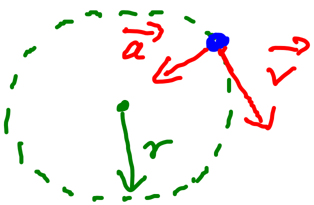
Recap

- Forces in uniform circular motion:



- $\sum \vec{F}_{\text{on object}} = m\vec{a}$ with $|\vec{a}| = \frac{v^2}{r}$

  and $\sum \vec{F}_{\text{on object}}$ and $\vec{a}$: point to center of circle, $\perp$ to path

- never show $m\frac{v^2}{r}$ on a FBD! $\vec{a}$ is the result of the external forces on the object!

- Kinetic Energy: $K = \frac{1}{2}mv^2$

- Work: energy transferred to or from an object by force

  for a constant force: $W = \vec{F}\cdot\vec{d} = Fd\cos\phi = F_{\parallel}\, d = F d_{\parallel}$

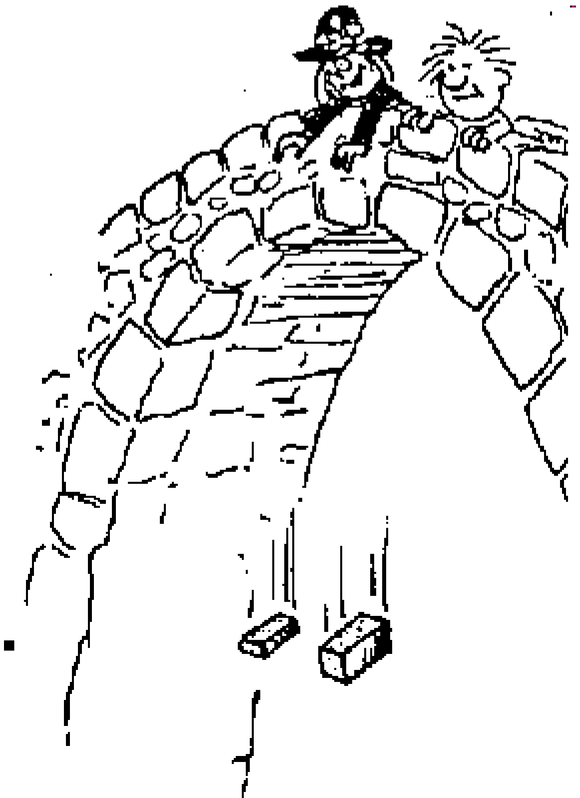
  "dot" product

  component of $\vec{F}$ $\parallel$ to $\vec{d}$

  component of $\vec{d}$ $\parallel$ to $\vec{F}$

Today:

- **Work and Energy:**
  - Work done by a single force
  - Work-kinetic energy theorem
  - Work done by gravity, friction…

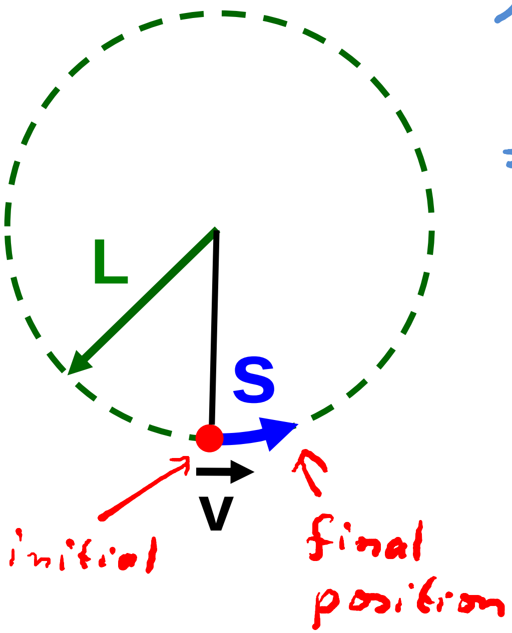


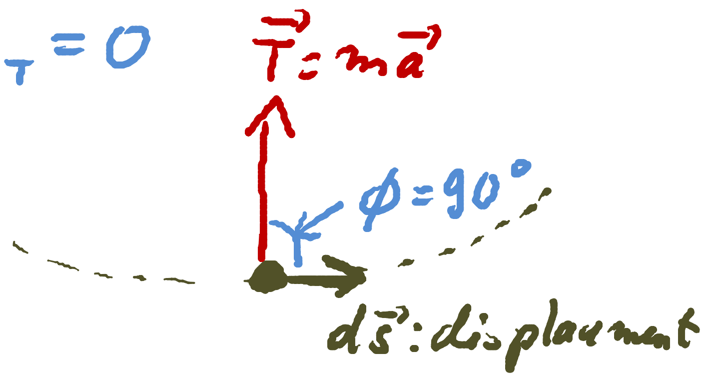
A **ball** of **mass *m*** is swung at the end of the rope in a **horizontal circle**. Its **speed *v*** is constant and the length of the rope is *L*. Note: $v = const \Rightarrow K = const \Rightarrow$ no work done

