Recap:

Lecture 19

- $\Delta K_{obj} = W_{net}$ $\qquad$ $\Delta E_{mech} = W_{by\ \underline{all}\ \underline{non}\text{-cons. forces}}$

$E_{mech} = K_{obj} + U_{obj}$

- $\Delta U = -W_{cons} = -\int_{x_i}^{x_f} F(x)\,dx \iff \frac{dU}{dx} = -F_x(x) = \left\{ \begin{array}{l} \text{slope of} \\ U-x \text{ graph} \end{array} \right\}$

- Equilibrium: $F(x_{equ}) = 0$ $\left\{ \begin{array}{ll} \text{if } \left.\frac{dF}{dx}\right|_{x=x_{equ}} < 0 & : \text{stable} \\ \text{if } \left.\frac{dF}{dx}\right|_{x=x_{equ}} > 0 & : \text{unstable} \end{array} \right.$

- **Simple Harmonic Motion:**

Special kind of oscillation: $x(t) = X_m \cos(\omega t + \phi)$

X_m : peak amplitude

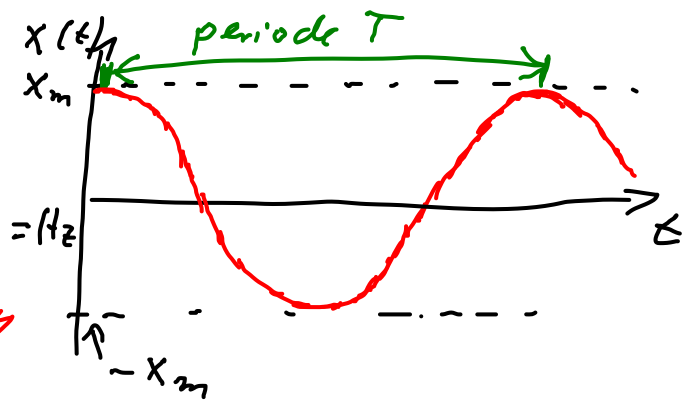
T : periode of motion

indep. of X_m!

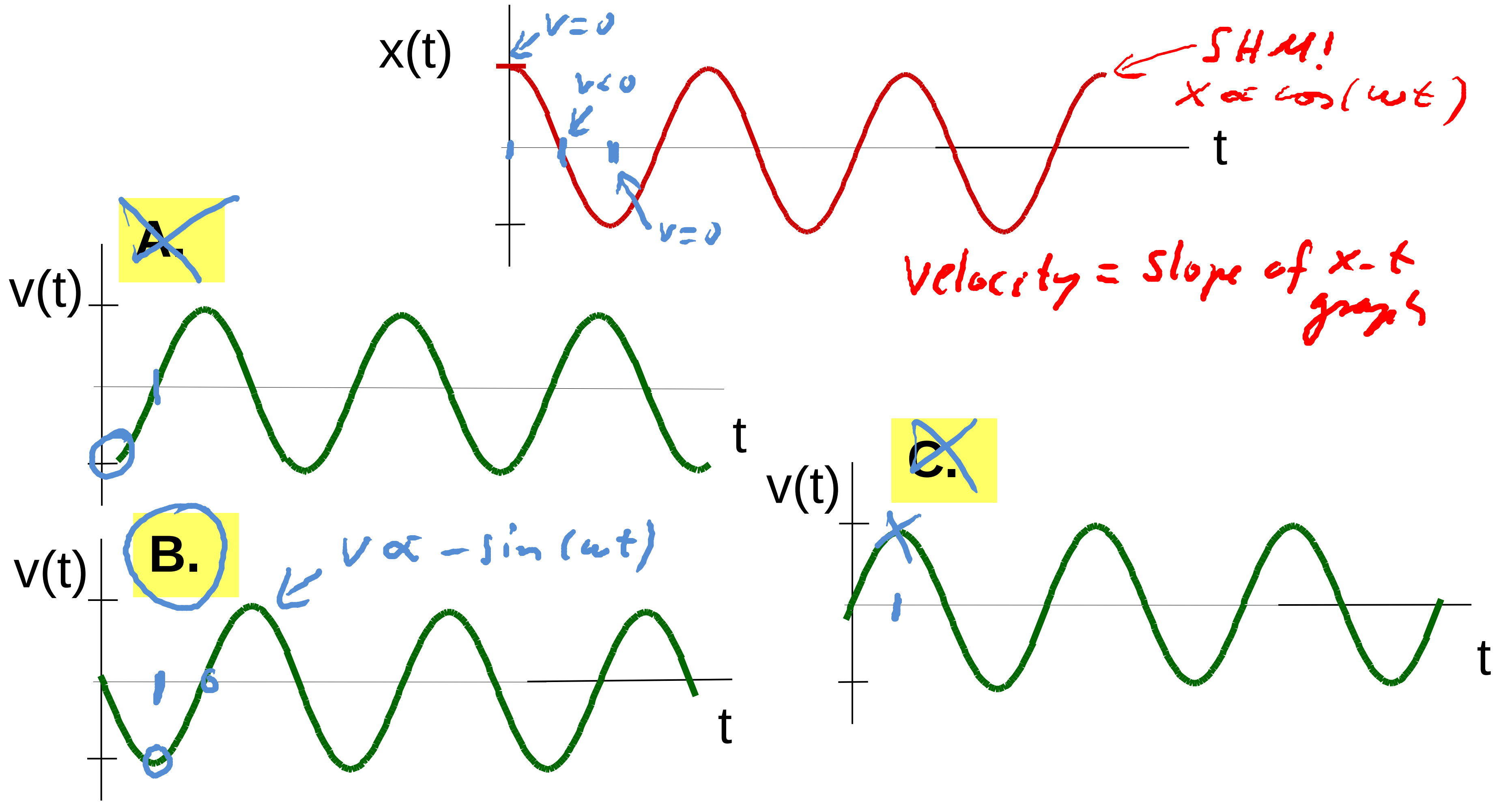
$f = \frac{1}{T}$: frequency of oscil. $[f] = \frac{1}{s} = Hz$

