Lecture 24

Recap:

- **Momentum**: $\vec{p} = m\vec{v}$

$$\vec{F}_{net,ext} = \sum \vec{F}_{ext} = m_{total}\,\vec{a}_{com} = \frac{d\vec{p}_{total}}{dt}$$

$\Rightarrow$ If $\vec{F}_{net,ext} = 0$: $\vec{p}_{total} = \underbrace{\vec{p}_1 + \vec{p}_2 + \vec{p}_3 + \ldots}_{\text{System of objects}} = \text{const}$ } Momentum is conserved!

- **Collisions**: Brief, intense interaction between two (or more) objects.

"Intense": external forces can usually be neglected during time of collision

$\Rightarrow$ If $\sum \vec{F}_{ext} = \dfrac{d\vec{p}_{total}}{dt} = 0$ $\Rightarrow$ $\boxed{\vec{p}_{total} = \text{const during any type of collision!}}$

Impulse: $\underbrace{\vec{J}_{on\ object}}_{\text{during collision}} = \underbrace{\int \vec{F}_{coll}(t)\,dt}_{\text{time of collision}} = \vec{F}_{avg}\,\Delta t_{coll} = \underline{\underline{\Delta\vec{p}}} = \left(\text{area under } F\text{-}t \text{ graph}\right)$

not F-x!

Example:

N III: $\vec{F}_{B\,on\,A} = -\vec{F}_{A\,on\,B} \Rightarrow \int_{\Delta t_{coll}} F_{BA}\,dt = -\int_{\Delta t_{coll}} F_{AB}\,dt$

$\Rightarrow \vec{J}_{on\,A} = -\vec{J}_{on\,B}$

$\Rightarrow \Delta\vec{p}_A = -\Delta\vec{p}_B$

$\Rightarrow \Delta\vec{p}_A + \Delta\vec{p}_B = 0 \Rightarrow \vec{p}_{total} = \text{const}$

Studying Collisions:

- General (always true):

If $\sum \vec{F}_{\text{ext on system during collision}} = \dfrac{d\vec{P}_{total}}{dt} = 0$ (or neglectable compared with internal forces during collision)

$\Rightarrow \vec{P}_{total}$ of system is conserved (constant)!

"system" = two (or more) objects, colliding

$\Rightarrow$

$$\vec{P}_{total,\,initial} = \vec{P}_{total,\,final}$$

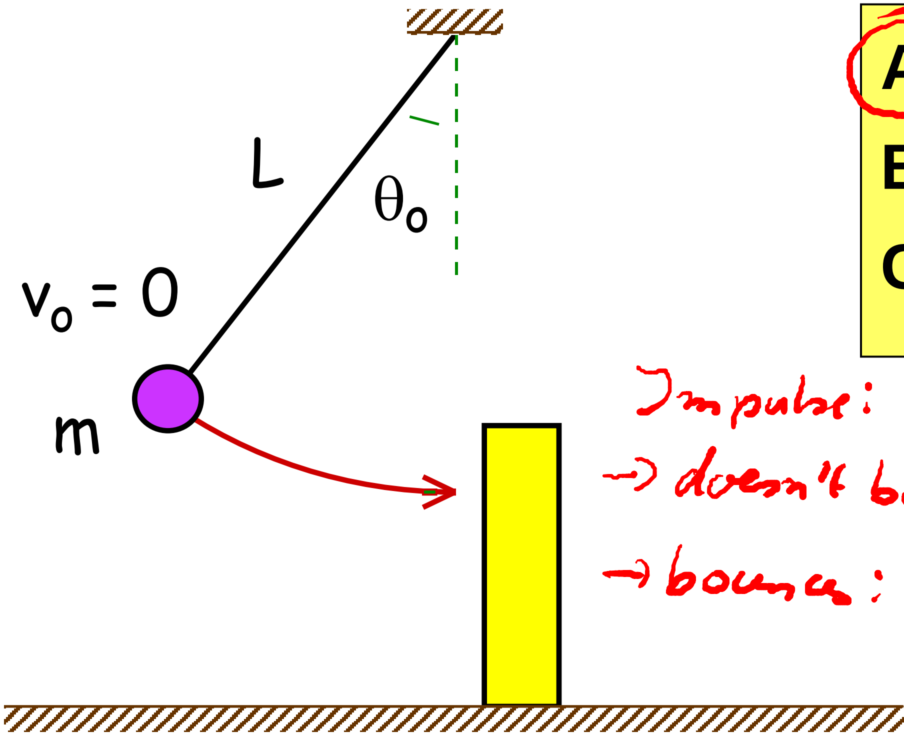
$$\vec{P}_{total,\,before\ coll.} = \vec{P}_{total,\,after\ coll.}$$

$$\vec{P}_{1,i} + \vec{P}_{2,i} + \ldots = \vec{P}_{1,f} + \vec{P}_{2,f} + \ldots$$

always true, for any type of collision

A ball of mass m, attached to a string of length L, is released from rest at angle θ_0 and then strikes a standing wooden block.

Is the block more likely to tip over if the ball bounces off of the block or if the ball doesn't bounce?



