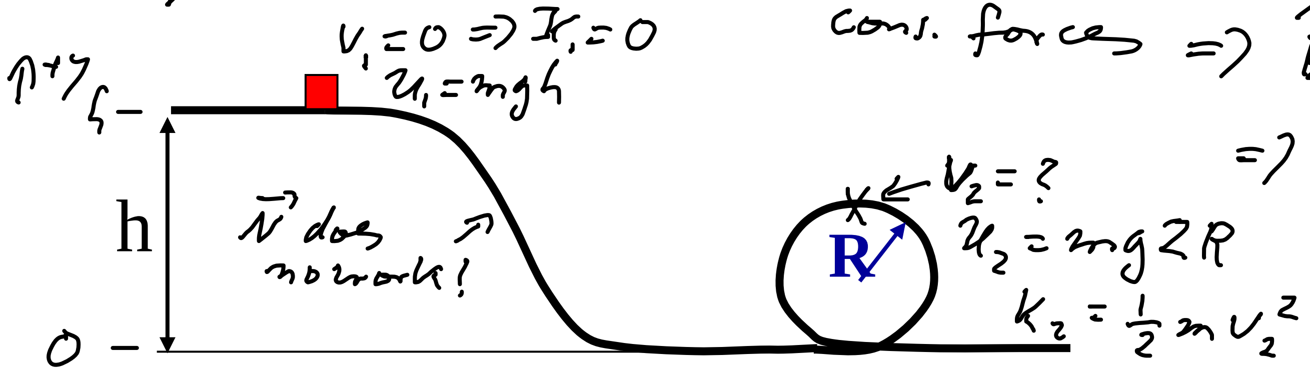
Recap:

Lecture 18

- $\Delta K_{obj} = W_{net\ on\ obj} = W_1 + W_2 + \ldots$ } always true
- Potential Energy: $\Delta U = U_f - U_i = -W_{cons.\ force} = -\int_{x_i}^{x_f} F(x)\,dx$
  defined for conservative forces only
  (work is path independent)
- gravity: $U_g = mgy$ for $\uparrow +y$ up
  (choose $U_g = 0$ at $y=0$)
  spring: $U_{sp}(x) = \frac{1}{2}kx^2$
  (choose $U_{sp} = 0$ at $x=0$)
- mechanical energy: $E_{mech} = K_{obj} + U_{obj}$
- if only cons. forces do work on object: $E_{mech,1} = E_{mech,2} = \text{const}$
- if also non-cons. forces do work: $\Delta E_{mech} = W_{by\ \text{all}\ \text{non cons. forces}}$

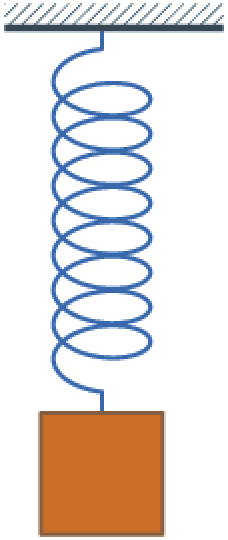
Example: without friction: all work done by cons. forces $\Rightarrow E_{mech,1} = E_{mech,2}$

