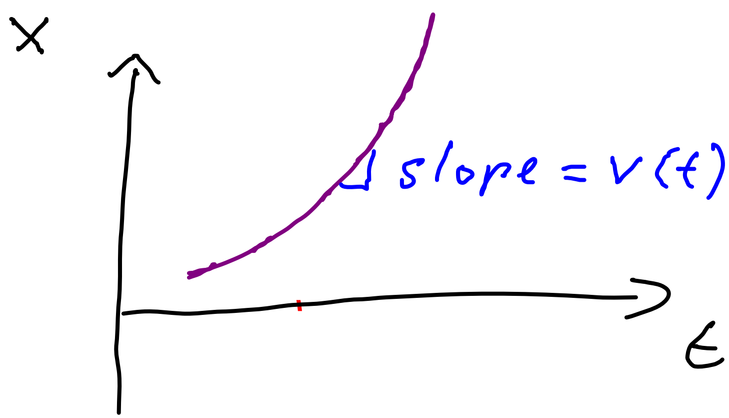
Lecture 2

Recap: Describing Motion in 1-D

- $x(t)$ = position $\qquad [x] = m$
- $\Delta x = x(t_2) - x(t_1)$ = displacement $\qquad [\Delta x] = m$
- $v(t)$ = velocity $= \frac{dx}{dt}$ = rate of change of position $\qquad [v] = \frac{m}{s}$

$$x(t) \underset{\longrightarrow}{\overset{?}{\longleftarrow}} v(t) \underset{\overset{\longrightarrow}{?}}{\overset{?}{\longleftarrow}} a(t)$$

slope of x-t graph

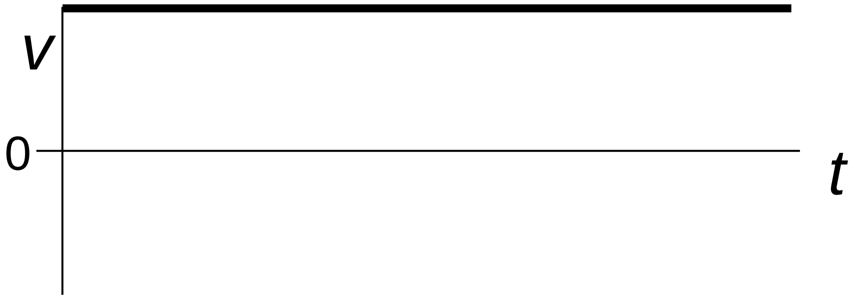


Today:

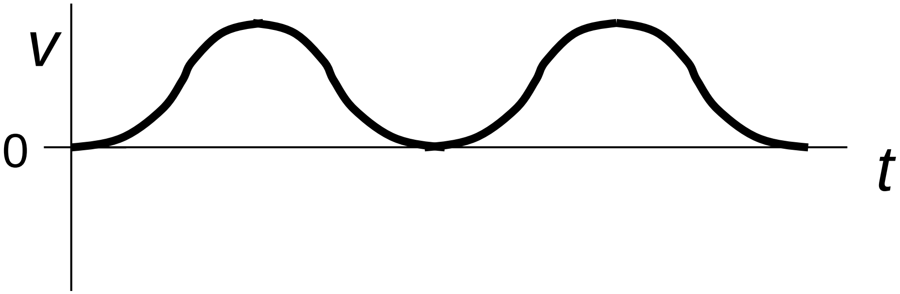
- Describing 1-D motion
  - Acceleration
  - $x(t) \underset{}{\overset{?}{\rightleftarrows}} v(t) \underset{}{\overset{?}{\rightleftarrows}} a(t)$
  - Visualizing motion, reading graphs

Which of the following *v*-*t* graphs best describes the **horizontal** motion of a foot relative to the ground during ordinary walking?

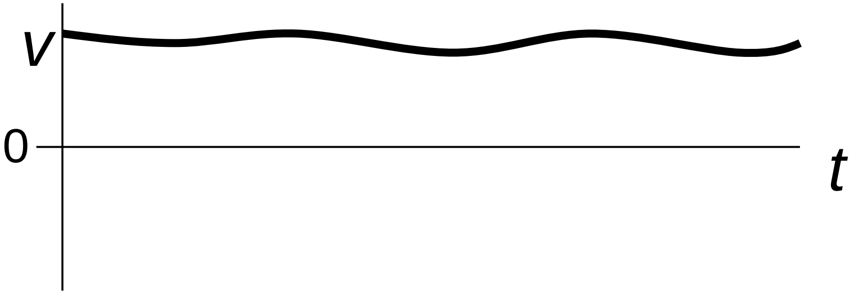
(A)



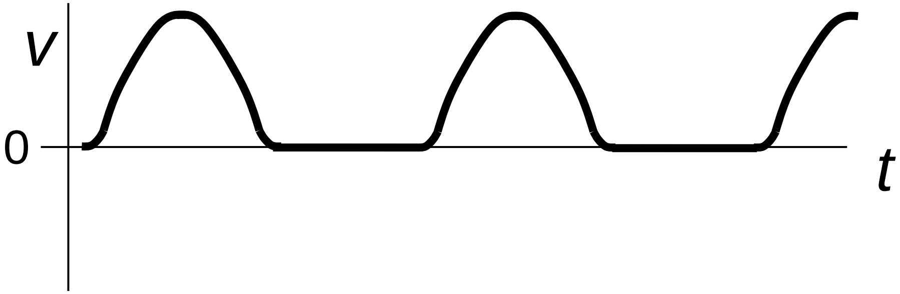
(C)



(B)



(D)



