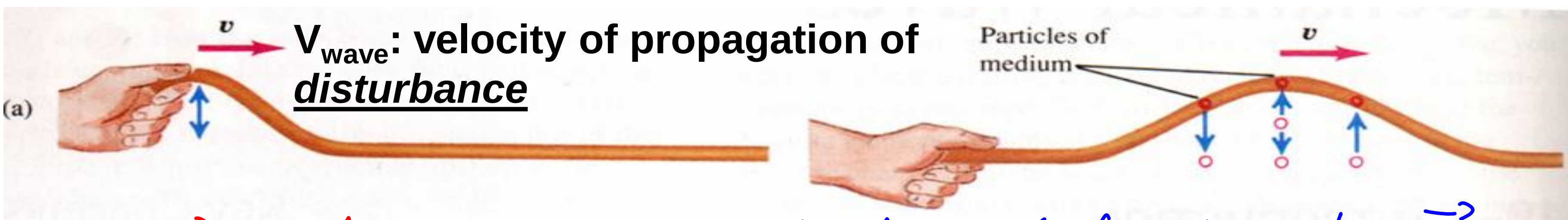
Recap: Waves

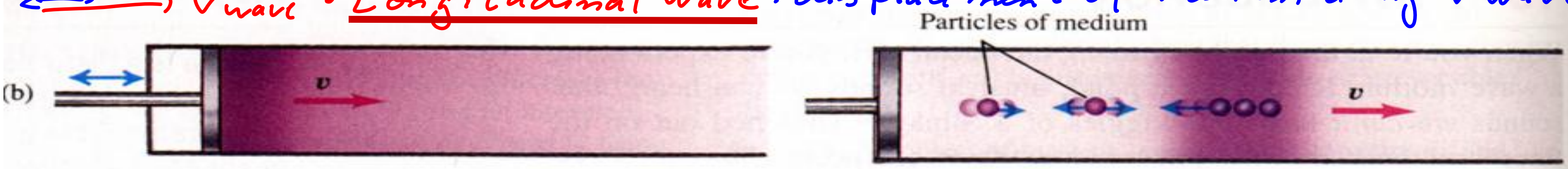
- Disturbance that propagates at wave speed v_{wave}
- Propagate energy and momentum, but not mass
- Reflected by boundaries

$\updownarrow$ $\longrightarrow \vec{v}_{wave}$ • **Transverse Wave**: displacement of medium $\perp$ to $\vec{v}_{wave}$



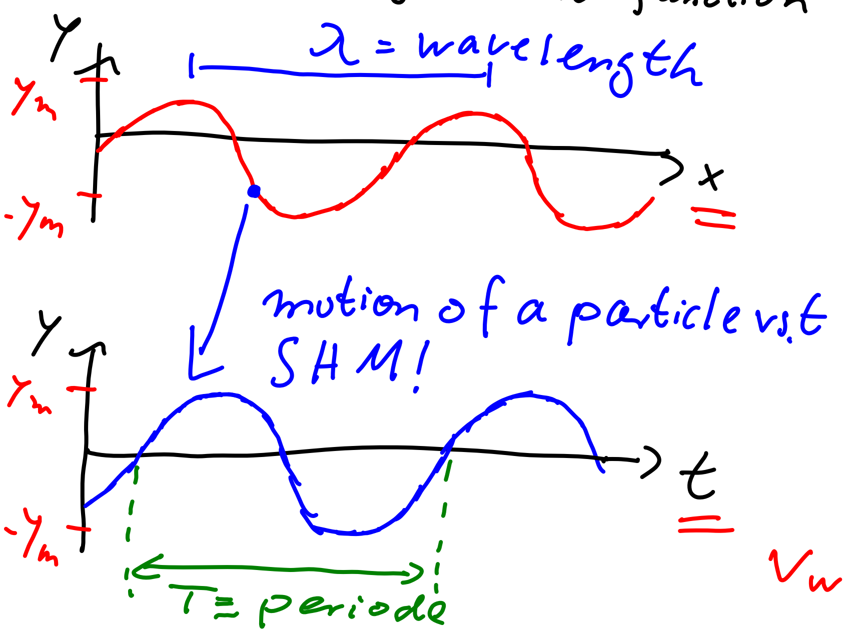
V_{wave}: velocity of propagation of ***disturbance***

$\leftrightarrow$ $\longrightarrow \vec{v}_{wave}$ • **Longitudinal Wave**: displacement of medium along $\vec{v}_{wave}$



• **Traveling sinusoidal transverse wave:**

Traveling → crest "moves"; sinusoidal → sine-function; transverse → motion of particles



sign gives direction of wave!

$$y(x,t) = y_m \sin[\underbrace{kx \mp \omega t}]$$

space and time dependence in argument of one sine-function!