A. Bounces

B. Doesn't bounce

C. Equally likely in both cases

Impulse: $\vec{J} = \vec{F}_{avg} \Delta t_{coll} = \Delta \vec{p}$

$\rightarrow$ doesn't bounce: $\Delta \vec{p} = \vec{p}_f - \vec{p}_i = -\vec{p}_i$

$\rightarrow$ bounce: $\Delta \vec{p} = \vec{p}_f - \vec{p}_i = -\vec{p}_i - \vec{p}_i$

larger change

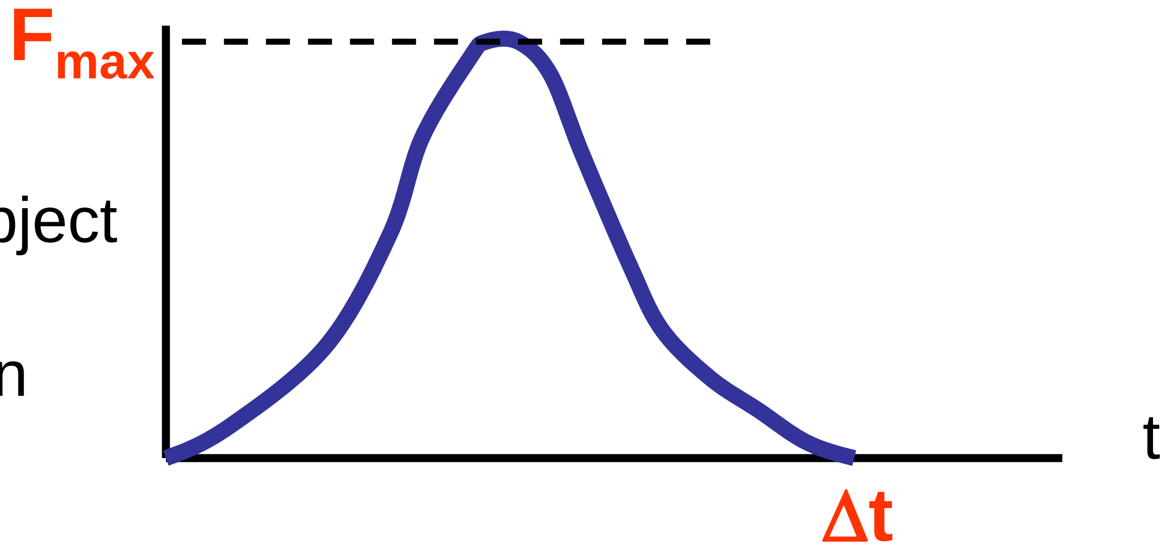
$\Rightarrow$ larger impulse on block

Today:

- Collisions, Collisions, Collisions…
- Airbags and shock absorbers
- The woodpecker (why don't they knock themselves silly)
- The gravitational "Slingshot" effect
- Types of collisions: elastic and inelastic

Shock Absorbers:

Impulse J = $\int$ F dt = $F_{avg}\,\Delta t$ = Δp



Key idea: Reduce F_{avg}, F_{max} by making Δt of collision longer!