- <u>acceleration</u>:

<u>average</u>: $a_{avg} = \bar{a} = \dfrac{\Delta v}{\Delta t} = \dfrac{v_2 - v_1}{t_2 - t_1}$ $[a] = \dfrac{m}{s^2}$

specific time interval

<u>instantaneous</u>:

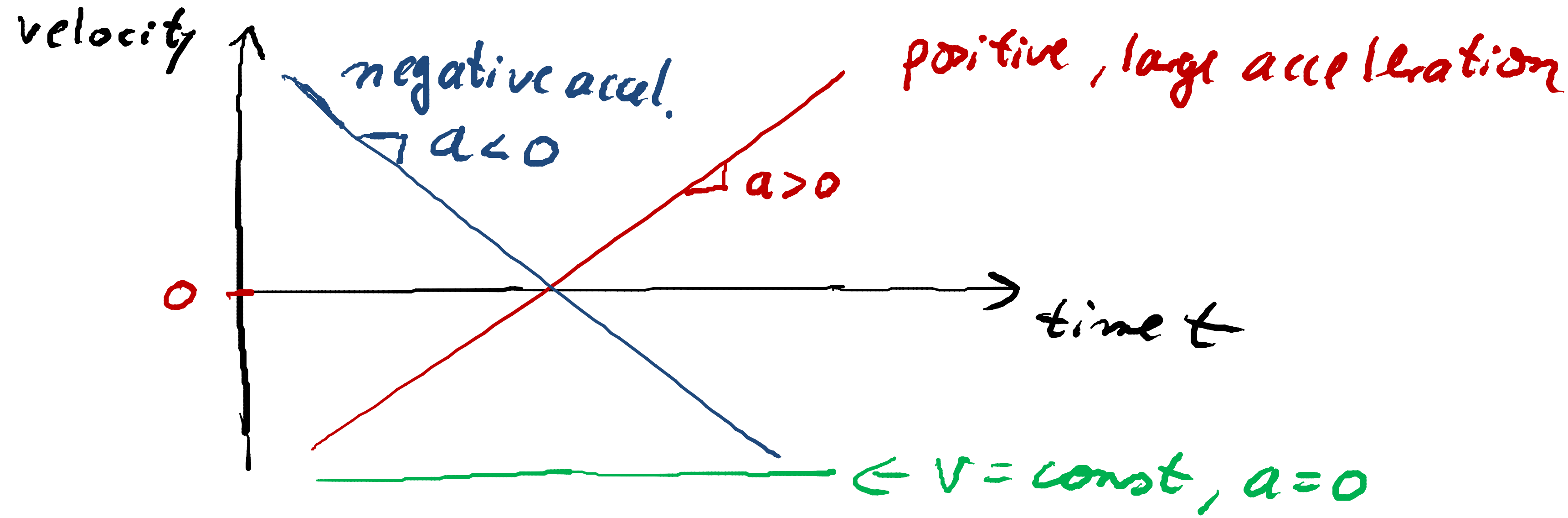
$a(t) = \lim\limits_{\Delta t \to 0} \dfrac{\Delta v}{\Delta t} = \dfrac{dv}{dt} = \begin{cases} = \text{rate of change of velocity} \\ = \text{slope of the velocity vs. time graph} \end{cases}$

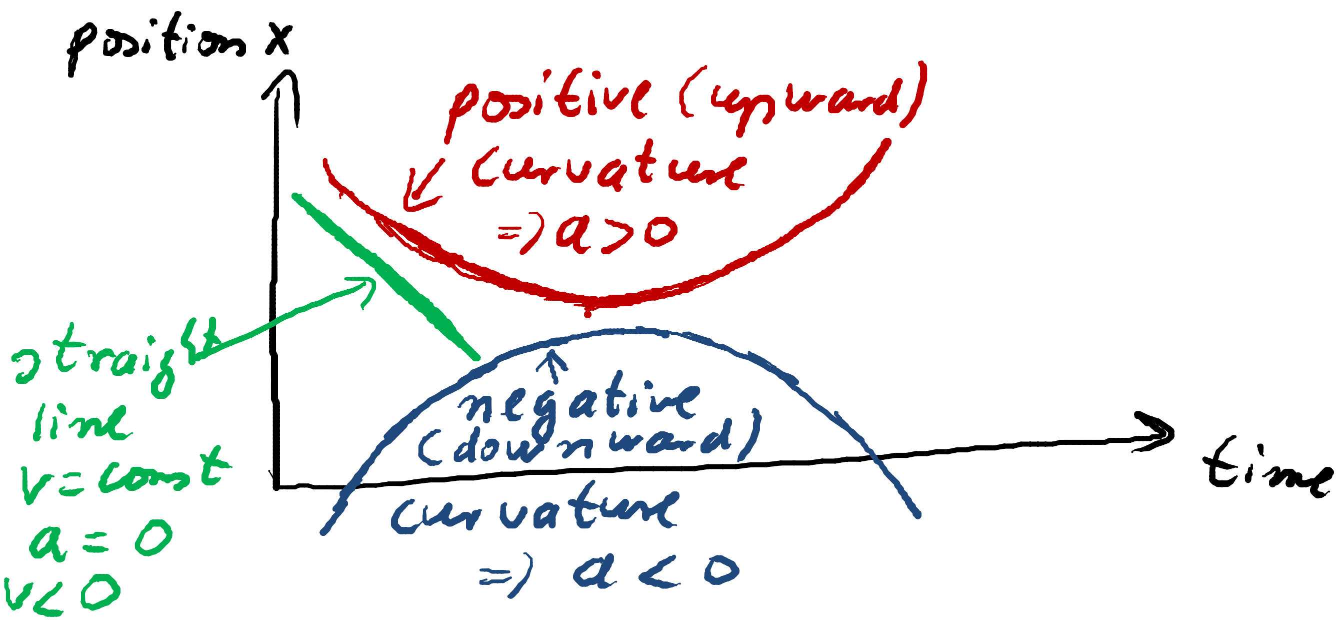
derivative of velocity wrt time

$a(t) = \dfrac{d}{dt} v = \dfrac{d}{dt}\left(\dfrac{dx}{dt}\right) = \dfrac{d^2x}{dt^2}$ ← second derivative wrt. time

$= \dfrac{\text{curvature}}{\text{time graph}}$ of the position vs.

