Recap: Energy and Work

Lecture 16

- Work: energy transferred to or from an object by force

for a constant force: $W = \vec{F} \cdot \vec{d} = Fd\cos\phi = F_{\parallel}\, d = F d_{\parallel}$

"dot" product — component of $\vec{F} \parallel$ to $\vec{d}$ — component of $\vec{d} \parallel$ to $\vec{F}$

$W = \vec{F} \cdot \vec{d} = F_x d_x + F_y d_y$

for any coordinate system

- Work - Kinetic Energy Theorem:

$\Delta K_{\text{of object}} = K_f - K_i = W_{\text{net done by all forces acting on object}}$

need to include all forces here!

$= W_1 + W_2 + W_3 + \ldots = \left(\sum_i \vec{F}_i\right) \cdot \vec{d} = F_{net}\, d\cos\phi$

Work / energy helps to solve some problems much easier than Newton's laws!

- Work by gravity: $W_{\text{by } g \text{ on obj}} = -mg\,(y_f - y_i)$

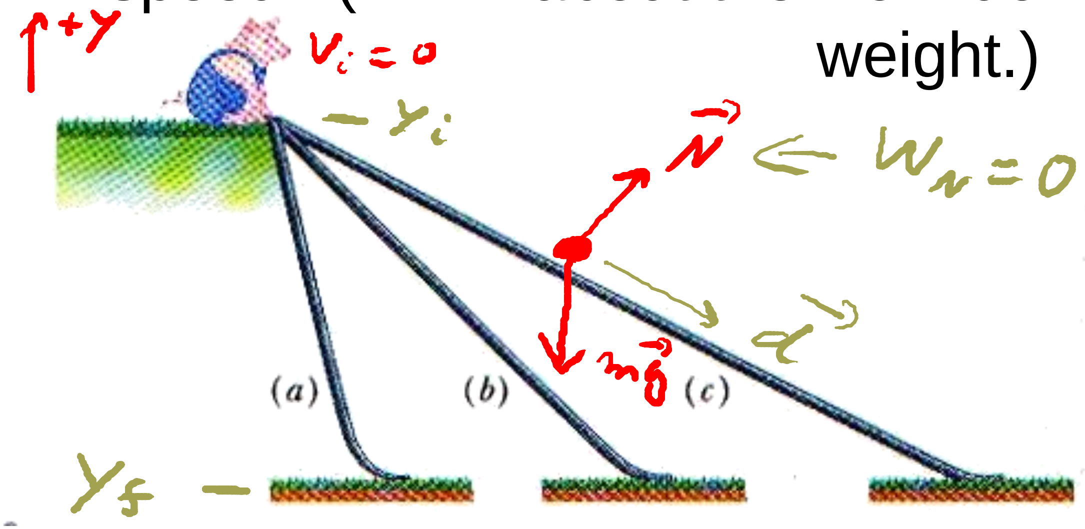
if $+y \uparrow$ up

Today:

- **Work done by friction, springs**
- **Compound bows**
- **Power**

A pig has a choice of three different *frictionless* slides as shown.

With which slide will the pig reach the ground with the greatest speed? (Think about the work done on the pig by the pig's weight.)



A. (a)

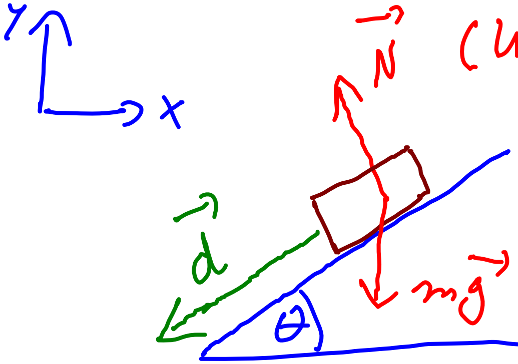
B. (b)