$\omega = 2\pi f = 2\pi/T$ = angular frequency

$[\omega] = \frac{1}{s} =$ "$\frac{rad}{s}$"



Which of the following corresponds to the velocity **v(t)** of the particle whose position x(t) is as shown below?

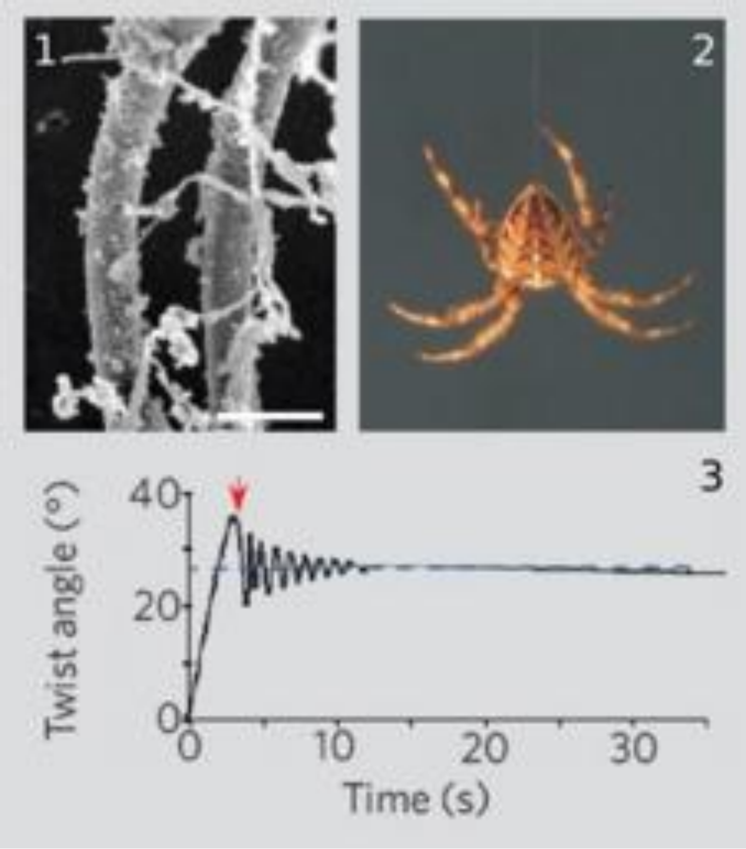


Today:

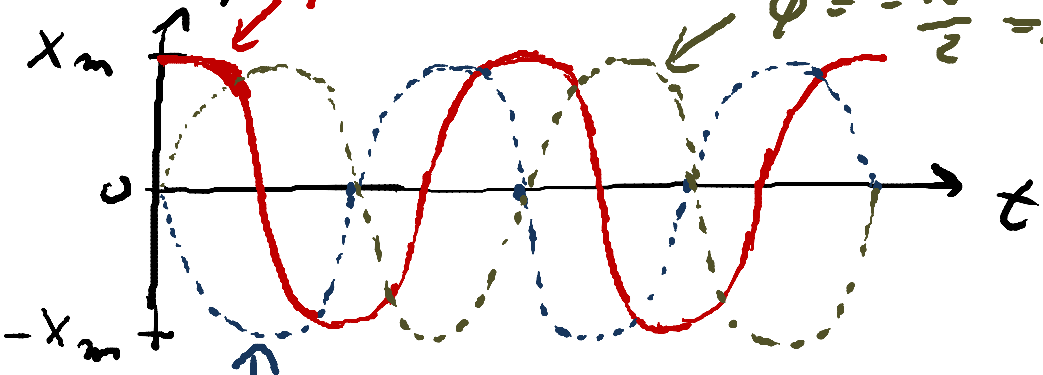
- **Simple Harmonic motion: forces, energy**

Twist angel of spider suspended from its silk thread

⇒ strong damping (that's why they hardly ever twist!)



$$x(t) = X_m \cos(\omega t + \phi)$$



$\phi = 0$

$\phi = -\frac{\pi}{2} \Rightarrow$ shift in $+t$ direction

$\phi = +\frac{\pi}{2} \Rightarrow$ shift in $-t$ direction

periode T (gets back where it started)

→ T = periode of oscillation $[T] = s$

→ f = frequency $\equiv \frac{1}{T}$ $\quad [f] = \frac{1}{s} = Hz$

→ ω = angular frequency $\equiv 2\pi f = \frac{2\pi}{T}$

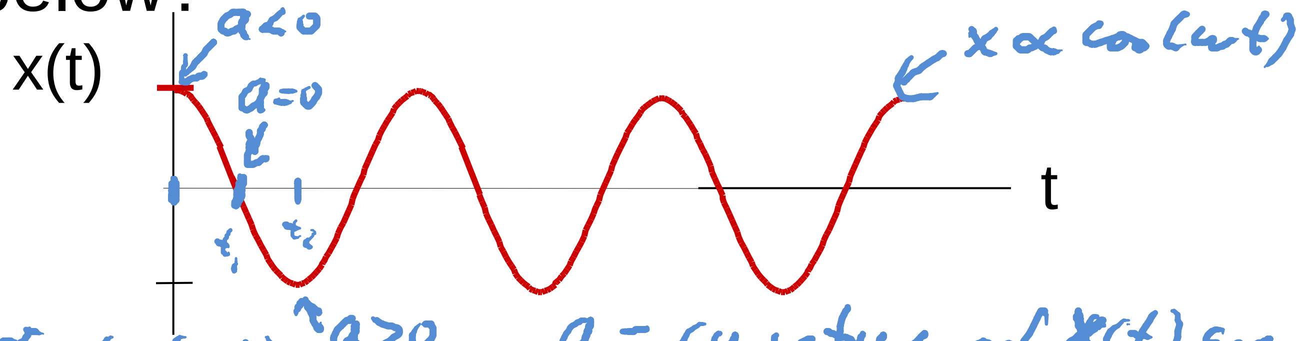
$[\omega] = \frac{1}{s} = \frac{\text{"rad"}}{s}$ "rad" is <u>not</u> a unit, just indicate factor of "2π" in equation so $\cos(\omega(t+T)+\phi) \overset{!}{=} \cos(\omega t+\phi)$ $\Big\}$ returns to where it started

$\Rightarrow \omega(t+T) = \omega t + \omega T \overset{!}{=} \omega t + 2\pi$

$\rightarrow$ argument of cos must increase by 2π during $\Delta t = T$

$\Rightarrow \omega T = 2\pi \Rightarrow \underline{\omega} = \frac{2\pi}{T} = \underline{2\pi f}$ as <u>before</u>