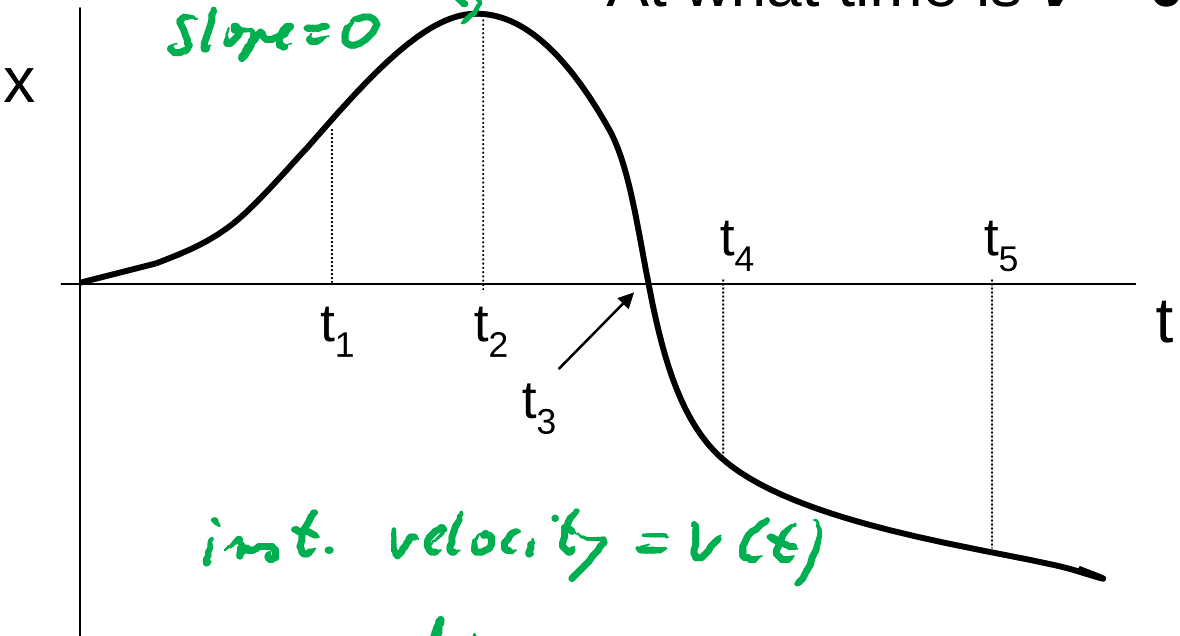
Note: acceleration is a vector ⇒ ± sign in 1-D.


velocity
negative accel.
a<0
positive, large acceleration
a>0
0
time t
⇐ V = const, a=0


position x
positive (upward) curvature
⇒ a>0
straight line
v=const
a=0
v<0
negative (downward) curvature
⇒ a<0
time

An object moves along the *x* axis as shown below.

At what time is ***v* = 0**?



$v=0$

slope $=0$

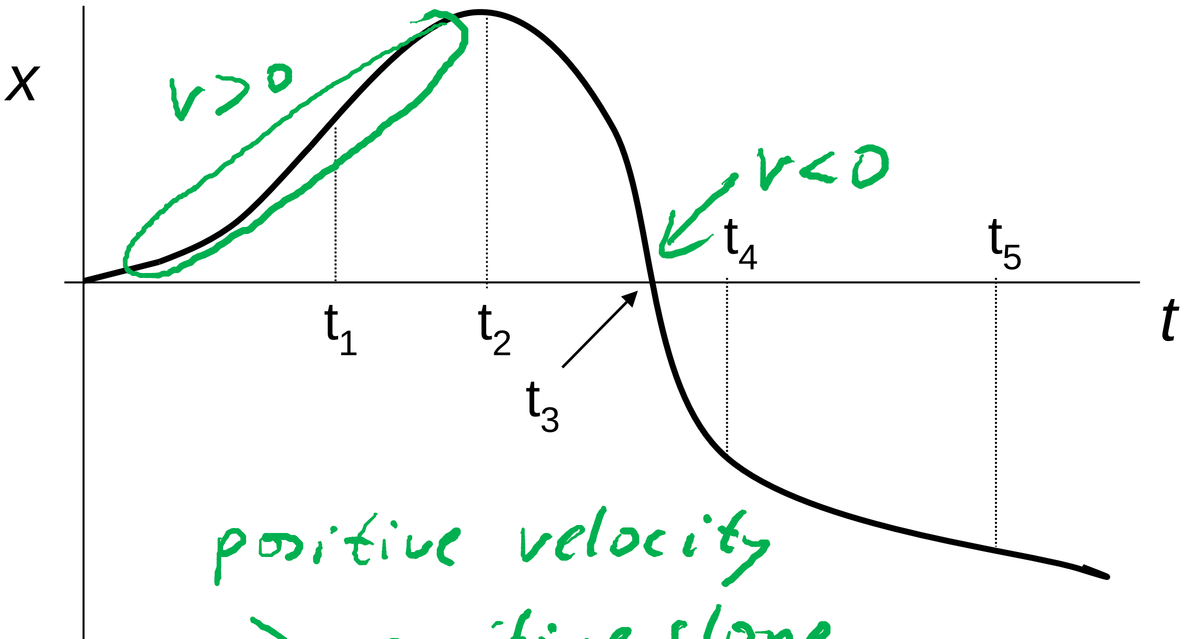
- A. t1
- B. t2
- C. t3
- D. t4
- E. t5

inst. velocity $= v(t)$

$v = \dfrac{dx}{dt}$ = slope of x-t graph

An object moves along the *x* axis as shown below.