C. (c)

D. all the same

$$\Rightarrow \Delta K = K_f - K_i = \frac{1}{2} m v_f^2 = W_{net} = W_{\text{by gravity on pig}} = -mg\underbrace{(y_f - y_i)}_{\text{same for all 3 slides!}}$$

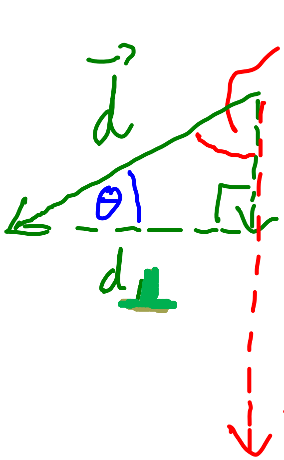
$v_i = 0 \Rightarrow K_i = 0$



$(W_N = 0)$

$W_{\text{by gravity}} > 0$

$\Rightarrow K_f > K_i$



$d_\parallel = -\Delta y$

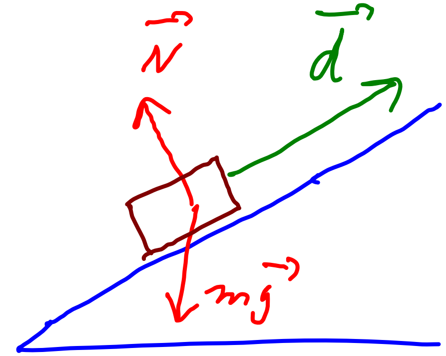
$\phi = 90° - \theta$

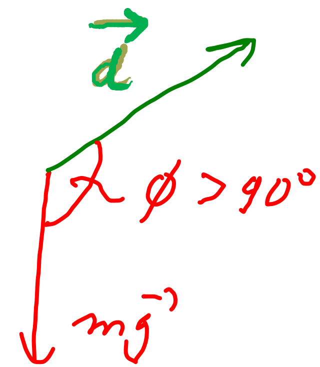
$W_{\text{by gravity on object}} = \vec F \cdot \vec d = F d_\parallel$

$= mg\, d \underbrace{\cos\phi}_{>0} = mg\, d_\parallel$

$= -mg\,\Delta y = -mg\,(y_f - y_i)$

as before, indep. of Δx!





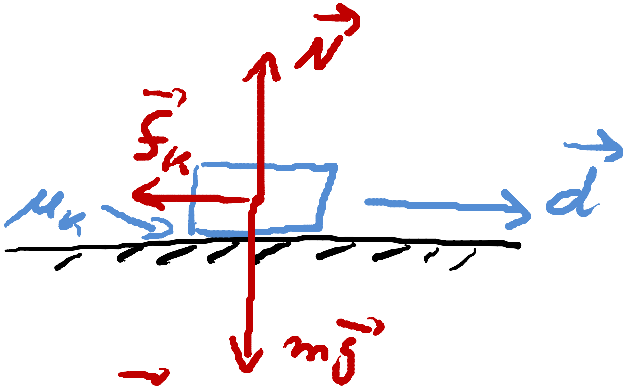
$\cos\phi < 0$ here

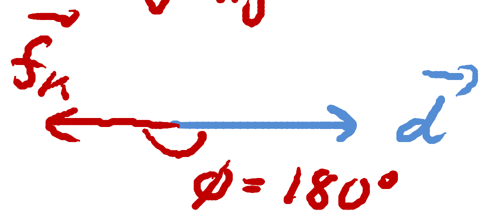
$\Rightarrow W_{\text{by gravity on object}} < 0$

$\Rightarrow K_f < K_i \quad (W_N = 0)$

(slows down)