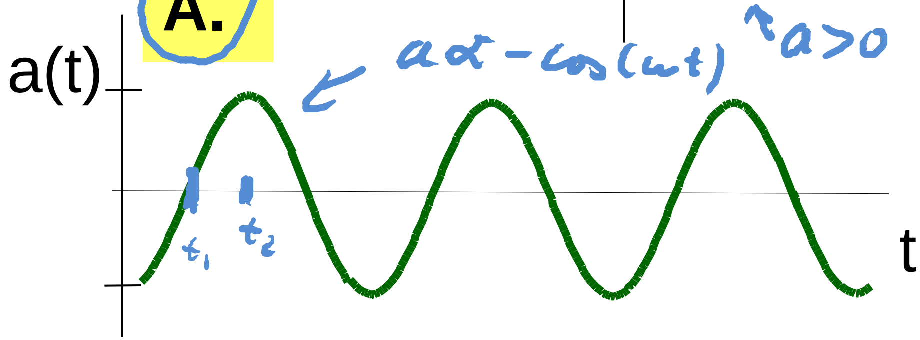
Which of the following corresponds to the acceleration **a(t)** of the particle whose position x(t) is as shown below?



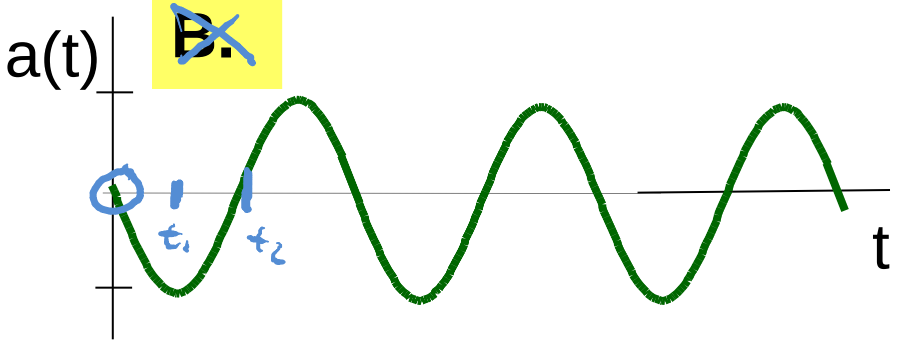
a = curvature of x(t) graph

$\Rightarrow a(t) \propto -x(t)$

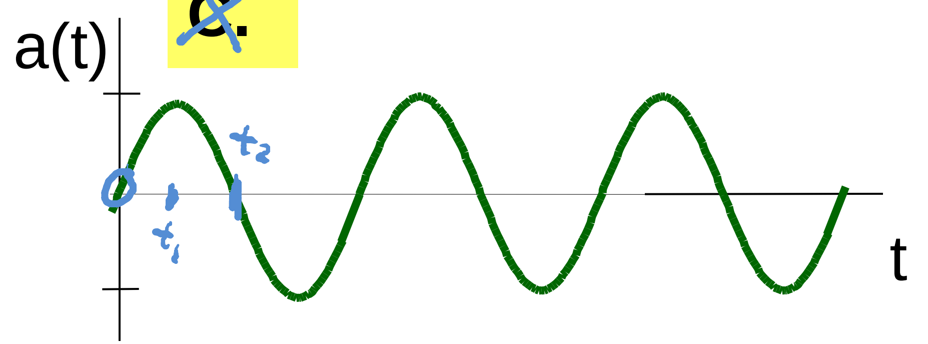
A.



~~B.~~



~~C.~~



$\Rightarrow a(t) \propto -x(t)$ in SHM why?

① SHM: $x(t) = X_{max} \cos(\omega t + \phi)$

derivative: $\frac{d}{dt} \cos(f(t)) = -\sin(f(t)) \cdot \overbrace{\frac{df(t)}{dt}}^{=\omega \text{ here}}$ } chain rule

here: $f = \omega t + \phi$

② velocity: $v(t) = \frac{dx}{dt} = -\omega X_{max} \sin(\omega t + \phi)$

$= -V_{max} \sin(\omega t + \phi)$

with max. speed $\boxed{V_{max} = \omega X_{max}}$

③ acceleration: $a(t) = \frac{dv}{dt} = -\omega^2 X_{max} \cos(\omega t + \phi)$

$= -a_{max} \cos(\omega t + \phi)$

with max. accel. $\boxed{a_{max} = \omega^2 X_{max}} = \omega(\omega X_{max})$

$= \underline{\omega V_{max}}$

$\Rightarrow$ compare ① and ③

$$\boxed{a(t) = -\omega^2 x(t)}$$

$\rightarrow a(t) \propto -x(t)$ ✓ in SHM!