At what time does the ***velocity v*** has its largest positive value?



A. t1

B. t2

C. t3

D. t4

E. t5

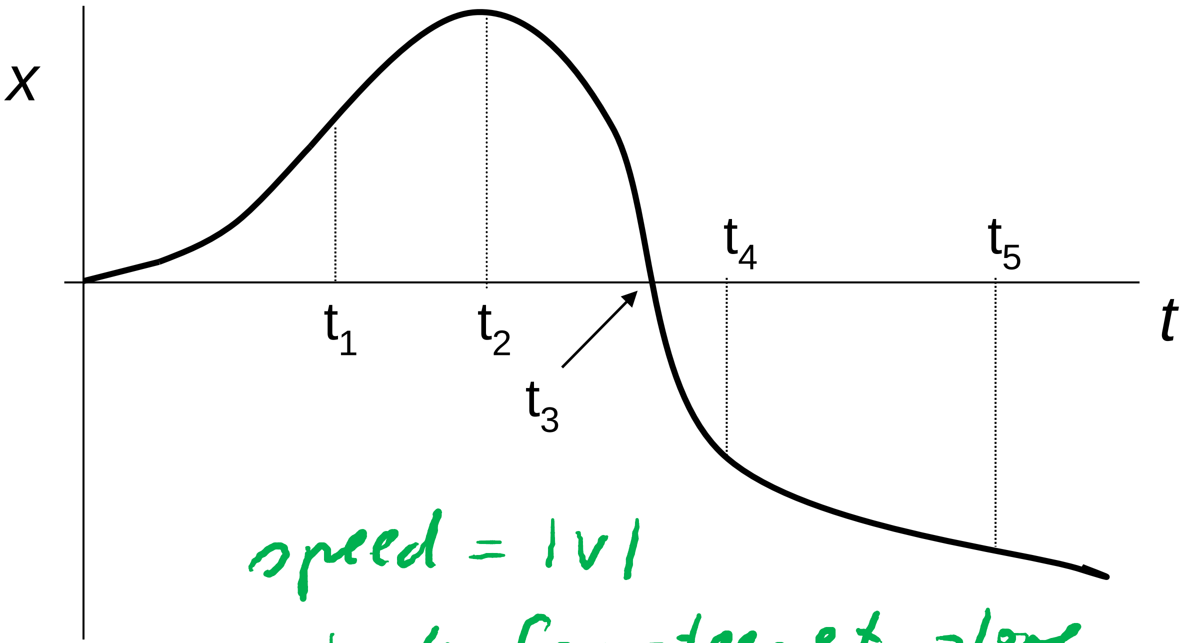
positive velocity

⇒ positive slope

⇒ velocity has direction!

An object moves along the *x* axis as shown below.

At what time is the **speed** of the object the **largest**?