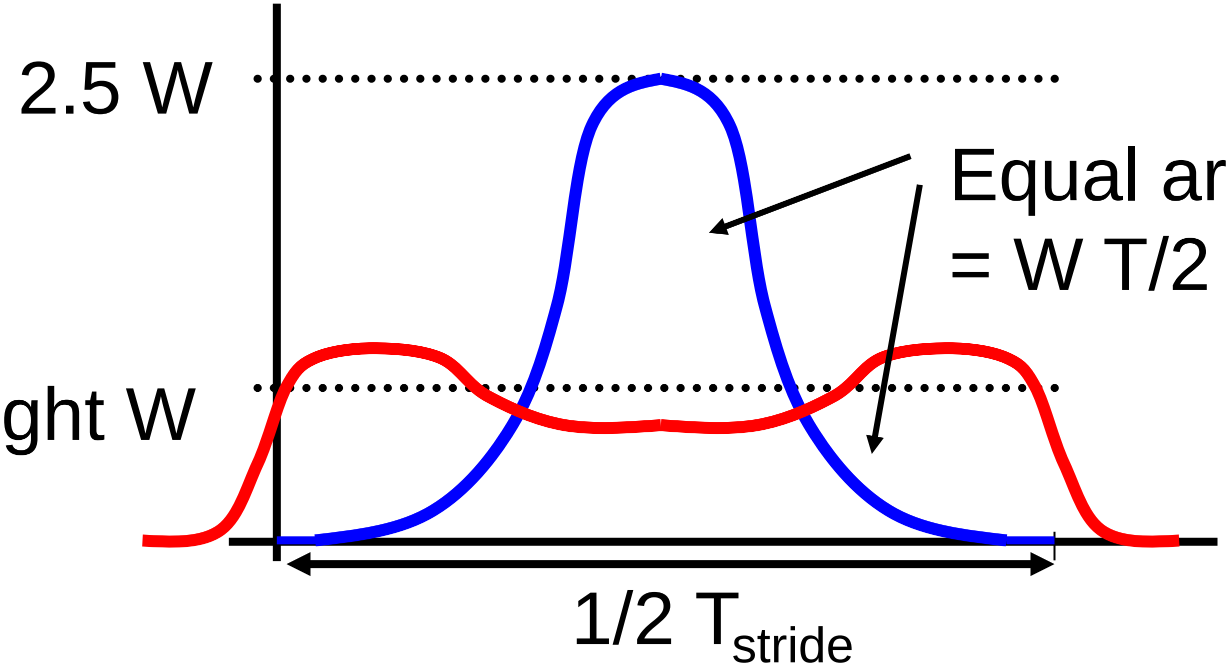
Foot arches

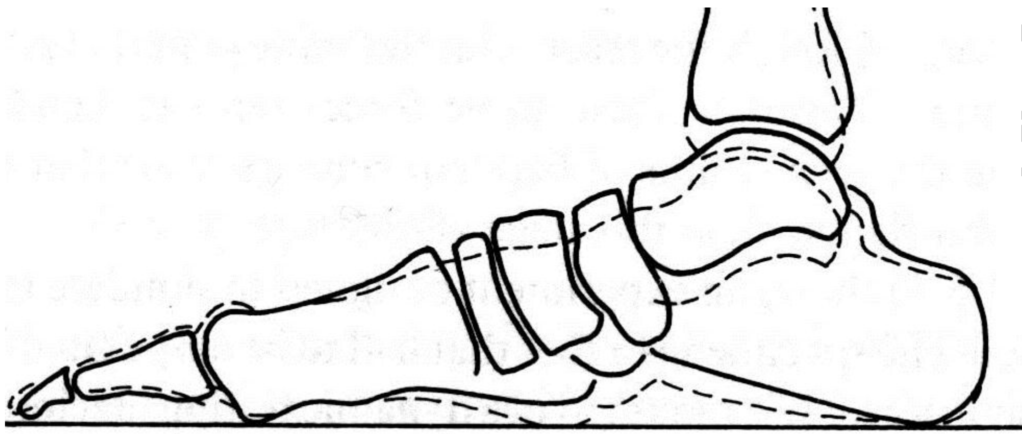
(padded running shoes, paw pads...)

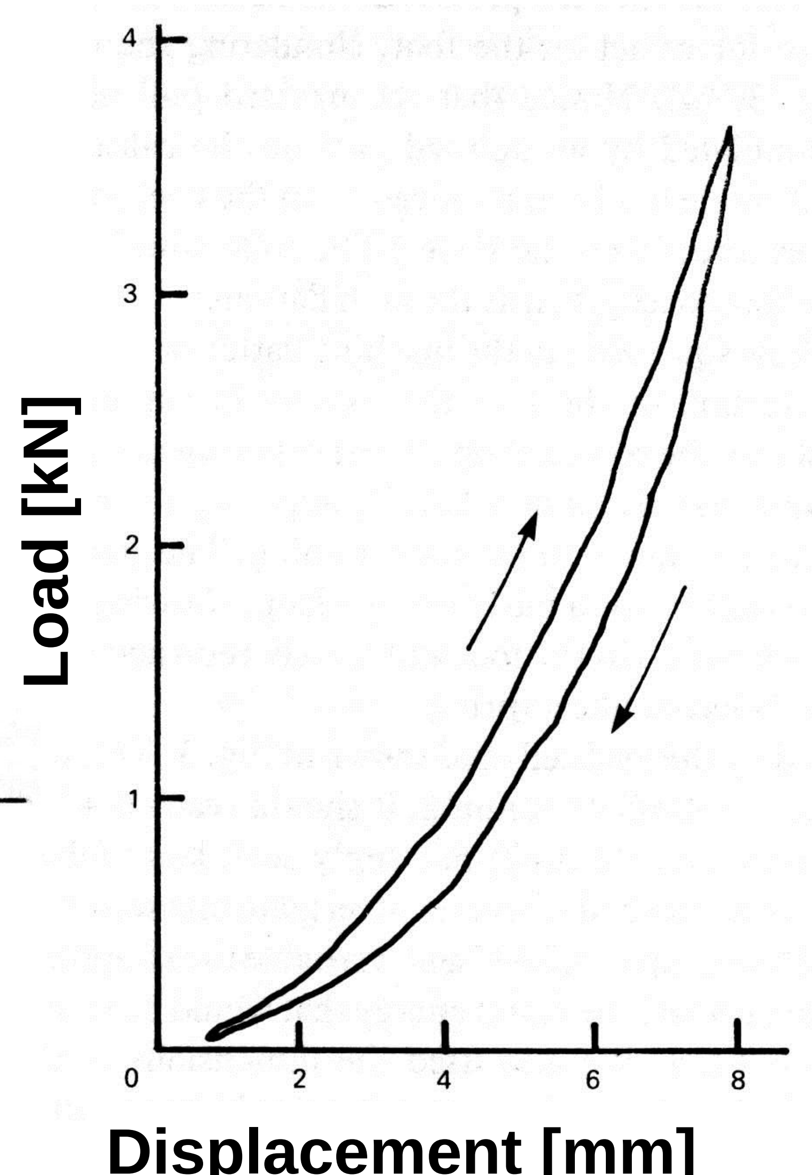
Walking: at least one foot in contact with ground at all times