What produces SHM? $\rightarrow$ i.e. $a(t)$ in SHM

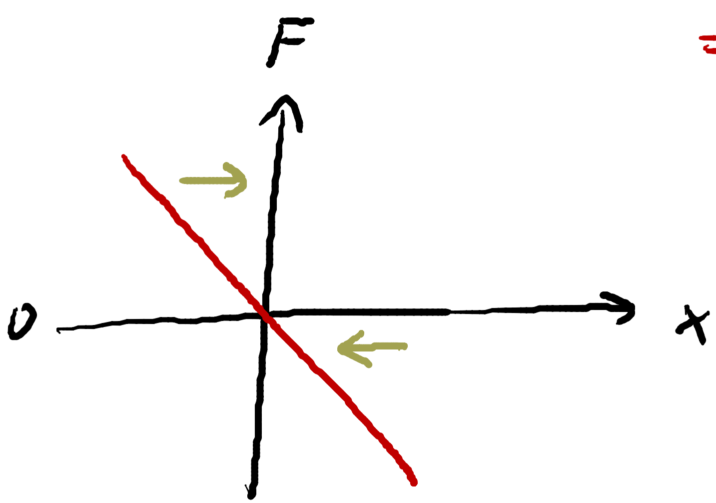
$\Rightarrow$ N II: Force causes acceleration!

N II: $\sum F = F_{net} = m\,a(t) = -m\,\omega^2 x(t)$ } for SHM

$a = -\omega^2 x(t)$

$\Rightarrow$ Object will do SHM, if $\boxed{F_{net}(t) = -m\,\omega^2 x(t)}$

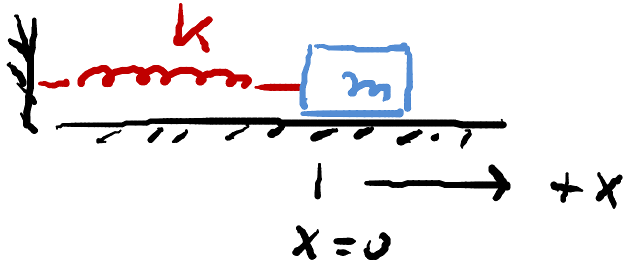
$\Rightarrow$ whenever: $F_{net} \propto -x \Rightarrow$ **SHM!**



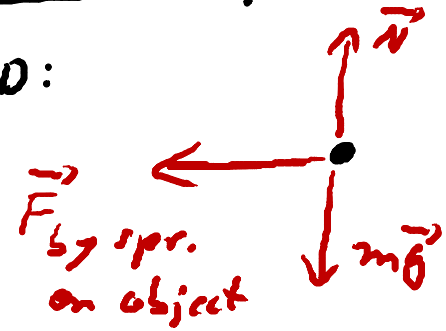
$x=0$: stable equilibrium

- restoring force (points to $x=0$ equilibrium)
- $|F|$ proportional to $|x|$ (and not x^2, $|x|^3$ ...)

• Special case 1: Mass on spring



FBD:



(no friction)

motion of m? $\Sigma F = F_{net} = F_{\text{by spring on obj}} = -kx$

linear restoring force $\Rightarrow$ SHM!