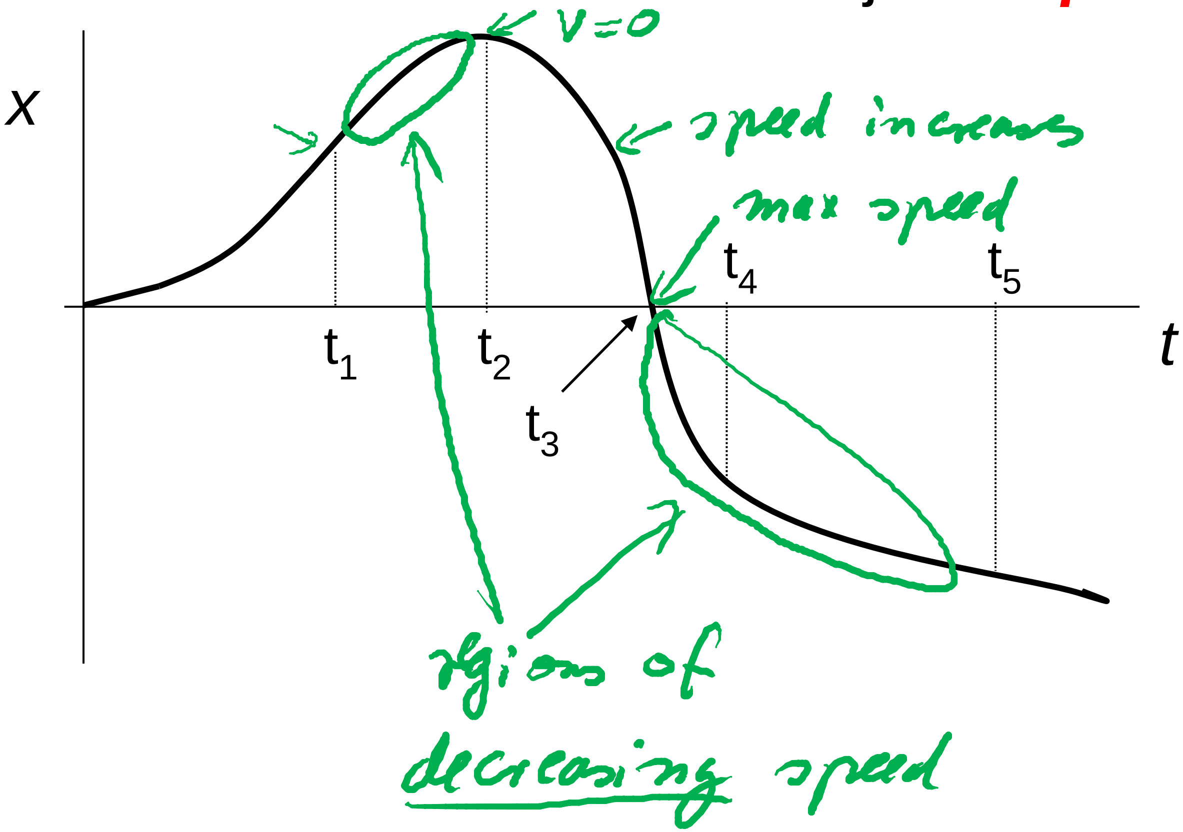


A. t1
B. t2
C. t3
D. t4
E. t5

speed = |v|
look for steepest slope
regardless of sign

An object moves along the x axis as shown below.

At what time is the object's ***speed decreasing***?



A. t1
B. t2
C. t3
D. t4
E. t5

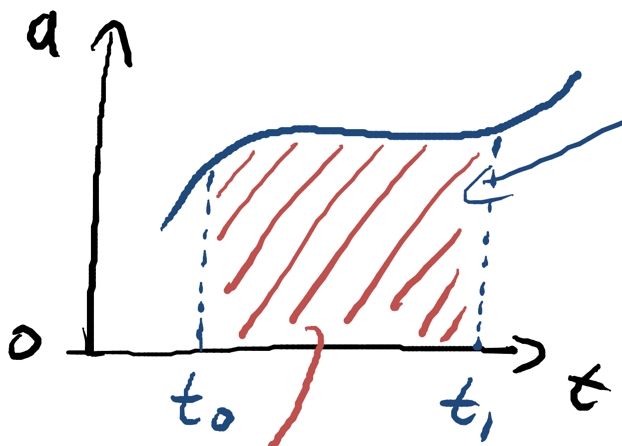
Next:

given either $x(t)$, or $v(t)$, or $a(t)$

$\Rightarrow$ can determine the graphs of the other two

$$\begin{tikzcd} x(t) \arrow[r, shift left, "\text{slope}"] \arrow[rr, bend left, "\text{curvature}"] & v(t) \arrow[l, shift left, "?"] \arrow[r, shift left, "\text{slope}"] & a(t) \arrow[l, shift left, "?"] \end{tikzcd}$$

$a(t) \xrightarrow{?} v(t)$:



area "under" $a-t$ graph

$$= \int_{t_0}^{t_1} a(t)\,dt = \int_{t_0}^{t_1} \frac{dv}{dt}\,dt = \int_{t_0}^{t_1} 1\cdot dv$$

$$= v(t_1) - v(t_0) = \underline{\Delta v}$$

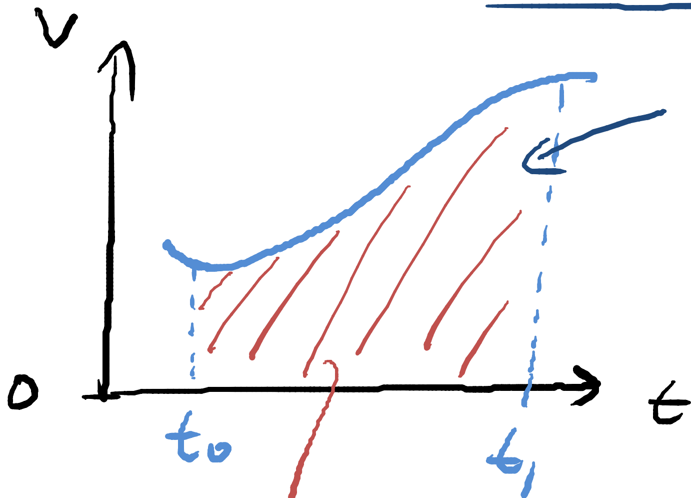
= change in velocity

$$[\text{area}] = \frac{m}{s^2}\cdot s = \frac{m}{s}$$

Note: To get $v(t_1)$, need initial condition
i.e. $v(t_0)$
and $a-t$ graph

$$\Rightarrow \boxed{v(t_1) = v(t_0) + \underbrace{\Delta v}_{=\text{area}}}$$

$$v(t) \xrightarrow{?} x(t):$$



area "under" v–t graph

$$= \int_{t_0}^{t_1} v(t)\,dt = \int_{t_0}^{t_1} \frac{dx}{dt}\,dt = \int_{t_0}^{t_1} 1\,dx$$