What **work *W* is done by the tension *T* in the rope** when the mass moves a small distance *s* along the circle?

$\Rightarrow$ at every point on path $\vec{T} \perp d\vec{s} \Rightarrow \phi = 90°$

$\Rightarrow W_{by\ T} = 0$





***W* = ?**

A. mg s

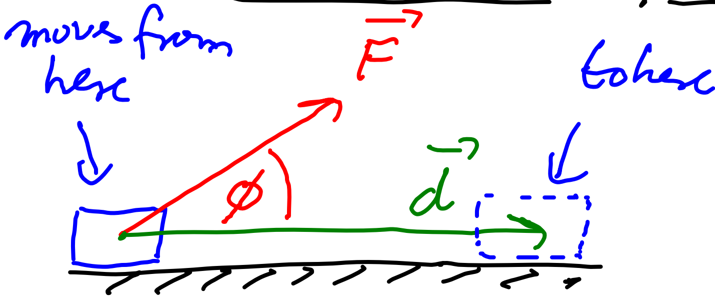
~~B.~~ T s

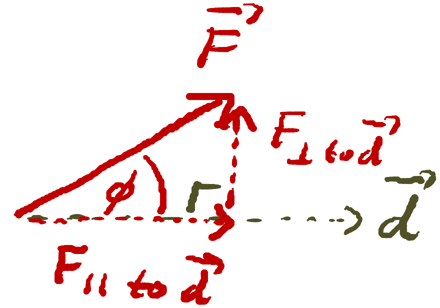
C. T L

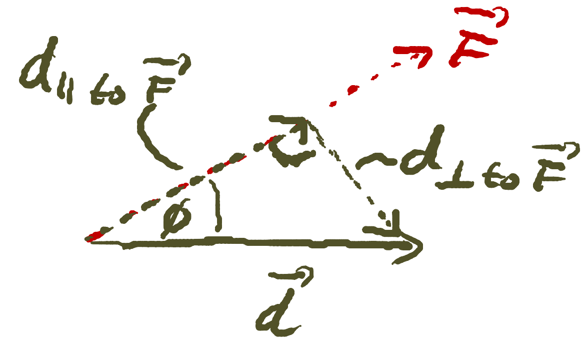
D. 0

$\Rightarrow$ force that acts $\perp$ to displacement does no work!

Work done by a single, constant force on object:







$\vec{d}$: displacement vector
$= \vec{r}_2 - \vec{r}_1$

ϕ : angle between $\vec{F}$ and $\vec{d}$

$F_{\parallel \text{ to } \vec{d}} = F\cos\phi$
$\Uparrow$
component of $\vec{F}$ along $\vec{d}$
$\Downarrow$

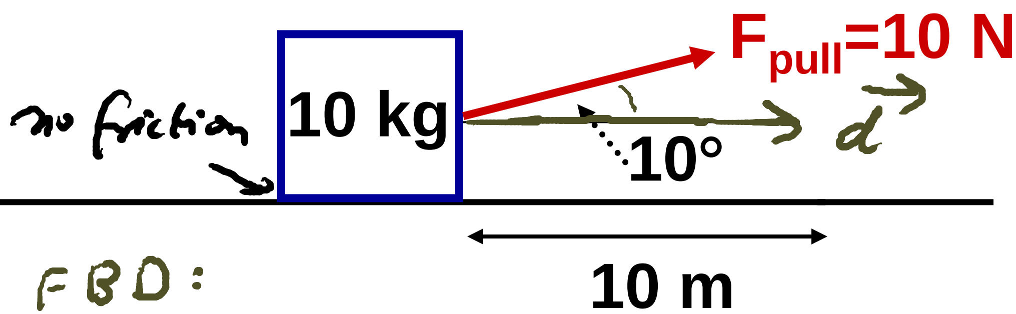
$d_{\parallel \text{ to } \vec{F}} = d\cos\phi$
$\Uparrow$
component of $\vec{d}$ along $\vec{F}$
$\Downarrow$