$\Rightarrow$ for SHM: need $F_{net} = -m\omega^2 x$

for mass on spring: $F_{net} = -k\ x$

} SHM! with $k = m\omega^2$

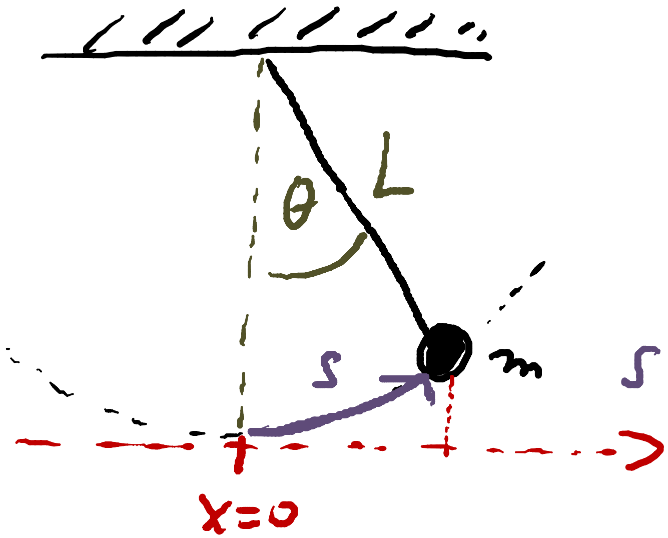
$\Rightarrow$ "natural angl. frequency" of oscillation: $\boxed{\omega_0 = \sqrt{\frac{k}{m}}} = 2\pi f_0$

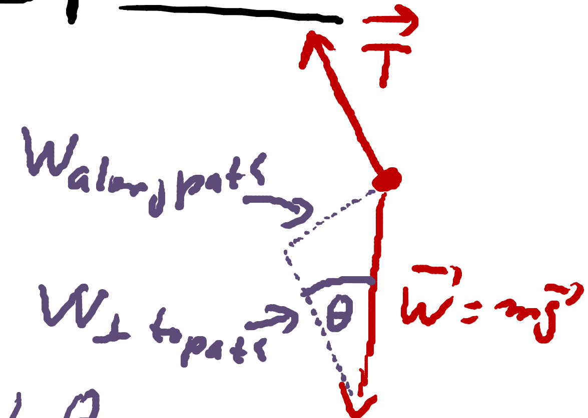
indicates "natural" frequency without damping / friction

Note: ω is independent of x_{max} (and so is $T = \frac{2\pi}{\omega}$)

$$\Rightarrow \boxed{T_0 = 2\pi\sqrt{\frac{m}{k}}}$$

Special case 2: Simple pendulum





s : arc length $= L\theta$

↑ in "rad"

x horizontal displacement

$\Rightarrow F_{\text{along path}} = -|W_{\text{along path}}| = -mg\sin\theta$

for <u>small</u> θ : $\sin\theta \simeq \theta = \frac{s}{L} \approx \frac{x}{L}$

