarrow \gamma \simeq 1$)

particle

CM (rest) frame	Lab frame
↕ stationary vibration associated with particle at rest (*Note*: the particle is <u>not</u> vibrating here!)	$\vec{v}$ → +x particle appears to be moving with speed v
Total Energy: $E = m_0 c^2 = h\nu_0$	$E^2 = p^2c^2 + m_0^2c^4 = (\gamma m_0 c^2)^2$
$\xi_0 = \sin(2\pi \nu_0 t_0)$	$\xi(x,t) = \sin[2\pi\nu_0\gamma(t - \frac{v}{c^2}x)]$
indep. of x_0 ; t_0: time in rest frame	$= -\sin[\frac{2\pi\nu_0\gamma v}{c^2}(x - \frac{c^2}{v}t)]$
	$= -\sin(kx - \omega t)$ ← traveling wave? associated with motion of a particle

Lorentz transformation (special relativity)

$$t_0 = \frac{1}{\sqrt{1-v^2/c^2}}\left(t - \frac{v}{c^2}x\right) = \gamma\left(t - \frac{v}{c^2}x\right)$$

t_0: in rest frame; t, x: in Lab frame

⇒ so we get for the "particle wave" in the lab frame

- $k = \frac{2\pi}{\lambda} = 2\pi \frac{\nu_0 V}{c^2} \gamma$ ⇒ wavelength $\lambda = \frac{c^2}{\nu_0 V \gamma}$

- $\omega = 2\pi\nu = 2\pi \frac{\nu_0 V}{c^2} \gamma \frac{c^2}{V} = 2\pi \nu_0 \gamma$

⇒ frequ.: $\nu = \nu_0 \gamma$

⇒ phase velocity: (velocity of crest) $V_{phase} = \lambda \cdot \nu = \frac{c^2 \nu_0 \gamma}{\nu_0 V \gamma} = \frac{c^2}{V} \geq c$!?

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⇒ $h\nu = \underbrace{h\nu_0}\gamma = m_0 c^2 \gamma = E$: total relativistic energy!

$h\nu_0 = m_0 c^2$ according to de Broglie for particle at rest

⇒ have: $\lambda = \frac{c^2}{\nu_0 V \gamma} = \frac{c^2 h}{\underbrace{h\nu_0} V \gamma} = \frac{c^2 h}{m_0 c^2 V \gamma} = \frac{h}{\gamma m_0 V} = \frac{h}{p}$

$h\nu_0 = m_0 c^2$

⇒ $\boxed{\lambda = \frac{h}{p}}$ 2