② Work done by friction force $\vec{f}$



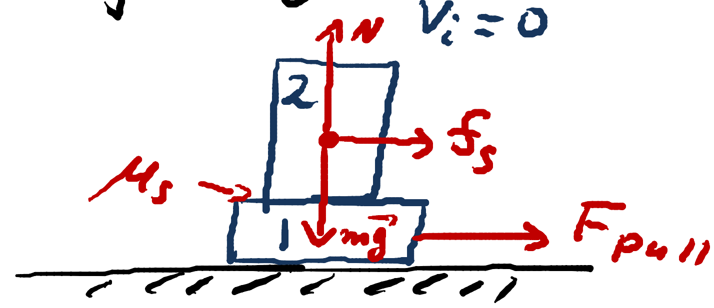


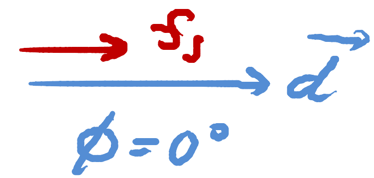
$$W_{\text{by } f_k \text{ on object}} = f_k\, d \underbrace{\cos(180°)}_{-1}$$

$$= -f_k\, d < 0$$

$\Rightarrow K_f < K_i$ here

$(W_g = 0,\ W_N = 0)$





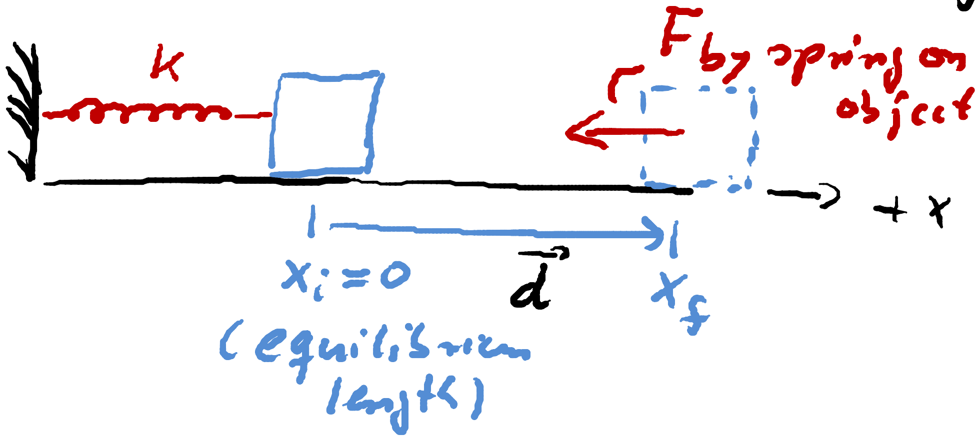
$$W_{\text{by } f_s \text{ on object "2"}} = f_s\, d \cos 0° > 0$$

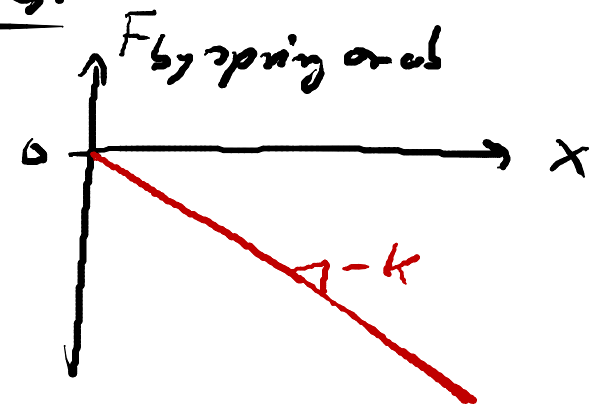
$\Rightarrow K_{f,2} > K_{i,2}$ here

$(W_g = 0,\ W_N = 0)$

$\Rightarrow$ Friction forces always oppose relative motion, but can lead to a decrease or an increase in the kinetic energy of an object.