↑ in "rad" not deg

$\Rightarrow F_{\text{along path}} \simeq -mg\frac{x}{L}$ for small θ

↑ linear restoring force $\Rightarrow$ SHM for small θ

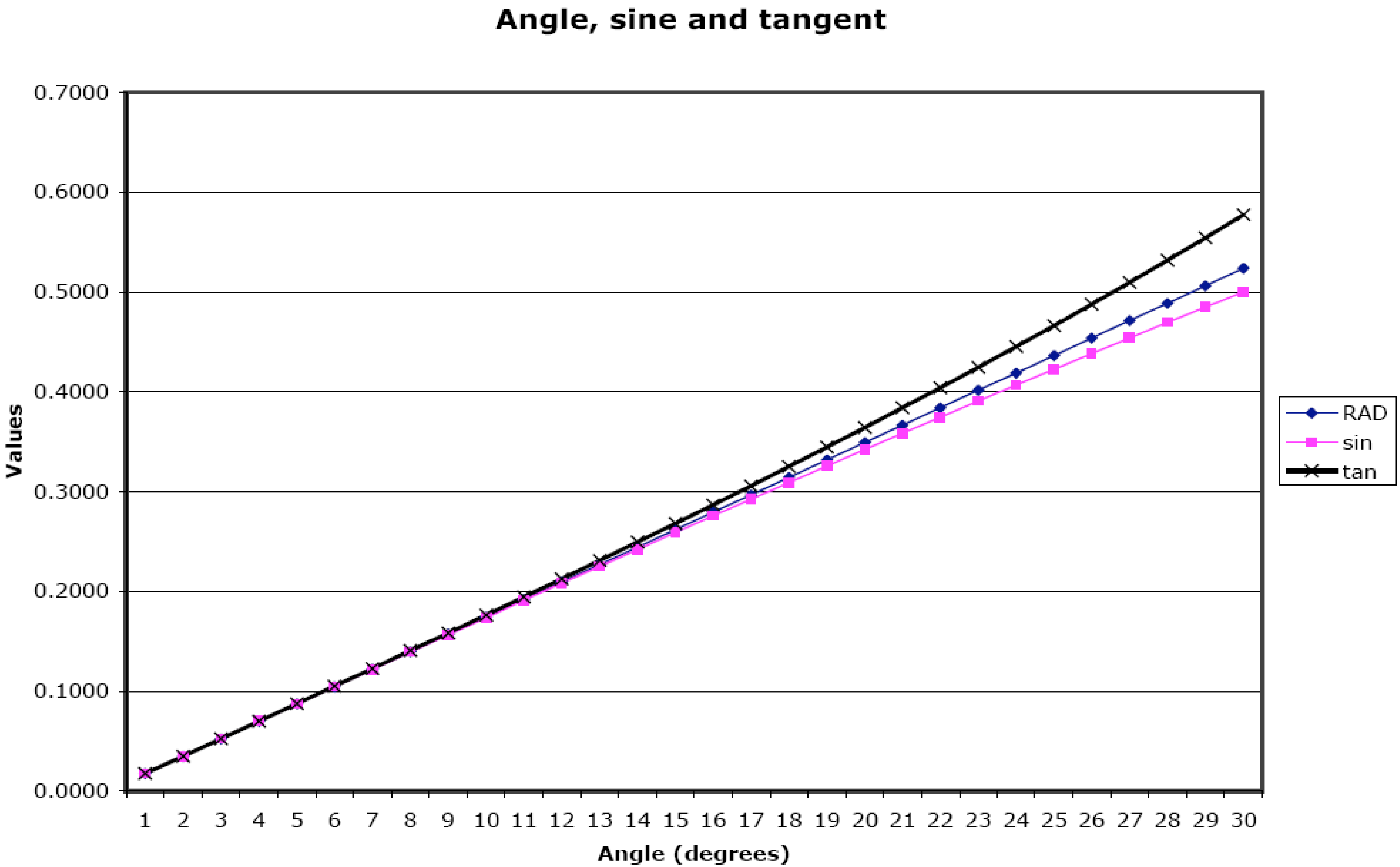
$\Rightarrow$ for SHM, need $F_{net} = -m\omega^2 x$

for simple pendulum at small θ $F_{net} = -m\frac{g}{L}x$

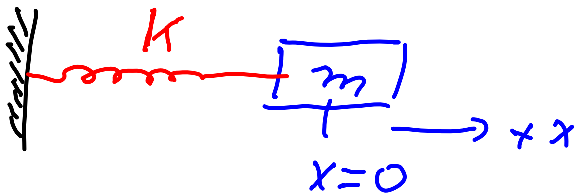
$\Bigg\}$ SHM with

$\omega_0 = \sqrt{g/L}$

$\Rightarrow T_0 = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{L}{g}}$


Angle, sine and tangent
0.7000
0.6000
0.5000
0.4000
0.3000
0.2000
0.1000
0.0000
Values
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30
Angle (degrees)
RAD
sin
tan

Energy in SHM:



assume that no work is done by non-cons. forces (i.e. no friction, no applied forces, ...)

$\Rightarrow$ for SHM, have:

$$E_{mech} = \mathcal{K} + \mathcal{U}_{sp} = \text{const} = \frac{1}{2} m v^2(t) + \frac{1}{2} k x^2(t)$$

$$\underline{\underline{E_{mech}}} = \frac{1}{2} m \left[-V_{max} \sin(\omega t + \phi)\right]^2 + \frac{1}{2} k \left[X_{max} \cos(\omega t + \phi)\right]^2$$

$V_{max} = \omega X_{max}$ $\qquad$ $k = m\omega^2$

$$= \frac{1}{2} m \omega^2 X_{max}^2 \sin^2(\omega t + \phi) + \frac{1}{2} m \omega^2 X_{max}^2 \cos^2(\omega t + \phi)$$

$$= \frac{1}{2} m \omega^2 X_{max}^2 \left(\underbrace{\sin^2(\omega t + \phi) + \cos^2(\omega t + \phi)}_{=1} \right]$$

$$= \frac{1}{2} m \omega^2 X_{max}^2 = \text{const}! = \underline{\text{indep. of time}}!$$

$$\Rightarrow E_{mech} = \mathcal{K}(t) + \mathcal{U}(t) = \frac{1}{2} m \omega^2 X_{max}^2 = \frac{1}{2} m V_{max}^2 = \frac{1}{2} k X_{max}^2 = \text{const}$$