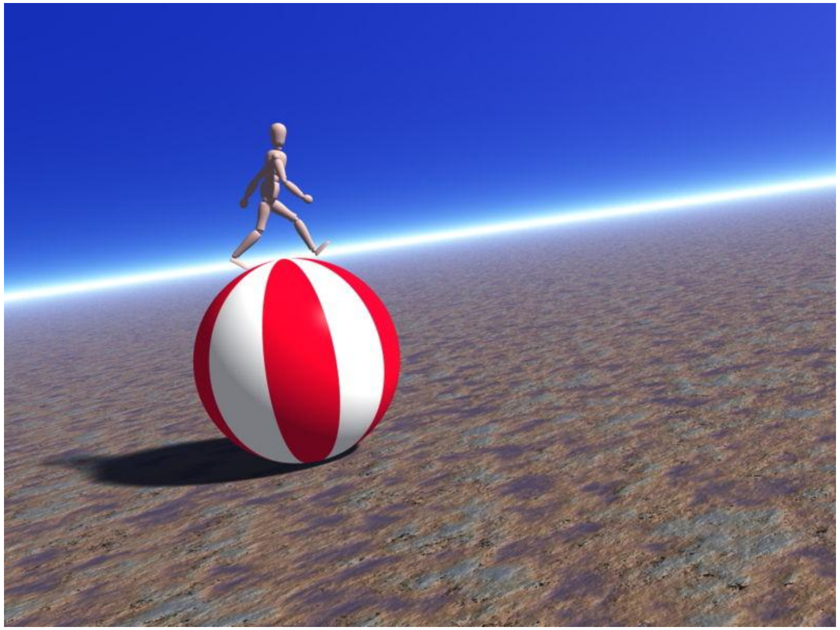
$\Rightarrow K_1 + U_1 = K_2 + U_2$



Today:

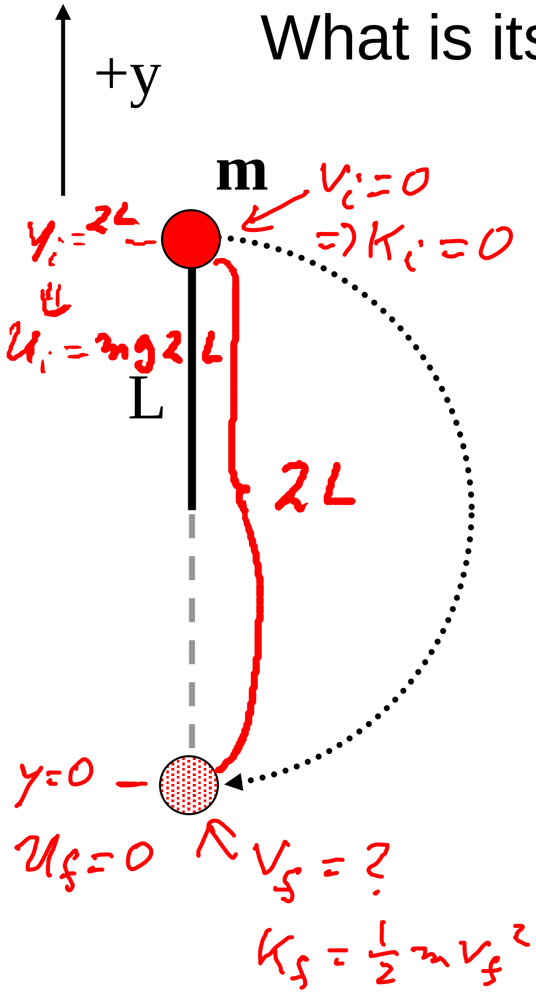
- Potential energy diagrams
- Stable and unstable equilibrium
- Oscillations, simple harmonic motion





A **mass *m*** is connected to a rigid *massless* rod of **length *L***. The mass is released from the vertical as shown.

What is its **speed *v* at the bottom** of its motion?



FBD of mass:

$\vec{T}$ changes direction ⇒ hard to solve with NII!

⇒ use energy

• $\vec{T} \perp$ to path ⇒ does no work!

⇒ all work done by cons. forces here (gravity)

⇒ $E_{mech,i} = E_{mech,f}$

$$E_i = K_i + U_i = E_f = K_f + U_f$$

$$0 + mg2L = \frac{1}{2} m v_f^2 + 0$$

$$\Rightarrow v_f = \sqrt{4gL}$$

v = ?