Running: ~ a series of leaps

Vertical force exerted on one foot during one stride:






Load [kN]
4
3
2
1
0
2
4
6
8
Displacement [mm]

Boxers:

$\mathbf{m_{arm}} \sim 7$ kg

$\mathbf{v_{arm}} \sim 7$ m/s

$\Rightarrow$ **Impulse $\mathbf{J = \Delta p}$** $\sim m_{arm}\, v_{arm} \sim 49$ kg m/s

impact time $\mathbf{\Delta t}$ ~ 0.01 s $\Rightarrow$ **$\mathbf{F_{avg} \sim J/\Delta t \sim 4900}$ N**

$\mathbf{m_{arm}} \sim 7$ kg $\mathbf{v_{arm}} \sim 7$ m/s

$\Rightarrow$ **Impulse J = Δp** $\sim m_{arm}\, v_{arm} \sim 49$ kg m/s

impact time Δt ~ 0.01 s $\Rightarrow$ $\mathbf{F_{avg} \sim J/\Delta t \sim 4900}$ **N**

$m_{head} \sim 6$ kg

$\Rightarrow$ $\mathbf{a_{head} = F/m_{head} \sim 800\ m/s^2 \sim 80\ g}$**!**

- **enough to cause unconsciousness**
  ~ 40% of fatal blow

Woodpeckers:

During "collision" with a tree,

a_{head} ~ 600 - 1500 g!!

How do they survive?

- $F_{brain} = m_{brain}\, a_{brain}$ is much smaller.
- m_{brain}/A_{brain} much smaller for woodpeckers
  $\Rightarrow F_{brain}/A_{brain}$ = "pressure" is much smaller
- Jaw muscles act as shock absorbers
- Straight head trajectory reduces damaging rotations

Studying Collisions:

- General (always true):

If $\sum \vec{F}_{\text{ext on system during collision}} = \frac{d\vec{p}_{total}}{dt} = 0$ (or neglectable compared with internal forces during collision)

$\Rightarrow$ $\vec{p}_{total}$ of system is conserved (constant)!

"system" = two (or more) objects, colliding

$$\Rightarrow \boxed{\begin{aligned} \vec{p}_{total, initial} &= \vec{p}_{total, final} \\ \vec{p}_{total, before\ coll.} &= \vec{p}_{total, after\ coll.} \\ \vec{p}_{1,i} + \vec{p}_{2,i} + \ldots &= \vec{p}_{1,f} + \vec{p}_{2,f} + \ldots \end{aligned}}$$

always true, for any type of collision

– Types of collisions:

- **Elastic Collisions:**

$$\boxed{\sum K_i = \sum K_f}$$

Kinetic energy is also conserved during the collision, in addition to total momentum

$$\frac{\text{total KE}}{\text{befor collision}} = \frac{\text{total KE}}{\text{after collision}}$$

- **Inelastic Collision:**

$$\sum K_i > \sum K_f$$

some of the initial KE is lost while total momentum is conserved

– special case: sticking collision:

$$\boxed{V_{s,1} = V_{s,2}}$$

have the largest possible loss in Kinetic energy ΔK_{loss}

- **Superelastic Collision:**

$$\sum K_i < \sum K_f$$

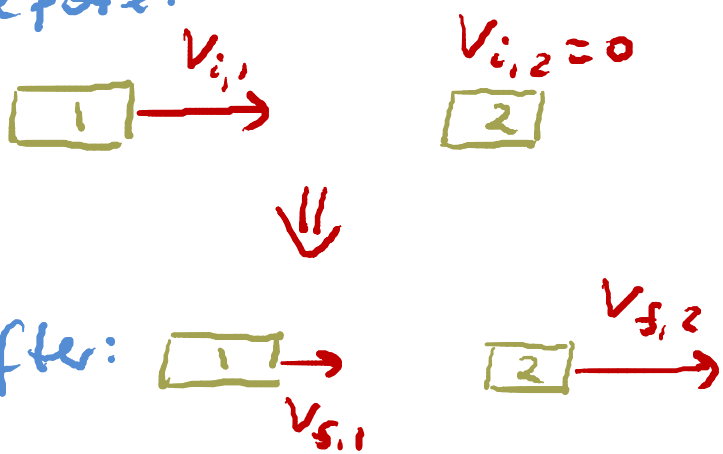
objects explode during collision...

Case A: Elastic collisions: kinetic energy is conserved in addition to momentum

$$\vec{P}_{total} = \text{const} \qquad \sum K_i = \sum K_f$$

Case A-1: 1-D elastic collision with object "2" at rest (stationary target)

before:



after:

- cons. of momentum:

$$m_1 v_{i,1} + m_2 v_{i,2} = m_1 \underline{v_{f,1}} + m_2 \underline{v_{f,2}}$$

- cons. of KE:

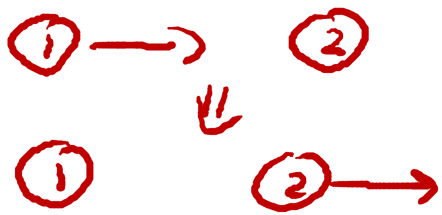
$$\frac{1}{2} m_1 v_{i,1}^2 + \frac{1}{2} m_2 v_{i,2}^2 = \frac{1}{2} m_1 v_{f,1}^2 + \frac{1}{2} m_2 v_{f,2}^2$$