$$= x(t_1) - x(t_0) = \underline{\underline{\Delta x}}$$

= change in position

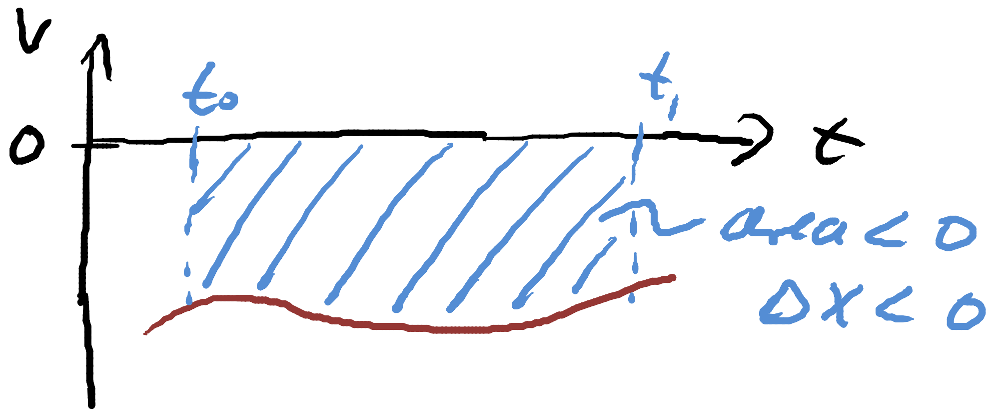
= displacement

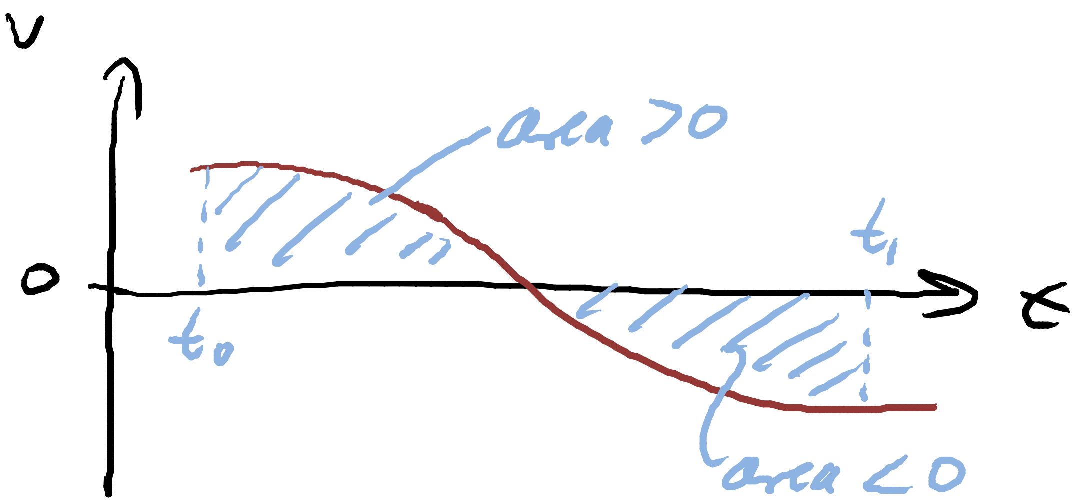
[area] $= \frac{m}{s} \cdot s = \underline{\underline{m}}$

Note: To get $x(t_1)$, need to have initial position $x(t_0)$:

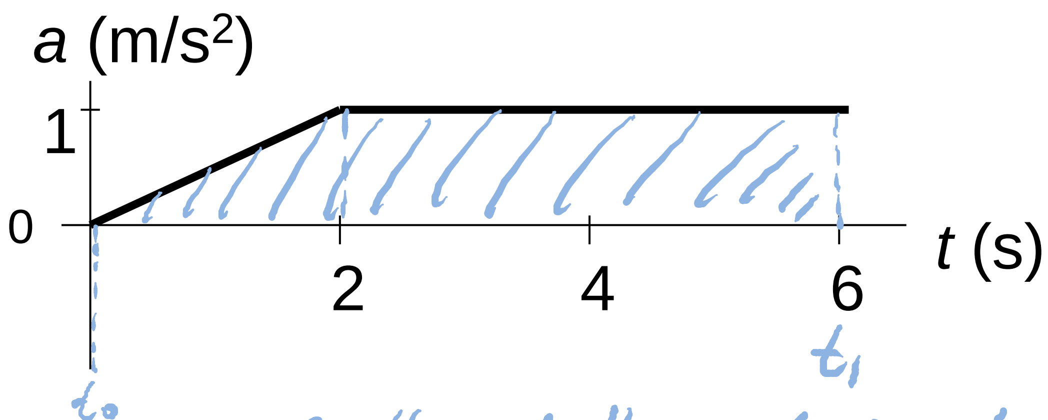
$$\Rightarrow \boxed{x(t_1) = x(t_0) + \underbrace{\Delta x}_{=\text{area}}}$$

• "area under" = area between curve and $v=0$ here





At t=0 the velocity of a particle is 2 m/s. If its acceleration a(t) is as shown, what is its **velocity at t = 6s**?



$v(t$ = 6s) =?

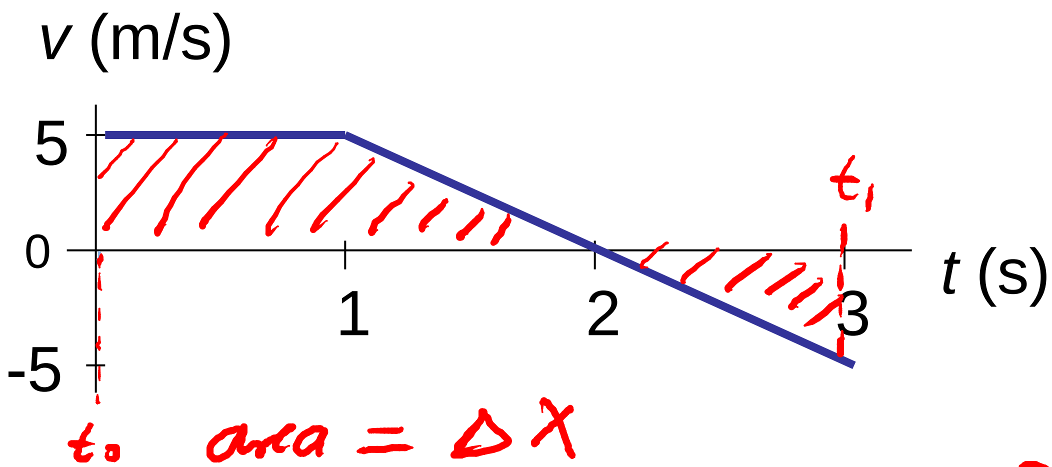
A. 2 m/s
B. 4 m/s
C. 6 m/s
D. 7 m/s
E. 8 m/s