A. **gL**

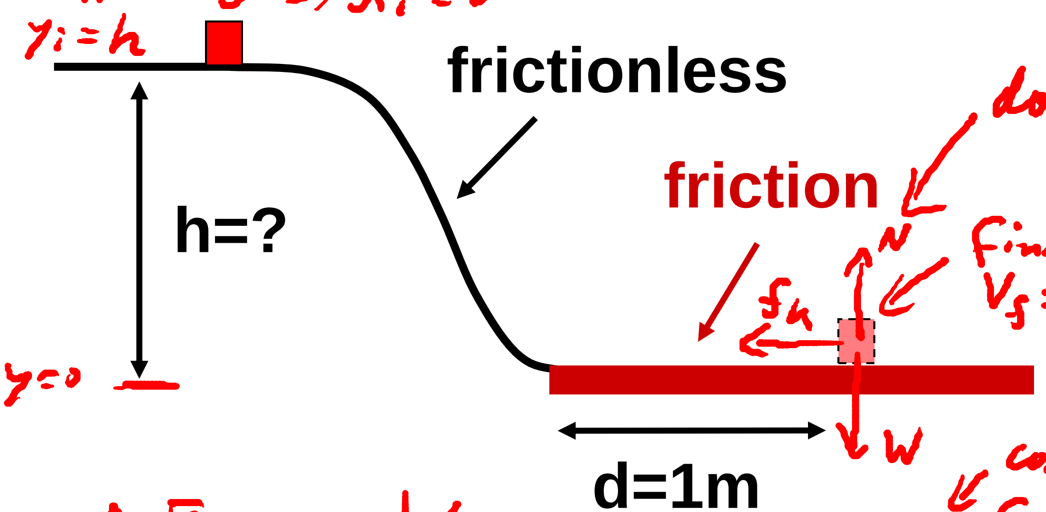
B. **$(gL)^{1/2}$**

C. **$(2gL)^{1/2}$**

D. **$(4gL)^{1/2}$**

A block, ***initially at rest,*** slides down a frictionless slide. At the bottom of the slide the block comes to a stop in a distance **d=1m** on a surface with friction (μ_k**=0.5**).

What is the **height h** of the slide?



h = ?

A. 0.5 m

B. 1 m

C. 1.5 m

D. 2 m

E. Not sure

cos 180°

$$\Delta E_{mech} = W_{\underline{non}\text{-}cons.\,forces} = -f_k d = -\mu_k N d = -\mu_k mg d$$

$N = W$ here

$$E_{mech,i} = K_i + U_i = 0 + mgh$$

$$E_{mech,f} = K_f + U_f = 0 + 0$$

$$\Delta E_{mech} = E_f - E_i$$

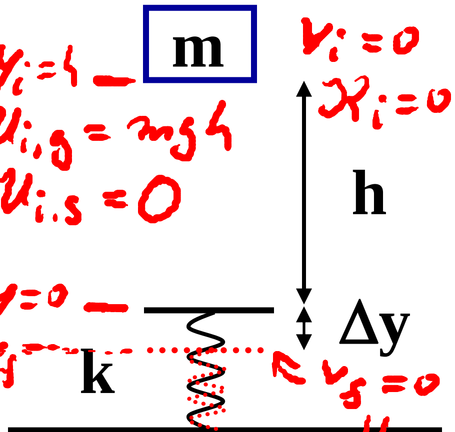
$$= -mgh = W_{non\text{-}cons} = -\mu_k mgd$$

$$\Rightarrow h = \mu_k d = 0.5 \cdot 1m$$

A block of mass *m* is dropped from a height *h* onto a spring with spring constant *k*, as shown.

If the spring compresses a **maximum distance *Δy***, from what **height *h* was the block dropped**?

$\uparrow y$



$y_i = h$

$U_{i,g} = mgh$

$U_{i,s} = 0$

$v_i = 0$

$K_i = 0$

$y = 0$

y_f

$v_f = 0 \Downarrow K_f = 0$

$y_f = -\Delta y$

$\Rightarrow U_{f,g} = mg(-\Delta y)$

$U_{f,s} = \frac{1}{2}k(\Delta y)^2$

Only forces that do work: gravity, spring ⇒ all cons.