$\Rightarrow$ 2 equ, 2 unknowns $(v_{f,1}, v_{f,2})$

$\Rightarrow$ solve:

$$\boxed{v_{f,1} = \frac{m_1 - m_2}{m_1 + m_2} v_{i,1} \qquad v_{f,2} = \frac{2 m_1}{m_1 + m_2} v_{i,1}}$$

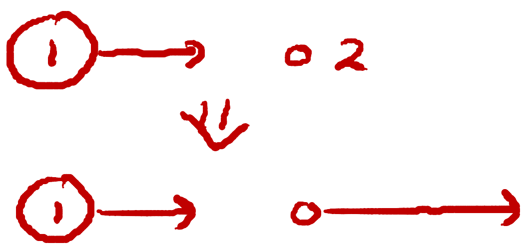
$\Rightarrow$ if $m_1 = m_2$



$$V_{f,1} = 0$$

$$V_{f,2} = V_{i,1}$$

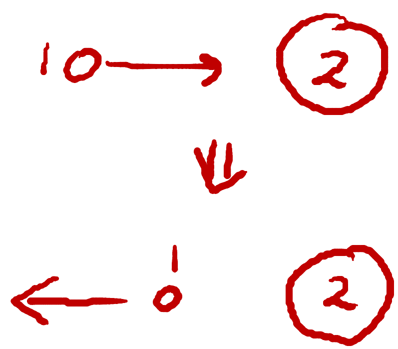
$\Rightarrow$ if $m_1 \gg m_2$



$$V_{f,1} = \frac{m_1 - m_2}{m_1 + m_2} V_{i,1} \approx V_{i,1}$$ } large mass keeps going

$$V_{f,2} = \frac{2m_1}{m_1 + m_2} V_{i,1} \approx 2 V_{i,1}$$

$\Rightarrow$ if $m_1 \ll m_2$



$$V_{f,1} = \frac{m_1 - m_2}{m_1 + m_2} V_{i,1} \approx -1 \cdot V_{i,1}$$ } bounces off mass 2

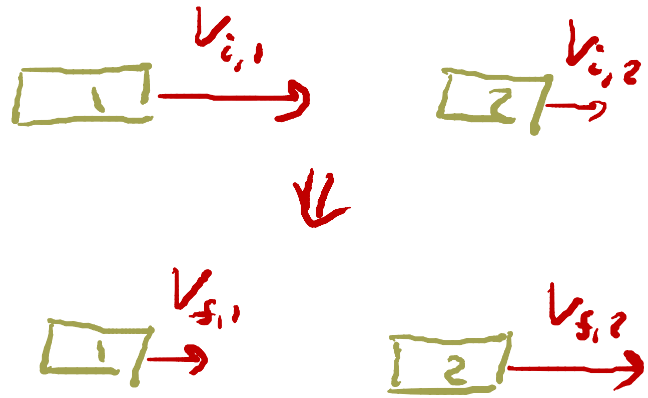
$$V_{f,2} = \frac{2m_1}{m_1 + m_2} V_{i,1} \to \text{tiny}$$


02 13:59:33 PLAY-001770
FWD30 -0001.7700sec
000000 A: 00.0 B: 00.0
1/1000 END 1000FPS

Case A-2: 1-D elastic, both objects moving before collision

before:

after:



• cons. of momentum:

$$\vec{p}_{i,1}+\vec{p}_{i,2}=\vec{p}_{f,1}+\vec{p}_{f,2}$$

• and cons. of KE:

$$K_{i,1}+K_{i,2}=K_{f,1}+K_{f,2}$$

$\Rightarrow$ solve:

(1) $$v_{f,1}=\frac{m_1-m_2}{m_1+m_2}v_{i,1}+\frac{2m_2}{m_1+m_2}v_{i,2}$$

(2) $$v_{f,2}=\frac{2m_1}{m_1+m_2}v_{i,1}+\frac{m_2-m_1}{m_1+m_2}v_{i,2}$$

$\Rightarrow$ subtract 2 from 1

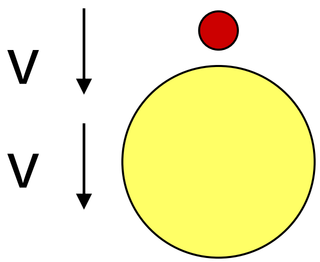
3 $\underbrace{V_{f,2} - V_{f,1}}_{} = -(V_{i,2} - V_{i,1})$ } for any 1-D **elastic** collision

(speed of separation) = (speed of approach) } same relative speed before and after collision

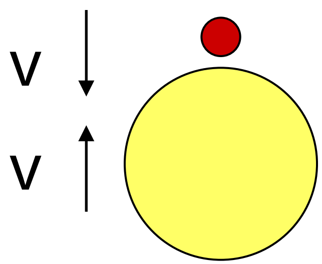
$\Rightarrow$ use equ. 3 + cons. of momentum to solve **elastic** collision problem!