③ Work by spring force:





$$F_{\text{by spring on obj.}} = -kx$$

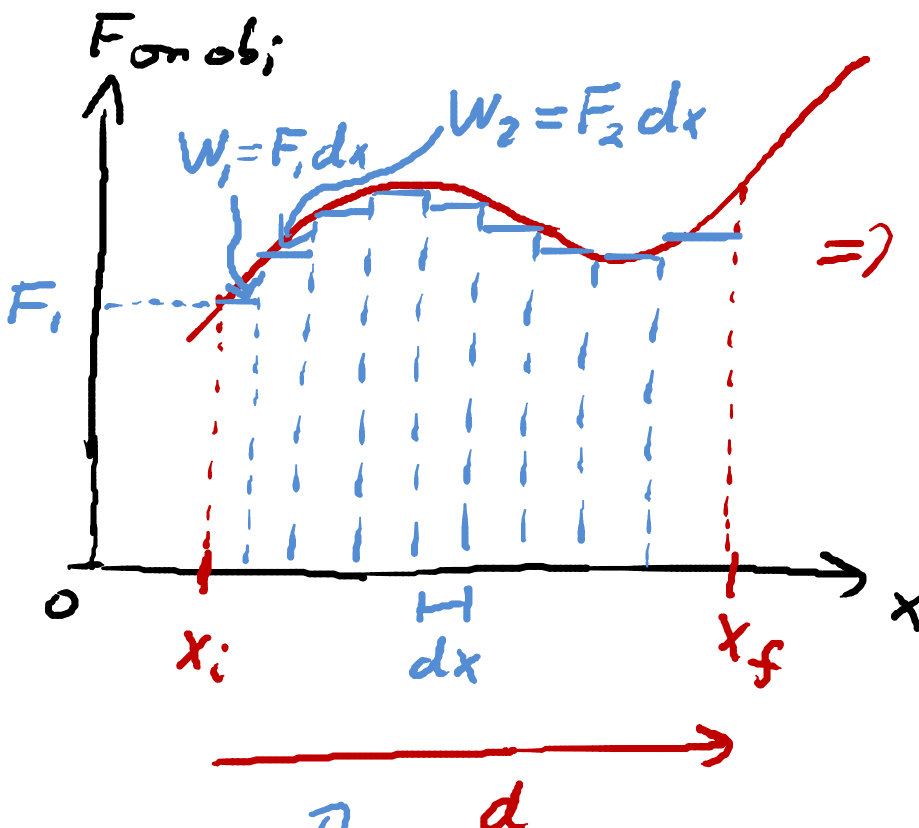
force varies with x, i.e. along the path

$$\Rightarrow W_{\text{by spring on obj}} \cancel{=} \underbrace{\vec{F}\cdot\vec{d}}_{\text{only for a constant force!}} \cancel{=} (-kx)\,x_f$$

what "x" to use here?

④ General case: Work done by a variable force:

for 1-D

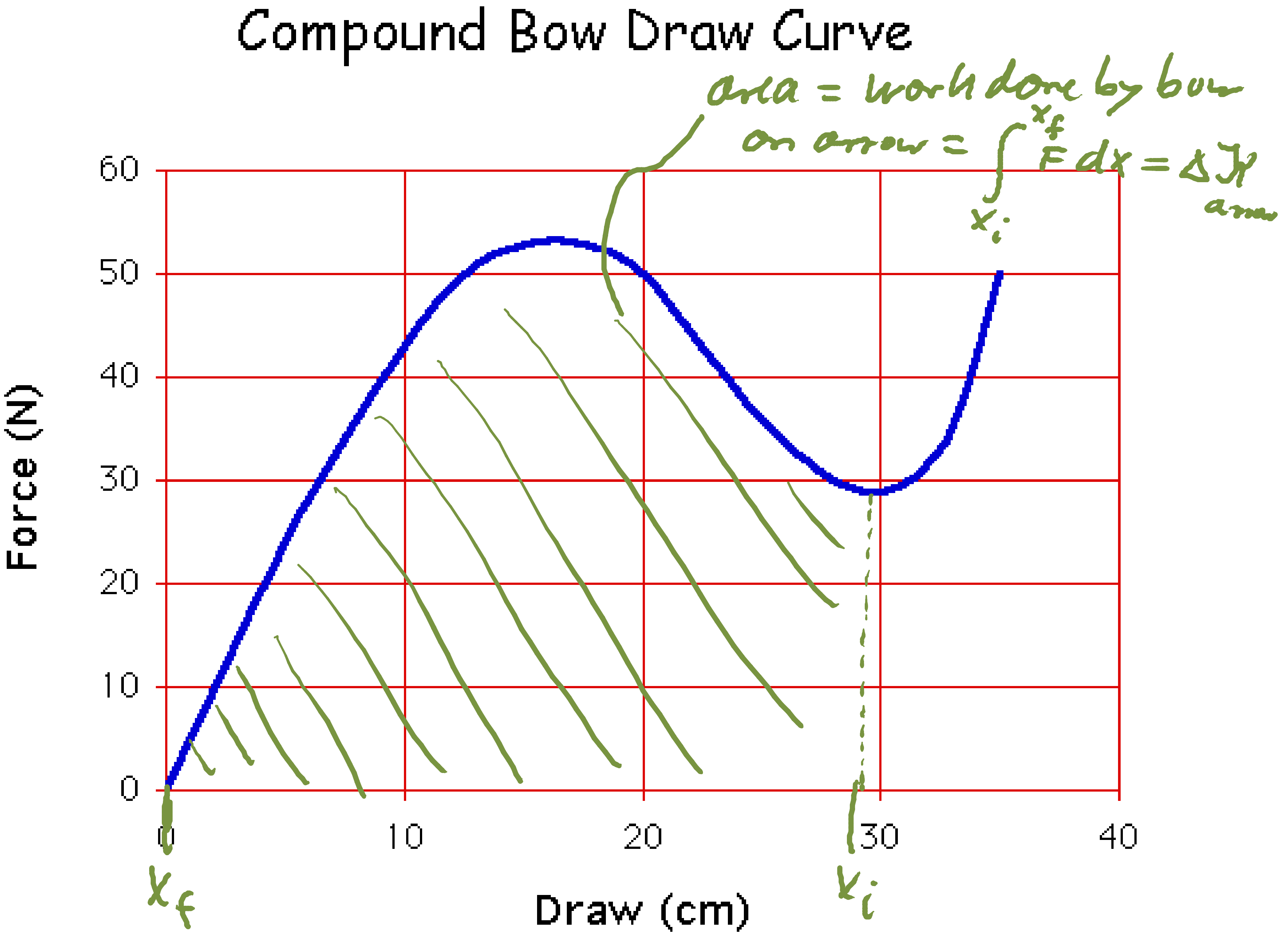


break into small intervals with $F \approx$ const

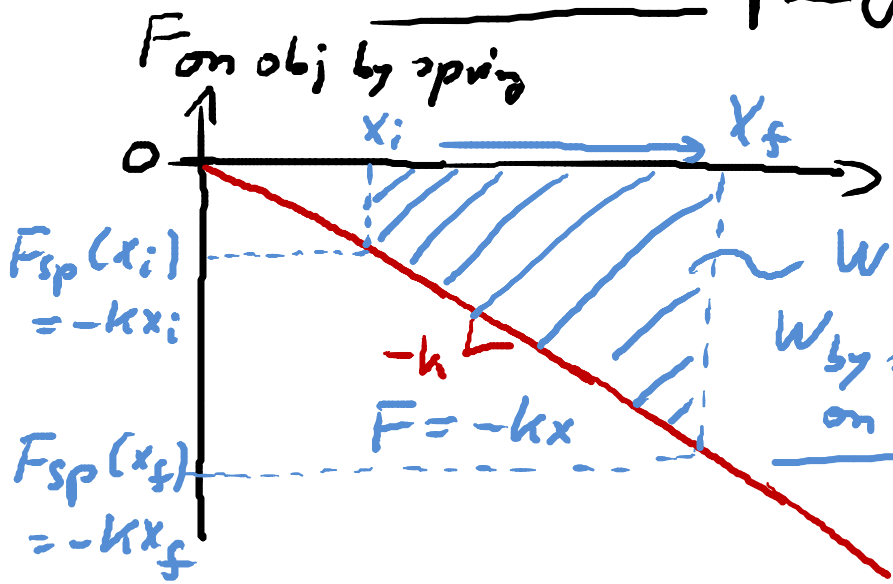
$$W = \sum_i W_i = \sum_i F_i\,dx$$

$$\Rightarrow W_{\substack{\text{on}\\\text{object}}} = \int_{x_i}^{x_f} F(x)\,dx$$

= area "under" $F(x)$ curve!