$\Rightarrow E_{mech} = \text{const}$

$\Rightarrow E_i = E_f$

$K_i + U_i = K_f + U_f$

$0 + \underline{mgh} + 0 = 0 - mg\Delta y + \frac{1}{2}k\Delta y^2$

$\Rightarrow h = \frac{1}{mg}\left(\frac{1}{2}k\Delta y^2 - mg\Delta y\right)$

h = ?

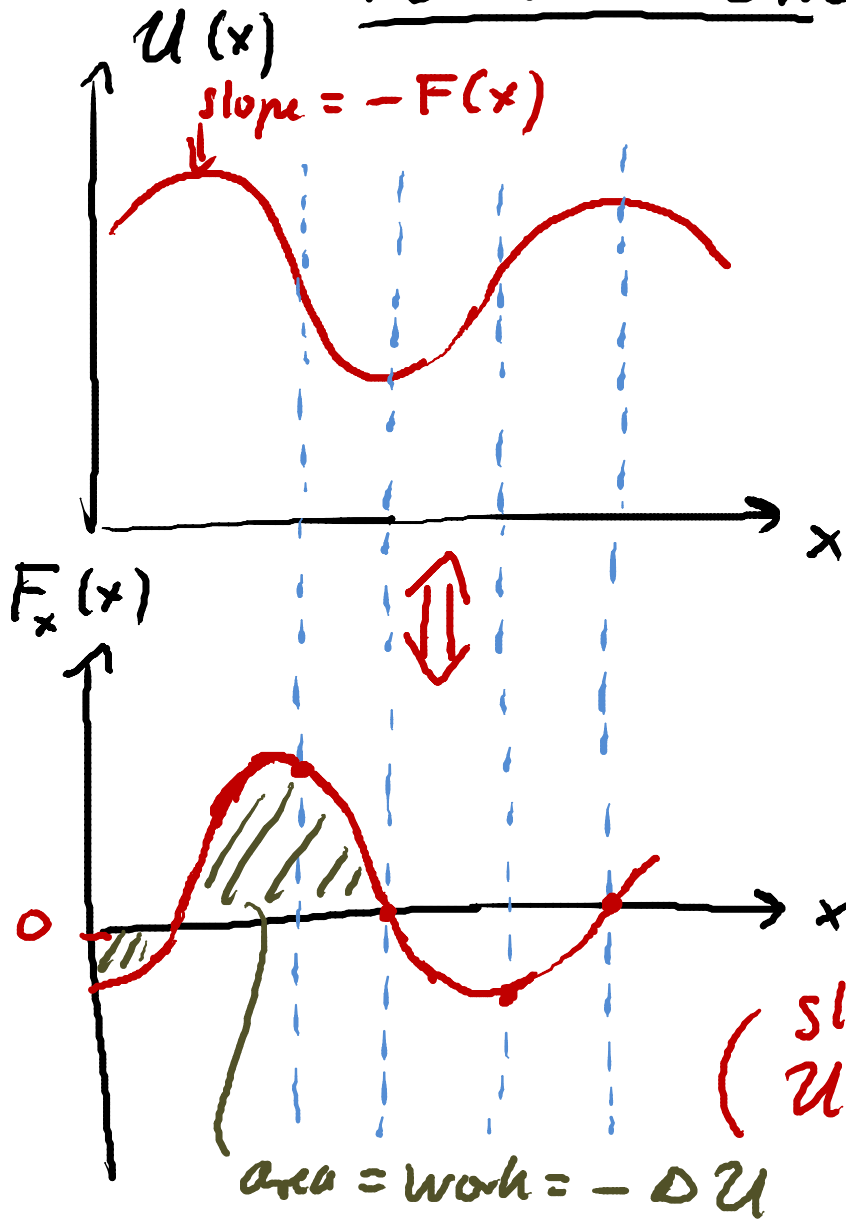
A. kΔy /mg

B. $k(\Delta y)^2$ / 2mg

C. $k(\Delta y)^2$ / 2mg - Δy

D. Not sure

Potential Energy Diagrams: $U - x$ graphs



- very useful to analyze motion
- for 1-D motion along x with cons. force along x $F_{cons,x}(x)$