$$\text{Work} = W \equiv F\cdot d\cos\phi = F_{\parallel \text{ to } \vec{d}}\cdot d = F\, d_{\parallel \text{ to } \vec{F}}$$

|displacement| |Force|

note: if $\phi = 90^\circ \Rightarrow W_{\text{by force on object}} = 0$

A **10 kg** crate is pulled **10 m** along a frictionless horizontal floor by a force $\mathbf{F_{pull}=10\ N}$ applied at an **angle of 10°** with respect to the horizontal, as shown. What is the **net work done on the crate by all the forces** that act on it?



W_{net} = ?

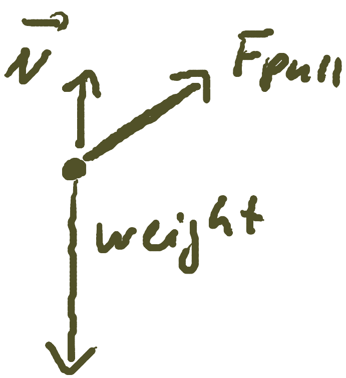
A. 100 cos 10° J

B. 100 sin 10° J

C. 1000 J+ 100 sin 10° J

D. 1000 J+ 100 sin 10° J

FBD:



$W = F d \cos\phi$

=> for F_{pull}:

$W_{pull} = 10N \cdot 10m \cdot \cos 10° = 100J \cdot \cos 10°$

=> for N: $W_N = N d \cos 90° = 0$

=> for weight: $W_w = 0$

sum: $W_{net} = 100J \cdot \cos 10° + 0 + 0$

• Work done by a single constant force on object

angle between $\vec{F}$ and $\vec{d}$

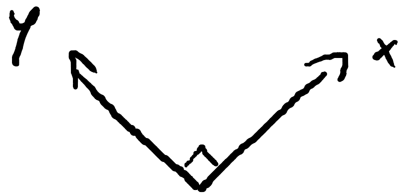
$$W_{\text{by force on object}} = F d \cos\phi = F_{\parallel} d = F d_{\parallel}$$

Mathematical Shorthand: $W = \vec{F} \cdot \vec{d}$

"dot" product of two vectors

$$= F d \cos\phi$$

angle between the two vectors



Use this if you know components of vectors

$$= F_x d_x + F_y d_y$$

note: components can be < 0 !

works for any coordinate system

- Work done by multiple forces (constant) acting on an object:

same object → same displacement $\vec{d}$

$$\underline{W_{net}} = W_1 + W_2 + \dots = \vec{F}_1 \cdot \vec{d} + \vec{F}_2 \cdot \vec{d} + \dots$$

$$= \left(\sum \vec{F}\right) \cdot \vec{d} = \vec{F}_{net} \cdot \vec{d} = F_{net}\, d \cos\phi$$

angle between $\vec{F}_{net}$ and $\vec{d}$

Work – Kinetic Energy Theorem:

- Work: Energy transferred to or from an object by force
- Kinetic energy: $K = \frac{1}{2} m v^2$

$$\Delta K_{\text{of object}} = K_f - K_i = W_{\text{net}} \text{ on object by } \underline{\underline{\text{all}}}$$

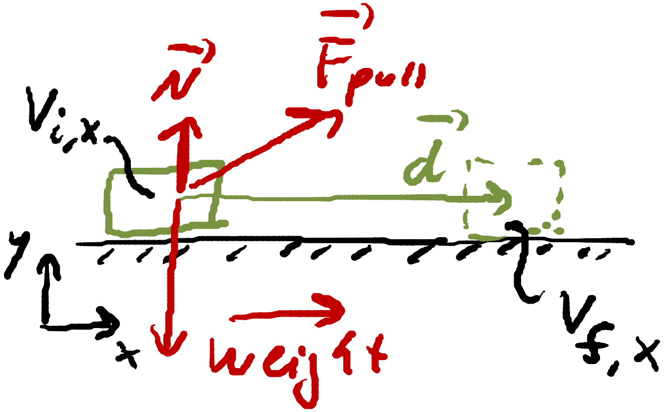
forces that act on the object while it moves from some initial to some final position

change!

$$\left(\begin{array}{c} \underline{\text{change}} \text{ in kinetic} \\ \text{energy of object} \end{array} \right) = \left(\begin{array}{c} \underline{\text{net}} \text{ work done} \\ \text{on object by forces} \end{array} \right) = \sum W_i$$