Compound Bow Draw Curve
Force (N)
Draw (cm)
0
10
20
30
40
50
60
0
10
20
30
40
area = work done by bow on arrow = $\int_{x_i}^{x_f} F\,dx = \Delta K_{arrow}$
x_f
x_i

Back to spring:



Work = area of trapezoid

$$W_{\substack{\text{by spring}\\\text{on obj}}} = \frac{1}{2}\left[F_{sp}(x_f)\,x_f - F_{sp}(x_i)\,x_i\right]$$

$$= -\frac{1}{2}k\left(x_f^2 - x_i^2\right)$$

$$W_{\substack{\text{by spring}\\\text{on obj}}} = \int_{x_i}^{x_f}(-kx)\,dx = -\frac{1}{2}k\overbrace{\left(x_f^2 - x_i^2\right)}^{\neq\,\Delta x^2}$$

$\Rightarrow$ if $x_i = 0$

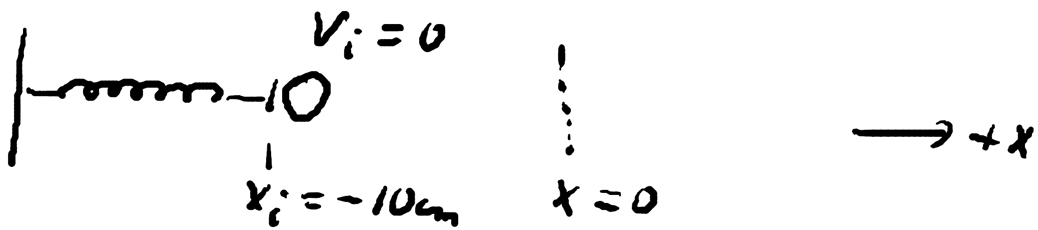
$$W_{\substack{\text{by spring}\\\text{on obj}}} = -\frac{1}{2}k x_f^2 \qquad W_{\substack{\text{by object}\\\text{on spring}}} = +\frac{1}{2}k x_f^2$$

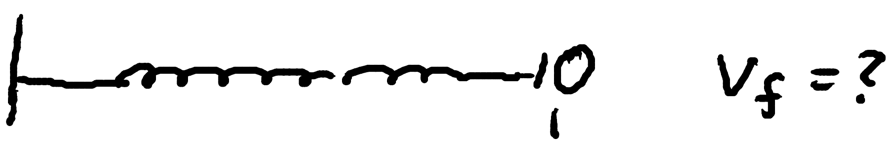
$$\vec{F}_{AB} = -\vec{F}_{BA}$$

A toy gun has a spring with spring constant ***k*=100 N/m**. The spring is **compressed a distance of 10 cm** and a **ball of mass *m*=10 g** is inserted in the gun.

$0.01\,kg = 1/100\,kg$

After the trigger releases the spring, what will be the ball's **speed** *v* when the spring has returned to its equilibrium length?





$W_{\text{by spring on ball}} = -\frac{1}{2}k(x_f^2 - x_i^2)$

$= -\frac{1}{2} 100 \frac{N}{m} (0^2 - (-0.1\,m)^2)$

$= 0.5\,J$

$\Rightarrow \Delta KE_{ball} = KE_f - KE_i = W_{net} = W_{\text{by spring}} = 0.5\,J = \frac{1}{2} m v_f^2$

$v_i = 0 \Rightarrow KE_i = 0$

$\Rightarrow v_f = 10\,m/s$

v_f=?