$$\Rightarrow \boxed{\Delta U = -W_{cons} = -\int_{x_i}^{x_f} F(x)\,dx}$$

$\Rightarrow$ take derivative:

$$\boxed{\frac{dU}{dx} = -F(x)}$$

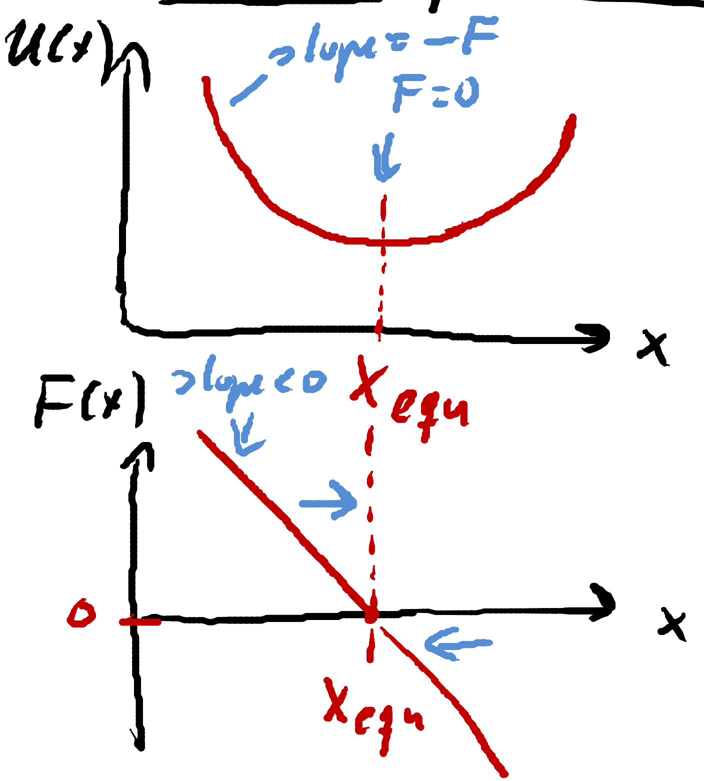
$$\left(\begin{array}{c}\text{slope of}\\ U\text{-}x\text{ graph}\end{array}\right) = -\left(\begin{array}{c}\text{force component}\\ \text{along } x\text{-direction}\end{array}\right)$$

Equilibrium points: $\boxed{F(x_{equ}) = 0}$

$\Rightarrow$ object placed at rest at these positions will remain at rest

2 kinds:

① Stable equilibrium:



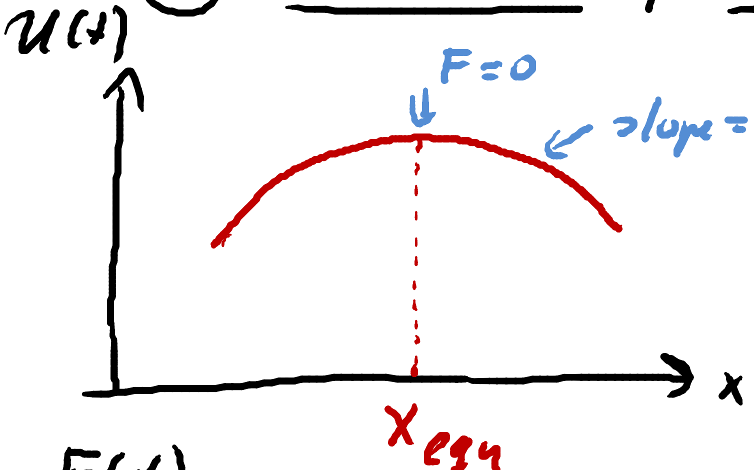
when displaced by some Δx from equilibrium, $F(x_{equ} + \Delta x)$ points toward x_{equ}.

