Recap:

Lecture 17

- Kinetic energy: $K = \frac{1}{2} m v^2$
- Work: $W_{\text{on obj}} = \vec{F} \cdot \vec{d} = F d \cos\phi$ } for a constant force

  $W_{\text{on obj}} = \int_{x_i}^{x_f} F(x)\, dx =$ area "under" F-x graph } 1-D case for a variable force

- $\Delta K = K_f - K_i = W_{\text{net on obj}} = W_1 + W_2 + \ldots$ } always true, but need to include all forces!
- Work by gravity: $W_{\text{by } g \text{ on obj}} = -mg\,(\underbrace{y_f - y_i}_{\text{if } +y \uparrow \text{ up}})$
- Work by a spring: $W_{\text{by spring on obj}} = -\frac{1}{2} k\,(\underbrace{x_f^2 - x_i^2}_{\neq \Delta x^2})$
- Power: rate at which work is done

  $\bar{P}$ = average power $= \frac{W}{\Delta t}$ $\iff$ $W_{\text{on obj}} = \bar{P}\,\Delta t$

  P = inst. power $= \frac{dW}{dt}$ = slope of W-t graph

  $[P] = \text{J/s} = \text{Watt} = \text{W}$

Power and Energy in Cycling

Assume $P_{cyclist} \approx$ Drag force $\times$ velocity (P = Dv)

$\therefore$ $P_{cyclist} = 1/2\ C\ \rho\ A\ v^2 \times v$ = **$1/2\ C\ \rho\ A\ v^3$**

- Assume C~ 0.4, r = 1.2 kg/m^3, A~0.7 m^2
- Assume human body ~ 25% efficient in converting food energy into mechanical energy.
- 1 Cal =4.2 J, 1 food Calorie =1 kCal = 4,200 J

Prof. Liepe:

v ~ 8 m/s (17 mi/h) ***P*** **~ 86 W (~0.12 hp)**

Burns ~ 344 W, 1.24 MJ/hour, **295 kCal/hour**

Professional distance cyclist:

v ~ 14 m/s (~30 mi/h) ***P*** **~ 460 W (~0.6 hp)**

Burns ~1.8 kW, 6.6 MJ/hour, **1580 kCal/hour**

Professional sprint cyclist:

v ~ 20 m/s (~45 mi/h) ***P*** **~ 1340 W (~1.8 hp)**

Burns ~ 5.4 kW, 19 MJ/hour, **4600 kCal/hour**

Average daily food energy intake for Tour de France Cyclists:

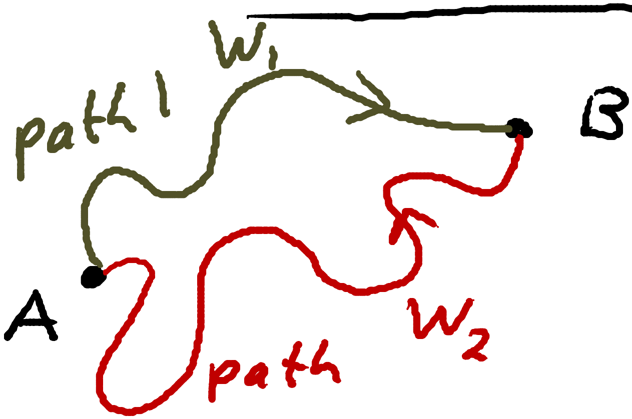
~10,000 kCal/ day

(~7 lb of uncooked pasta)

Today:

- Conservative forces
- Potential energy
- Mechanical energy
- Loop-the-loop

Conservative Forces:



object under the influence of a force F

$\Rightarrow$ for a conservative force: $W_1 = W_2$ always

$\rightarrow$ work done between A and B on object is path-independent!

$\rightarrow$ Net work done on object by a conservative force when it goes around a closed path $= 0$ ($W_1 - W_2 = 0$)

$\rightarrow$ Key: don't need to care about path between A and B!

$\Rightarrow$ for a <u>non</u>-cons. force: Work done does depend on path!

Which of the following forces are conservative forces?

A. ✓ **Gravity** $W_g \propto \Delta y \Rightarrow$ cons. force

B. ✓ **Spring force** $W_{sp} \propto (x_f^2 - x_i^2) \Rightarrow$ cons. force

~~C.~~ **Friction** W_f depends on path $\Rightarrow$ <u>non</u>-cons. force

D. **two of the above**

E. **three of the above**

Examples:

- Cons. force: Gravity, spring force, electrostatic force
- Non-cons. forces: Friction, applied force, tension, ...