A. 1 m/s

B. 5 m/s

C. 10 m/s

D. 50 m/s

E. none of the above

Power: What is power?

P = rate at which work is done by a force

$$\bar{P} = \text{avg power} = \frac{W_{\text{by force}}}{\Delta t}$$ ← time interval during which work was done

$$P = \text{instantaneous power} = \frac{dW}{dt} = \text{slope of the } W-t \text{ graph}$$

Units? $[P] = \frac{J}{s} = \text{Watt} = W$

$$1\ hp = 746\ W$$

Example: 100 hp = 74,600 W = 1000 × 75 W light bulbs

$$\Rightarrow \text{Power} = P = \frac{dW}{dt} = \frac{d}{dt}\big(\overbrace{\vec{F}\cdot\vec{d}}^{\text{for constant force}}\big) = \frac{d}{dt}\left(Fd\cos\phi\right)$$

displacement

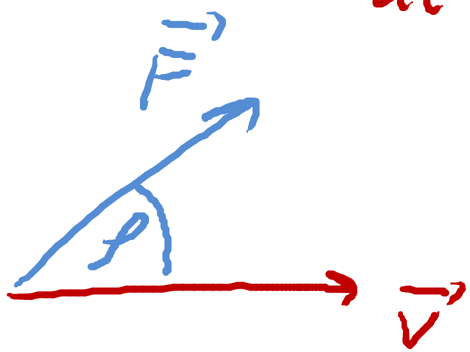
$\Rightarrow$ if $\vec{F}$ = const, then:

$$P = \vec{F}\cdot\frac{d\vec{d}}{dt} = \vec{F}\cdot\vec{v}$$

angle between $\vec{F}$ and $\vec{v}$

$$= Fv\cos\phi$$

$$= F_x v_x + F_y v_y$$



A **cyclist** travels along a flat road at constant speed v. The power produced by the cyclist all goes to overcoming **air drag**.

How does the power the cyclist must produce to travel at **54 km/h** compare with that required to travel at **27 km/h**?

$P_{cyclist} = -P_{drag}$

$P_{cyclist} = -P_{drag} = -(\vec{D} \cdot \vec{v})$

$= +\left(\frac{1}{2} C \rho A v^2\right) v \propto v^3$

$\Rightarrow \dfrac{P(54\,km/h)}{P(27\,km/h)} = \left(\dfrac{2}{1}\right)^3$

P(54 km/h)/P(27 km/h) = ?

A. **1**

B. **2**

C. **4**

D. **8**

E. **16**

Power and Energy in Cycling

Assume $P_{cyclist} \approx$ Drag force $\times$ velocity (P = Dv)

$\therefore$ $P_{cyclist} = 1/2\ C\ \rho\ A\ v^2 \times v$ = $\mathbf{1/2\ C\ \rho\ A\ v^3}$

- Assume C~ 0.4, r = 1.2 kg/m^3, A~0.7 m^2
- Assume human body ~ 25% efficient in converting food energy into mechanical energy.
- 1 Cal =4.2 J, 1 food Calorie =1 kCal = 4,200 J

Prof. Liepe:

v ~ 8 m/s (17 mi/h) ***P* ~ 86 W (~0.12 hp)**

Burns ~ 344 W, 1.24 MJ/hour, **295 kCal/hour**

Professional distance cyclist:

v ~ 14 m/s (~30 mi/h) ***P* ~ 460 W (~0.6 hp)**

Burns ~1.8 kW, 6.6 MJ/hour, **1580 kCal/hour**

Professional sprint cyclist:

v ~ 20 m/s (~45 mi/h) ***P* ~ 1340 W (~1.8 hp)**

Burns ~ 5.4 kW, 19 MJ/hour, **4600 kCal/hour**

Average daily food energy intake for Tour de France Cyclists:

~10,000 kCal/ day

(~7 lb of uncooked pasta)