Using collisions with large masses to increase speed:

Just before basketball hits ground:



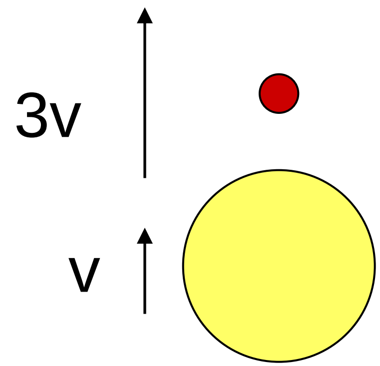
Just after basketball hits ground and before collision with tennis ball:



speed of approach

$\mathbf{v_{app}} = v - (-v) = \mathbf{2v}$

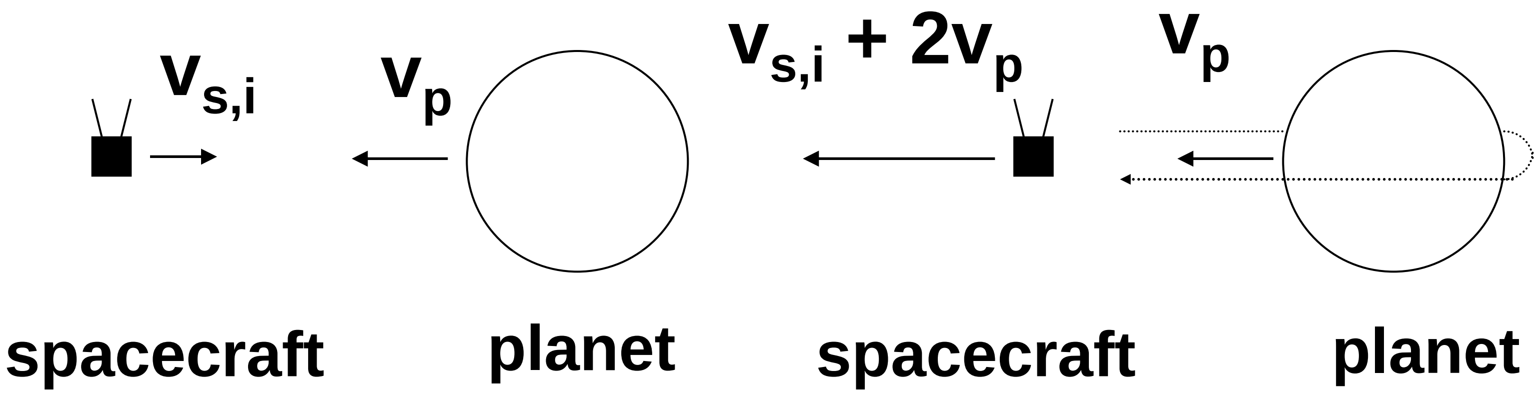
Just after collision between basket-ball and tennis ball:



speed of separation

$\mathbf{v_{sep}} = 3v - v = \mathbf{2v}$

Gravitational "Slingshot" Effect:



speed of approach

$v_{app} = |v_{s,i}| + |v_p|$

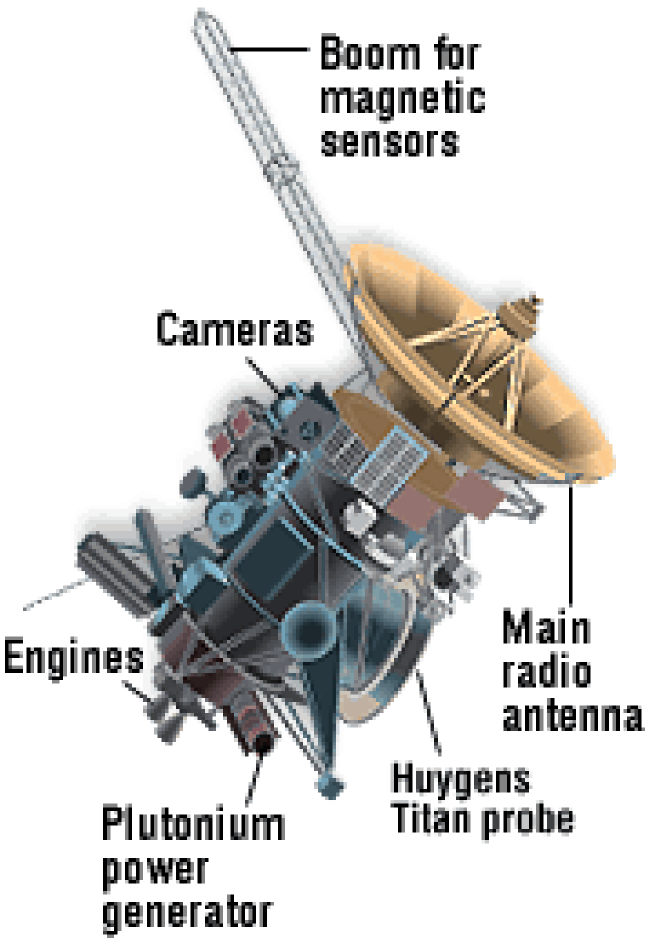
speed of separation

$v_{sep} = v_{s,f} - v_p$

$= v_{app} = v_{s,i} + v_p$

$\Rightarrow v_{s,f} = v_{s,i} + 2v_p$

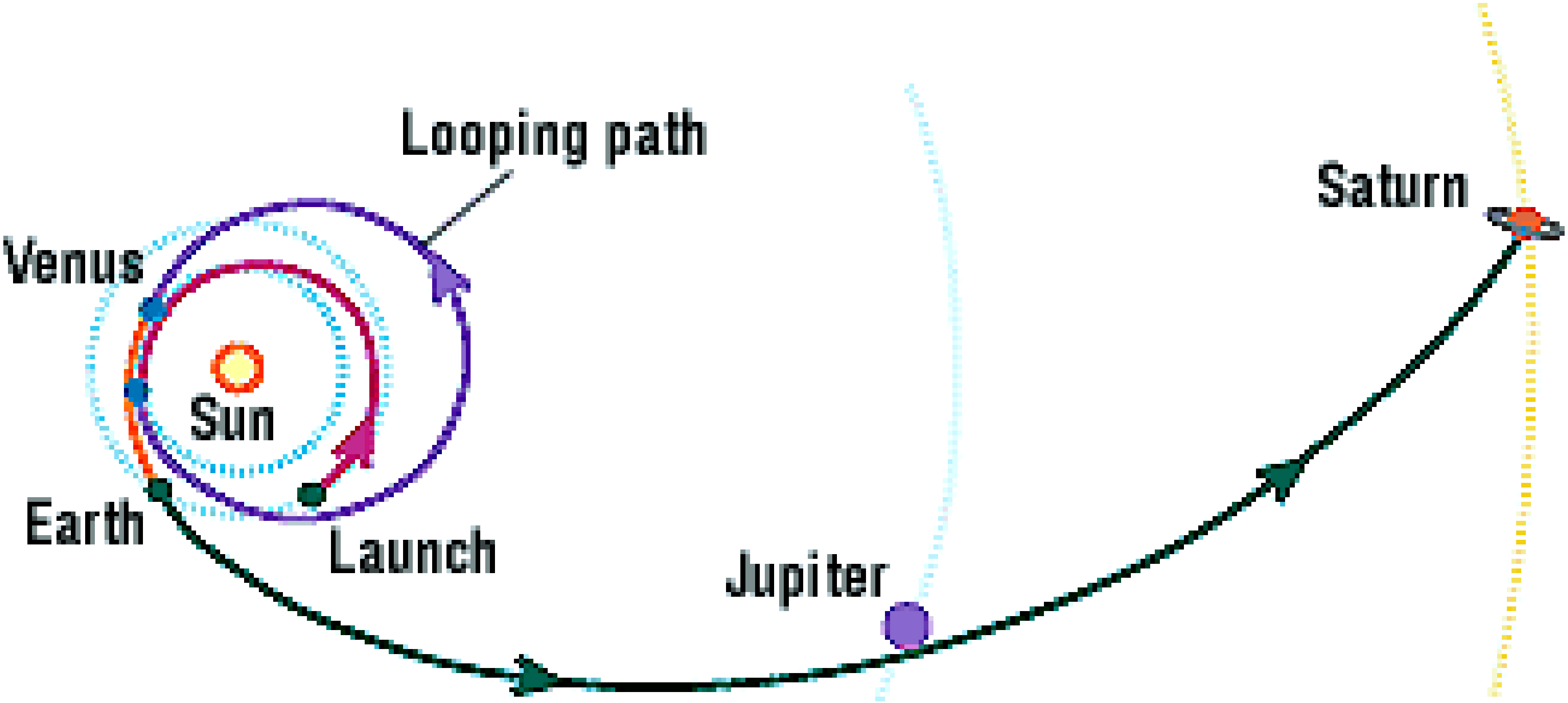
Cassini Mission to Saturn:



Four "Slingshot" Gravity Assists:

Venus: June 1998
Venus: April 1999
Earth: August 1999
Jupiter: December 2000

Arrive at Saturn: July 2004


Looping path
Saturn
Venus
Sun
Earth
Launch
Jupiter

A moving object (A) strikes a stationary object **(B).**

A ○ $\longrightarrow v_{i,A}$ $\boxed{B}$

Is it possible for **B to end up with more momentum than A had before the collision?**

$\Rightarrow$ cons. of total momentum $\vec{p}_{total} \Leftarrow$ vector!