  $\Rightarrow$ work depends on path

Potential Energy U:

U = Energy associated with the configuration of a system of objects that exerts force on one another

- only defined for conservative force
- Energy which is a function of position

Define as:

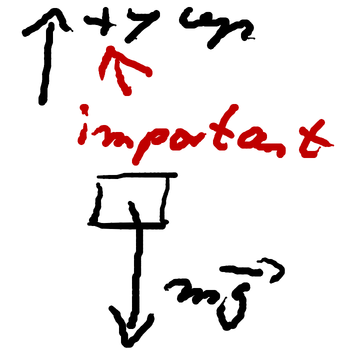
$$\underline{\underline{\Delta}}\, U_{object} = U_f - U_i \equiv \overset{!}{\downarrow} - W_{\substack{\text{by cons. force} \\ \text{on object}}} \overset{\text{1-D}}{\downarrow}= -\int_{x_i}^{x_f} F(x)\,dx$$

$$\begin{pmatrix}\text{change in} \\ \text{potential energy} \\ \text{of object}\end{pmatrix} = -\begin{pmatrix}\text{work done by the} \\ \text{cons. force on object}\end{pmatrix}$$

Key: W_{cons} depends on initial and final positions of the object, not on path!

⇒ <u>for gravity (cons. force!)</u>

$$\Delta U_g = -W_{\text{by gravity on object}} = -\int_{y_i}^{y_f} (-mg)\,dy$$



$$\Rightarrow \boxed{\Delta U_g = +mg(y_f - y_i)}$$

⇒ $\Delta U_g > 0$ if $y_f > y_i$, i.e. object moves up

⇒ choice of $U_g = 0$ is arbitrary: $\Delta U_g \Leftrightarrow W_g \Leftrightarrow \Delta K$

choose: $U_g = 0$ at $y = 0$

$$\Rightarrow \boxed{U_g(y) = mgy} \quad \Big\} \text{ for } \uparrow +y \text{ up}$$

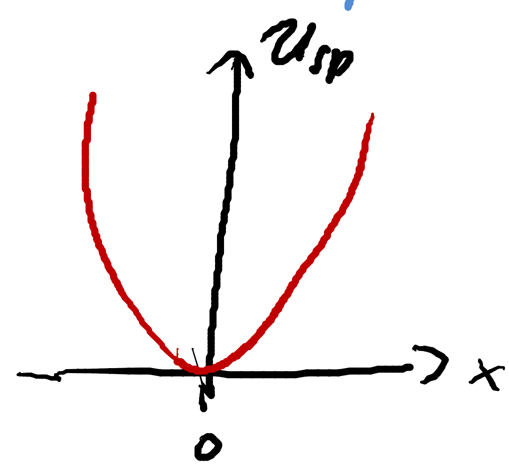
$\Rightarrow$ <u>for ideal spring force:</u> $F_{\substack{\text{by spring} \\ \text{on obj.}}} = -kx$

$$\Delta U_{sp} = -W_{\substack{\text{by spring} \\ \text{on obj}}} = -\int_{x_i}^{x_f} (-kx)\, dx$$

$$\Rightarrow \boxed{\Delta U_{sp} = +\frac{1}{2} k \left(x_f^2 - x_i^2\right)}$$

$\Rightarrow$ choose $U_{sp} = 0$ at $x = 0$ (at relaxed position, when $F_{sp} = 0$)

$$\Rightarrow \boxed{U_{sp}(x) = +\frac{1}{2} k x^2}$$



2 Cases:

- **Case I**: The only forces that do work on object are conservative.

$\Rightarrow$ work-kinetic energy theorem : $\Delta \mathcal{K}_{obj} = W_{by\ cons.\ forces}$

$\Rightarrow$ $\Delta \mathcal{U} = -W_{by\ cons.\ forces}$

$$\Rightarrow \boxed{\Delta \mathcal{K}_{obj} = -\Delta \mathcal{U}_{obj}} \Big\} \text{ for case I only!}$$

$\Rightarrow$ Define Mechanical Energy of object

$$\boxed{E_{mech,\,obj} = \mathcal{K}_{obj} + \mathcal{U}_{obj}}$$

$\Rightarrow$ at point 1: $E_1 = \mathcal{K}_1 + \mathcal{U}_1$

at point 2: $E_2 = \mathcal{K}_2 + \mathcal{U}_2$

$$\Delta E = E_2 - E_1 = (\mathcal{K}_2 - \mathcal{K}_1) + (\mathcal{U}_2 - \mathcal{U}_1)$$
$$= \Delta \mathcal{K} + \Delta \mathcal{U}$$
$$= W_{cons} - W_{cons} = \underline{\underline{0}}$$