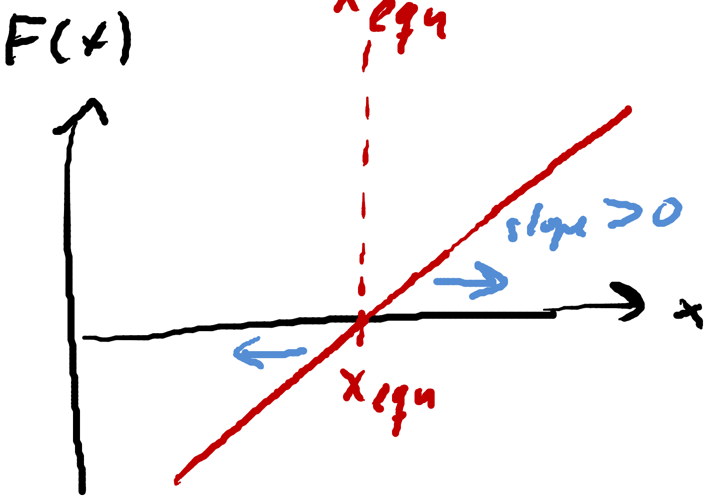
$\Rightarrow$ $F(x_{equ}) = 0$

and $\left.\dfrac{dF}{dx}\right|_{x = x_{equ}} < 0$ at equilibrium

$\Rightarrow$ curvature $U(x) > 0$ at x_{equ}.

② Unstable equilibrium:





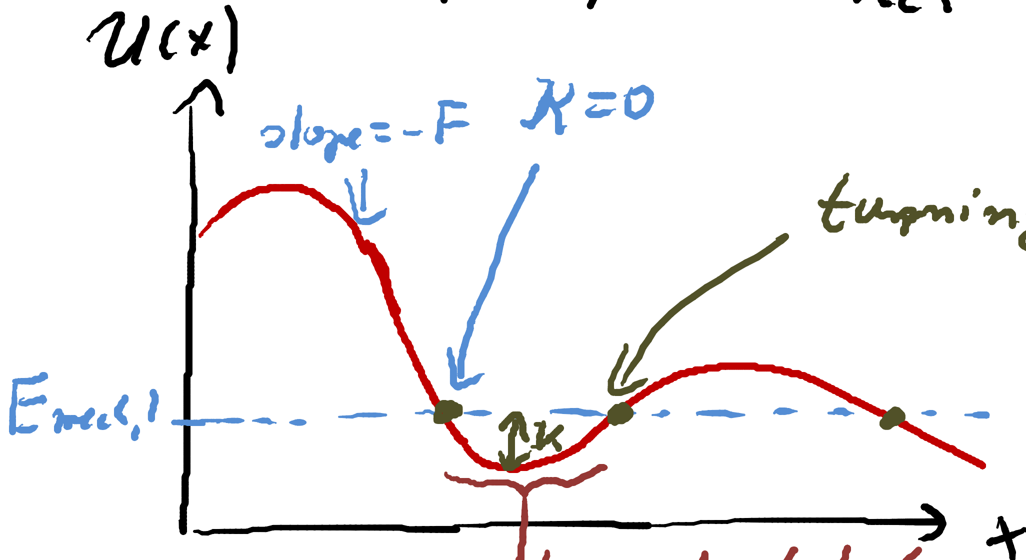
when displaced some Δx from equilibrium, $F(x_{equ}+\Delta x)$ points <u>away</u> from x_{equ}.

$$\boxed{\Rightarrow F(x_{equ}) = 0 \quad \text{and} \quad \left.\frac{dF}{dx}\right|_{x=x_{equ}} > 0}$$

$\Rightarrow$ curvature of $U(x) < 0$ at x_{equ}.