area "under" a-t graph = $\underline{\Delta V}$

$= 1\ \frac{m}{s} + 4\ \frac{m}{s} = 5\ m/s$

$v(6s) = v(t=0) + \Delta V = 2\ \frac{m}{s} + 5\ m/s = 7\ m/s$

At t=0 the position of a particle is x = -5 m. If its velocity $v(t)$ is as shown, what is its **position at t = 3 s**?



$x(t$=3s) = ?

A. **-5 m**

B. **0 m**

C. **2.5 m**

D. **5 m**

E. **7.5 m**

area $= \Delta X$

$= 5\,m + 2.5\,m - 2.5\,m = 5\,m$

$\Rightarrow X(t=3s) = x(t=0) + \Delta X = -5\,m + 5\,m = \underline{\underline{0\,m}}$

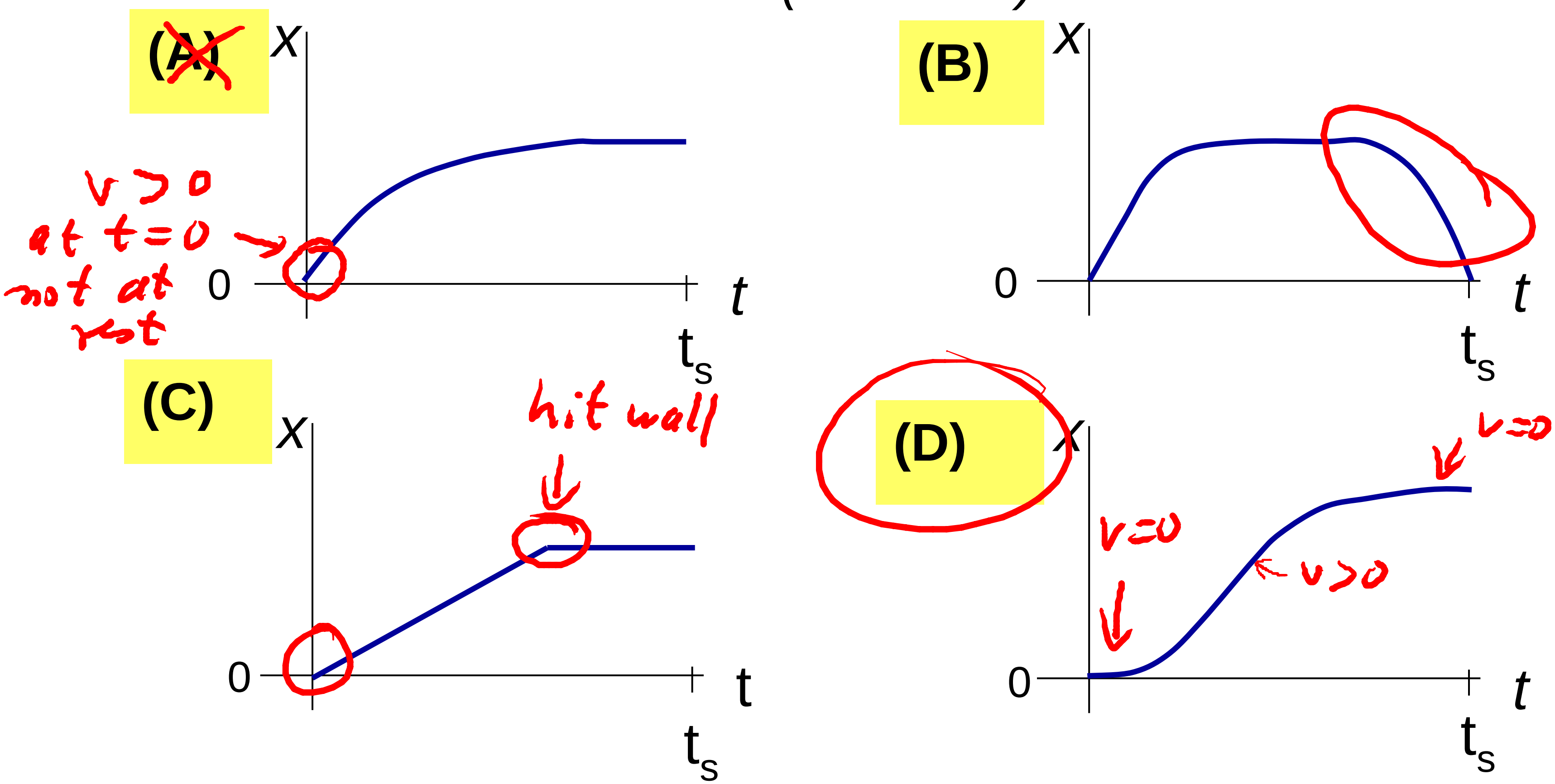
Next:

Transfer out thinking among:

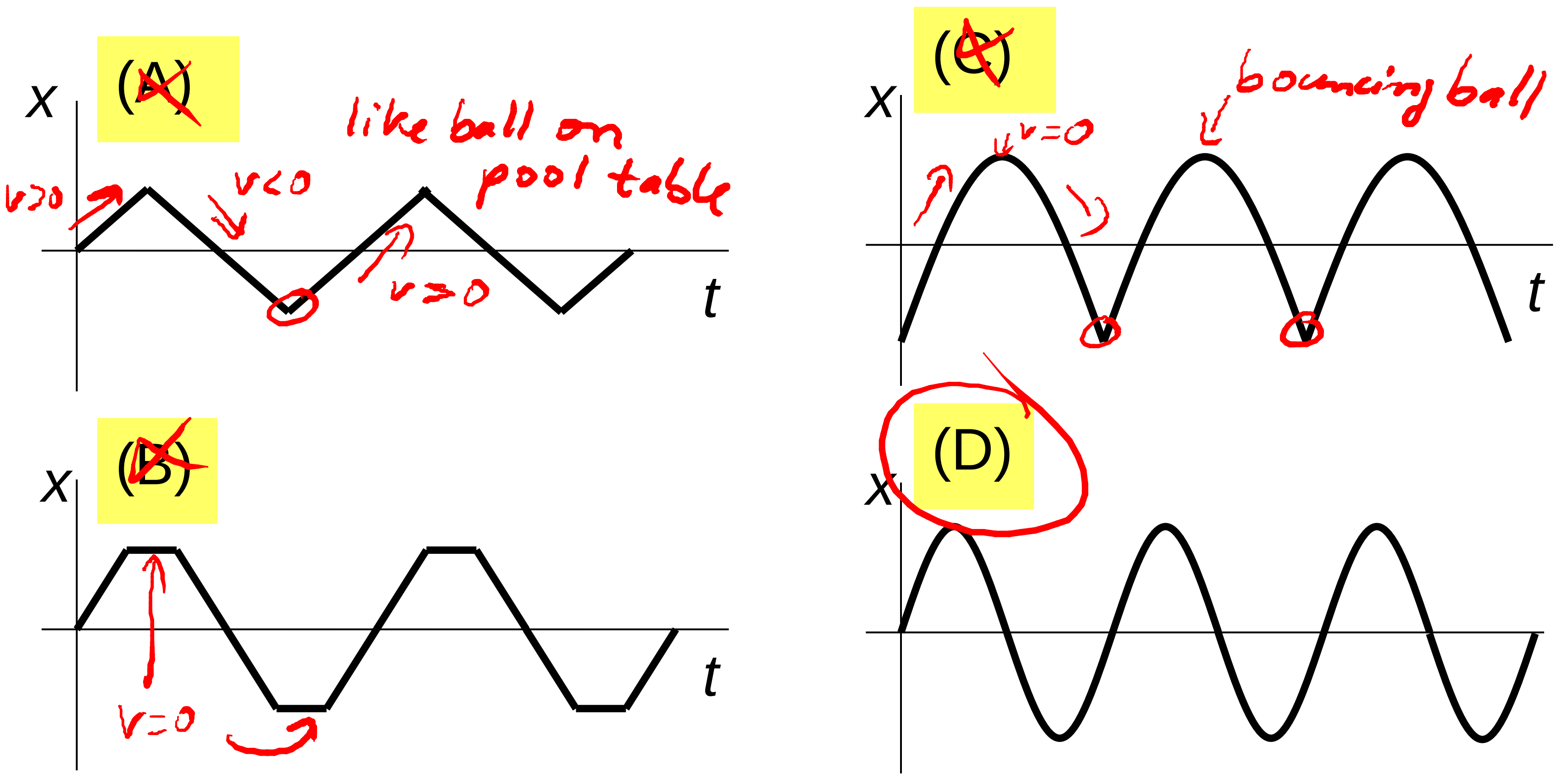
mental picture $\rightleftarrows$ verbal description of motion $\rightleftarrows$ $x(t)$, $v(t)$, $a(t)$ graphs

When a traffic light turns green at t=0, a car moves forward from rest at x=0 and eventually comes to a stop at the next red light at t =t_s.

Which of the following x-t graphs could describe the motion of the car? *(Pick one.)*

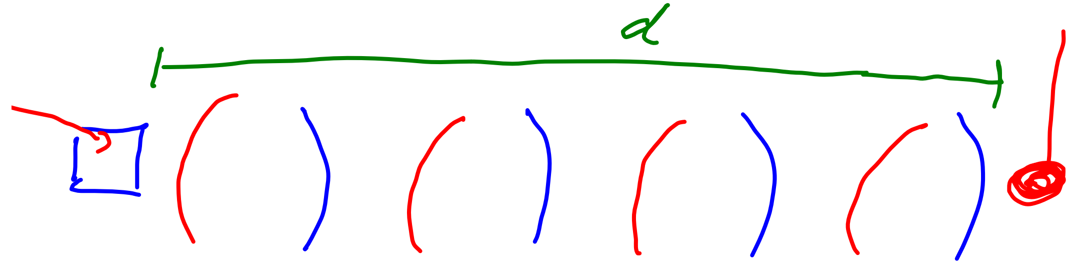


Which of the following *x*-*t* graphs could describe the motion of a simple pendulum?



Pendulum: Measuring distance using echos of reflected ultrasound

Ultrasonic ranger: sends very short pulses of ultrasound, then listens for time of echo



$$\Delta t_{echo} = \frac{2d}{V_{sound}}$$

$$V_{sound} = 343 \frac{m}{s}$$