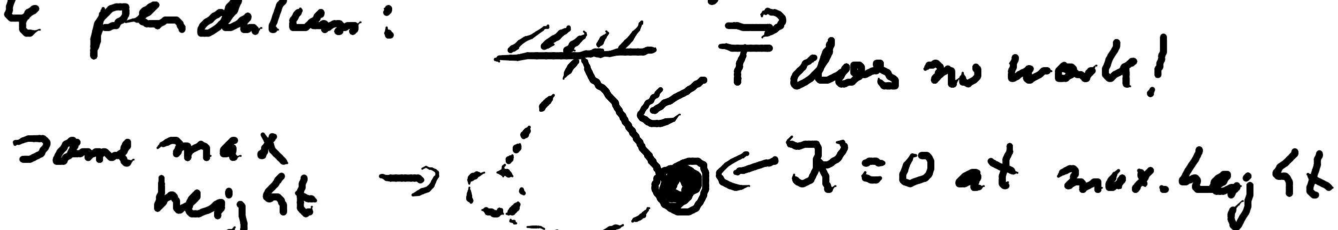
$\Rightarrow E_{mech,2} = E_{mech,1} = K_{obj} + U_{obj} = \text{const}$

($\Delta E_{mech} = 0$): Mechanical energy is conserved (constant) throughout motion if there is <u>no</u> work done by <u>non</u>-cons. forces!

<u>Example</u>: If only gravity does work on object:

$$E_{mech} = K_{obj} + U_{obj} = \text{const} = \frac{1}{2}mv^2 + mgy$$

– simple pendulum:



– sloping track (without friction)

Case II: Work is done by conserv. and non-cons force (e.g. friction, applied forces)

$$\boxed{\Delta K_{obj} = W_{net} = \sum W_i} \quad \text{\} always true}$$

$$\Rightarrow \Delta K_{obj} = \underbrace{W_{by\ appl.\ forces} + W_{by\ friction} + \ldots}_{} + \underbrace{W_{by\ gravity} + W_{by\ spring}}_{}$$

$$= W_{non-cons.\ forces} + W_{cons.\ forces}$$

$$= W_{non-con.\ forces} + (-\Delta U_g - \Delta U_{sp})$$

$$\Rightarrow \Delta K_{obj} + \Delta U_g + \Delta U_{spr} = W_{\underline{non}-cons.\ forces}$$

$$\Rightarrow \boxed{\Delta E_{mech} = \Delta K_{obj} + \Delta U_{obj} = W_{\underline{\underline{non}}-cons.\ forces}}$$

always true!

A **block of mass *m*** is released from the top of the frictionless track shown.

What is its **speed *v* at the top** of the loop-the-loop?

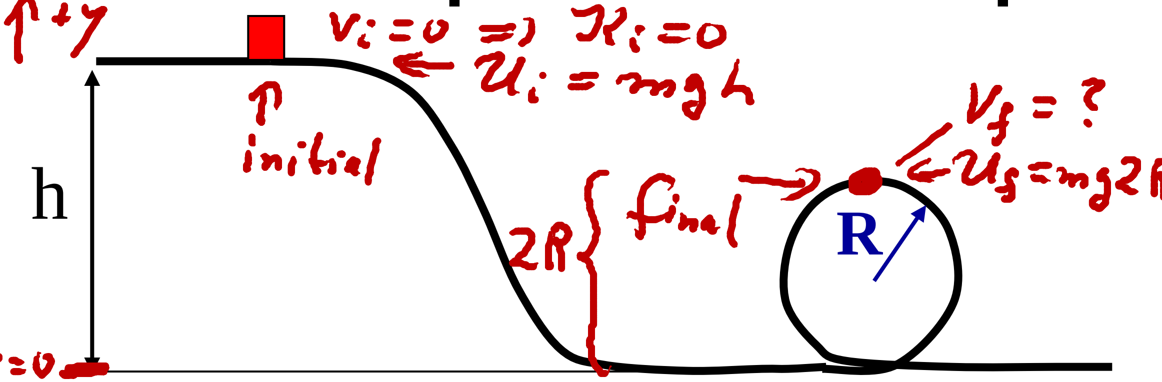
$\uparrow +y$

$v_i = 0 \Rightarrow K_i = 0$

$\leftarrow U_i = mgh$

$v_f = ?$

$\leftarrow U_f = mg2R$



v = ?

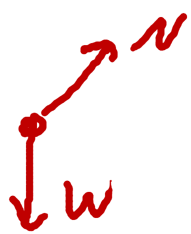
A. 2gh

B. $(2gR)^{1/2}$

C. $[2g(h-R)]^{1/2}$

D. $[2g(h-2R)]^{1/2}$

FBD:



$N \perp$ path always

$\Rightarrow$ no work

$\Rightarrow$ only cons. forces do work here (gravity) $\Rightarrow$ case I

$E_{mech} = \text{const}$

$E_i = E_f = K_i + U_i = K_f + U_f$

$= 0 + mgh = \frac{1}{2} m v_f^2 + mg2R$

$\Rightarrow v_f = \sqrt{2g(h-2R)}$