$\Rightarrow$ If $E_{mech} = K + U$ is conserved $\Rightarrow$ $E_{mech} = \text{const}$ for all x!

$\Rightarrow$ $K(x) = E_{mech} - U(x)$



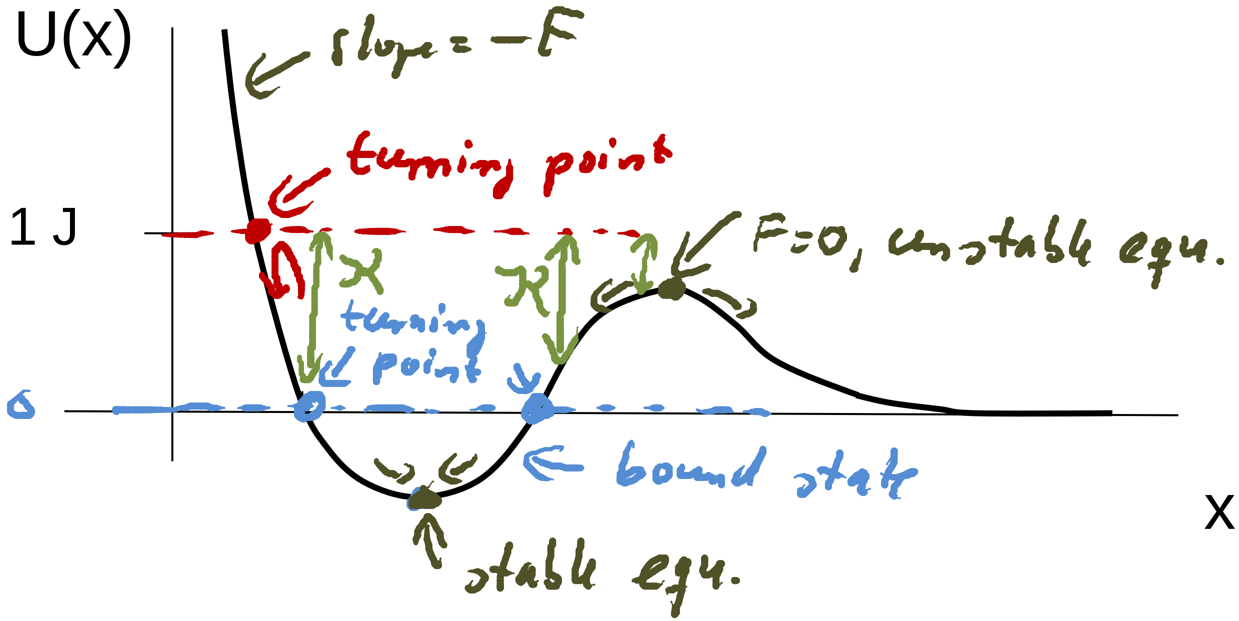
turning point: $K(x) = E_{mech} - U(x) = 0$

$\Rightarrow V = 0$

bound state (confined by two turning points)

for $E_{mech,1}$: 3 turning points
1 bound state

Consider the potential energy curve U(x) shown below:



Stable equilibrium?
Unstable equilibrium?

1. **E=0 J**: Turning points?

2. **E=1 J** : Turning points?

Oscillations:

- any repetetive motion
- SHM: Simple Harmonic motion

$\equiv$ motion that is sinusoidial in time

$$x(t) = X_{max} \cos(\omega t + \phi)$$

or $$y(t) = Y_{max} \cos(\omega t + \phi)$$

or $$\theta(t) = \underbrace{\theta_{max}}_{\text{peak amplitude of motion}} \cos(\omega \underset{\text{time}}{t} + \underset{\text{phase}}{\phi})$$

$2 \cdot X_{max}$: "peak to peak" amplitude

(cos goes from -1 to $+1$)