$$\vec{p}_{i,A} + \vec{p}_{i,B} = \vec{p}_{f,A} + \vec{p}_{f,B}$$

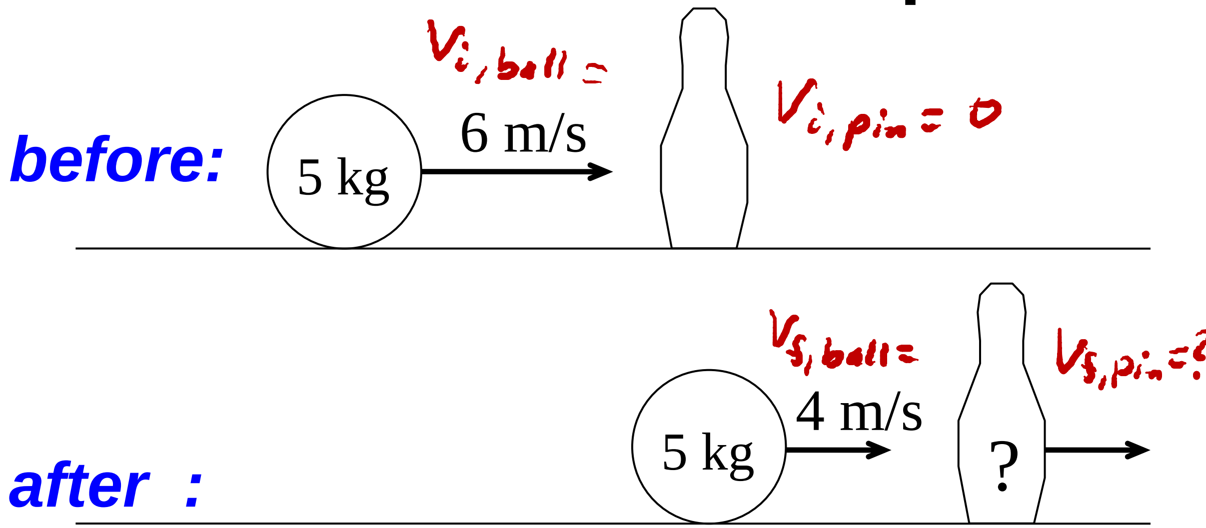
$$\longrightarrow + 0 = \longleftarrow + \underbrace{\longrightarrow}_{|p_{f,B}| > |p_{i,A}| \text{ if } m_B > m_A}$$

$$v_{f,B} = \frac{2m_A}{m_A + m_B} v_{i,A} \Rightarrow \underline{p_{f,B}} = m_B v_{f,B} = \underbrace{\frac{2m_B}{m_A+m_B}}_{>1 \text{ if } m_B > m_A} \underbrace{m_A v_{i,A}}_{p_{i,A}}$$

A.	**Yes**
B.	**No**

A 5 kg bowling ball collides *elastically* with a stationary pin of unknown mass.

What is the **final speed** of the pin?



$v_{pin,final}$ = ?

A. **2 m/s**

B. **4 m/s**

C. **6 m/s**

D. **10 m/s**

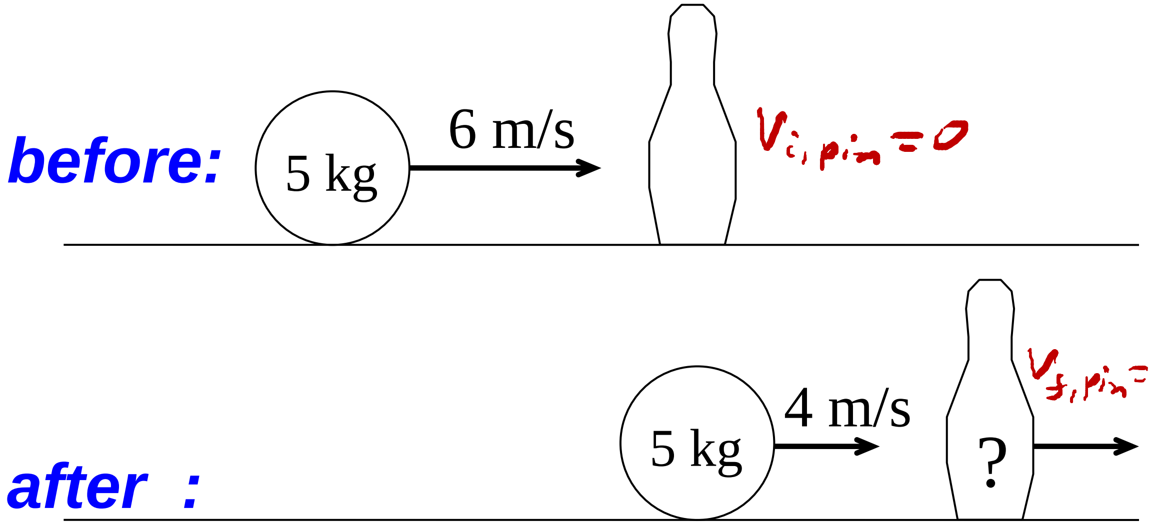
E. **not sure**

1-D elastic collision: speed of appr. = speed of separation

$$6\ m/s = \underbrace{10\ m/s - 4\ m/s}_{V_{f,pin} - V_{f,ball}}$$

A 5 kg bowling ball collides *elastically* with a stationary pin of unknown mass.

What is the **mass** of the pin?



m_{pin} = ?

A. 1 kg

B. 5 kg

C. 10 kg

D. not sure

=> use cons. of momentum:

$$m_b V_{i,b} + m_p V_{i,p} = m_b V_{f,b} + m_p V_{f,p}$$

$$\Rightarrow \underbrace{5\,kg \cdot 6\,m/s}_{30\,kg\,m/s} + 0 = \underbrace{5\,kg \cdot 4\,m/s}_{20\,kg\,m/s} + \underbrace{m_p \cdot 10\,m/s}_{10\,kg\,m/s} \Rightarrow \underline{m_p = 1\,kg}$$