Note: $\Delta K < 0$ or $\Delta K > 0$ possible!

check Work – Kinetic Energy theorem:



$$\Delta K = K_f - K_i = \underline{\frac{1}{2} m v_{f,x}^2 - \frac{1}{2} m v_{i,x}^2}$$

$$= W_{net} = \underbrace{W_N + W_{weight}}_{= 0 \text{ here}} + W_{pull}$$

$$= \vec{F}_{pull} \cdot \vec{d} = F_{pull,x}\, d$$

↑ $\vec{d}$ is along x

use NII: $\sum F_x = \underbrace{m a_x = F_{pull,x}}$

$$\Rightarrow \frac{1}{2} m \left(v_{f,x}^2 - v_{i,x}^2 \right) = F_{pull,x}\, d = m a_x d$$

$$\Rightarrow \underline{v_{f,x}^2 - v_{i,x}^2} = 2 a_x d = \underline{2 a_x \Delta x} \Big\} \text{ nothing new...}$$

$\Rightarrow$ same as Newton's laws, but work/kinetic helps to solve some problems much easier than Newton's Laws!

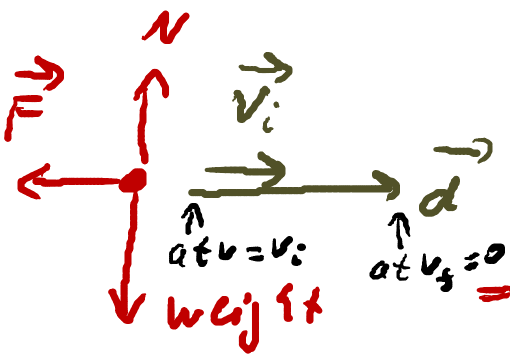
A car of mass *m* traveling at a speed v_i is braked to a stop by a constant force *F*.

What is the stopping distance *d* of the car?

(Use energy/work concepts to solve, not NII.)

Use $\Delta K = W_{net} = \sum W_i$ (work-kinetic energy theorem)

FBD of car



$W_{weight} = 0 \quad (\phi = 90^\circ)$

$W_N = 0$

$W_F = F d \underbrace{\cos 180^\circ}_{-1} = -F d$

$\Rightarrow \Delta K = K_f - K_i = -F d$

$= 0 - \frac{1}{2} m v_i^2 = -F d$

$v_f = 0$ here

$\Rightarrow d = \dfrac{m v_i^2}{2F}$

d = ?

A. F/v_i

B. mv_i/F

C. $mv_i^2/2F$

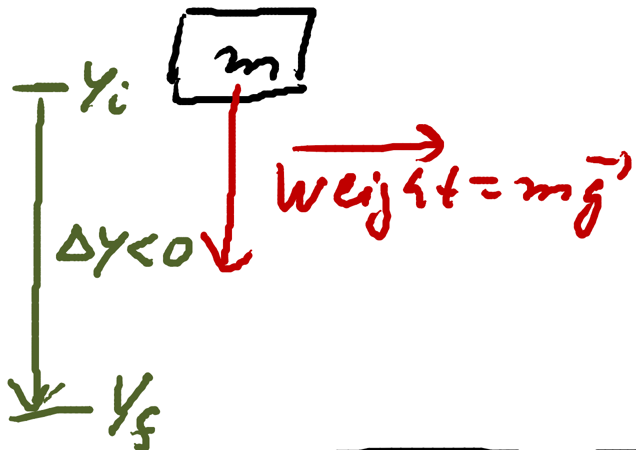
D. mv_i^2/F

E. $2F/mv_i^2$

Work done by specific forces:

① Work done by gravity:

↑ +y ⇐ important!



$$\underbrace{W}_{\text{work}}{}_{\text{by gravity on object}} = \vec{F} \cdot \vec{d}$$

$$= F\, d \cos \underbrace{\phi}_{0^\circ \text{ for down motion}} = F\, d$$

$$= -mg\,\Delta y \Leftarrow \Delta y < 0 \text{ here}$$

$$= -mg\,(y_f - y_i)$$

$$= -mg\,(\text{vertical displacement})$$

↑ for +y up!

$$\boxed{\begin{array}{l} \text{moves down} \Rightarrow W_g > 0 \\ \text{moves up} \Rightarrow W_g < 0 \end{array}}$$