$V_{max} = \omega X_{max}$ $\qquad$ $k = m\omega^2$

⇒ for SHM:

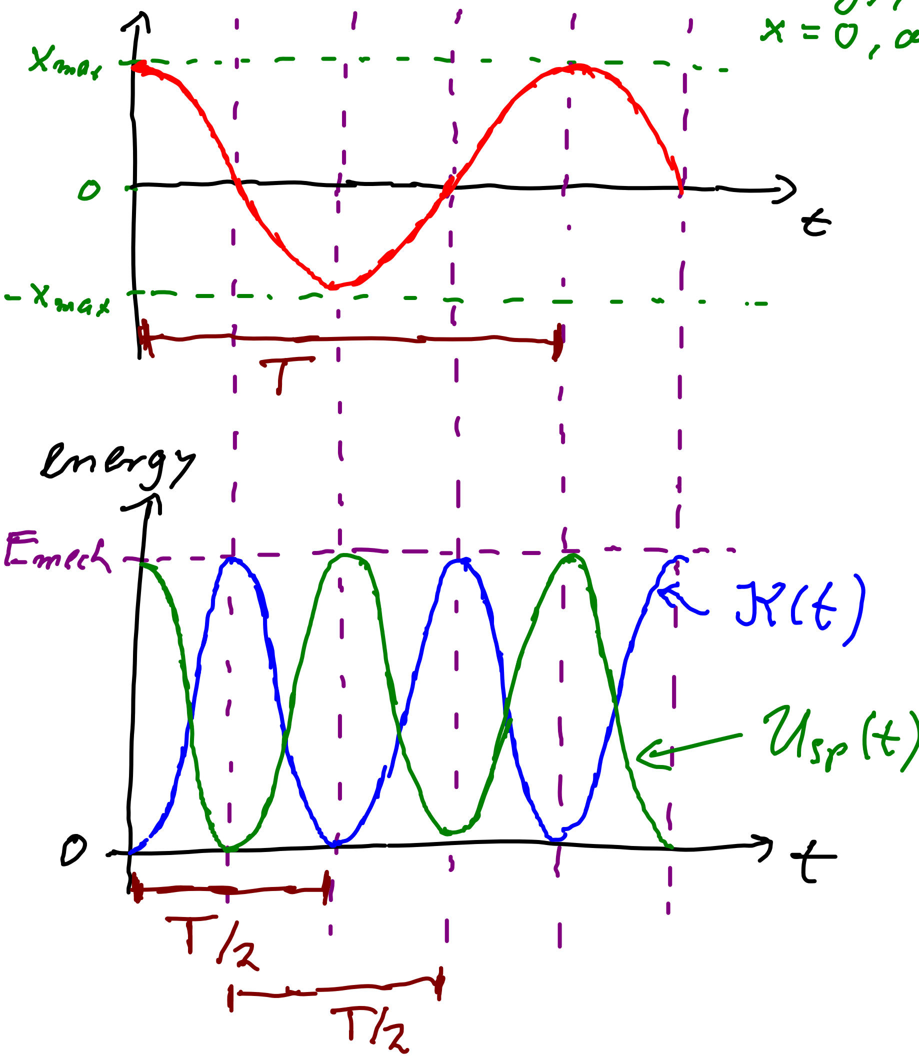
kinetic energy

spring const.

$$E_{mech} = K(t) + U_{sp}(t) = \underbrace{\frac{1}{2} m v_{max}^2}_{\substack{\text{maximum kinetic} \\ \text{energy, when} \\ x = 0 \text{, and } U = 0}} = \underbrace{\frac{1}{2} k x_{max}^2}_{\substack{\text{maximum potential} \\ \text{energy, when} \\ x = \pm x_{max} \text{ and } K = 0}} = const$$

potential energy

$x(t)$



Note:

- $K(t)$ and $U(t)$ are 180° out of phase, so that $K(t) + U(t) = const$
- $x(t)$ oscillates at $\omega = 2\pi/T$

  K, U oscillate at $2\omega = \frac{2\pi}{T/2}$