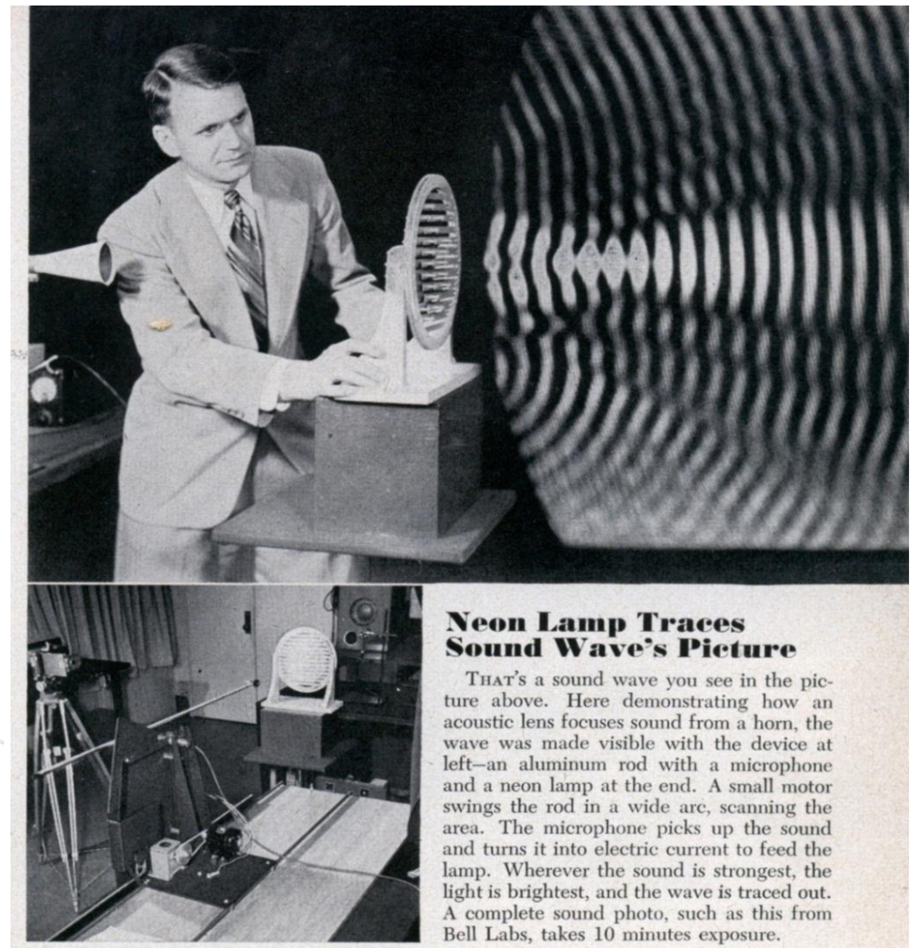
$$k = \frac{2\pi}{\lambda} = \text{wave number}$$

$$\omega = 2\pi f = \frac{2\pi}{T} = \text{angular frequ.}$$

$$v_{wave} = \frac{\omega}{k} = \frac{\lambda}{T} = \lambda f = \text{"speed of crest"}$$

Today:

- Wave velocity
- Longitudinal traveling waves
    - Sound waves and intensity
    - Beats



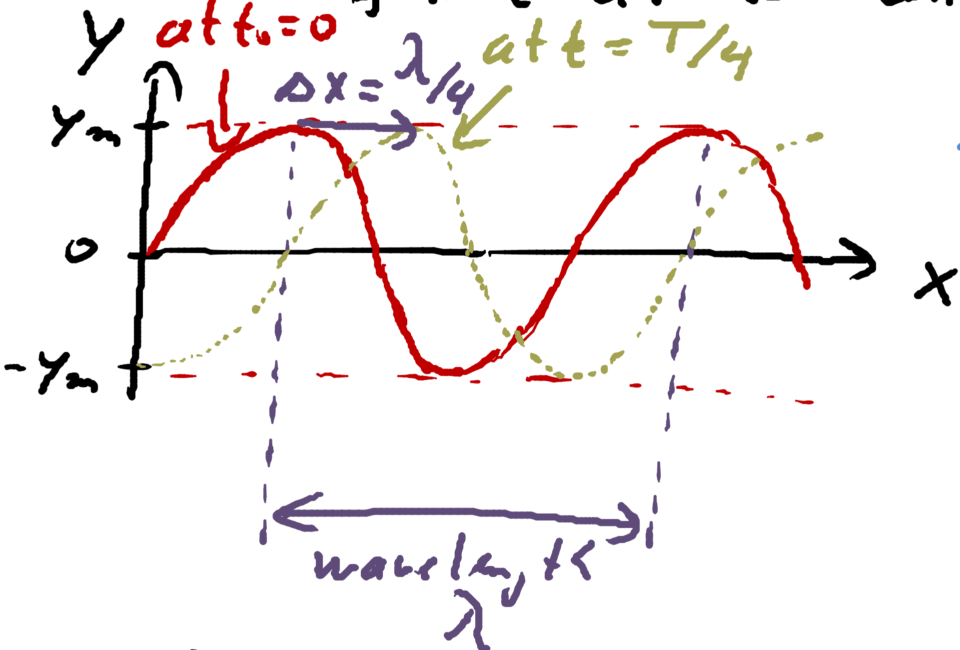
Neon Lamp Traces Sound Wave's Picture

THAT'S a sound wave you see in the picture above. Here demonstrating how an acoustic lens focuses sound from a horn, the wave was made visible with the device at left—an aluminum rod with a microphone and a neon lamp at the end. A small motor swings the rod in a wide arc, scanning the area. The microphone picks up the sound and turns it into electric current to feed the lamp. Wherever the sound is strongest, the light is brightest, and the wave is traced out. A complete sound photo, such as this from Bell Labs, takes 10 minutes exposure.

→ "Snapshot" of the wave:

fix t at t_0 and plot y vs. x

"−" ⇒ moves in $+x$ direction



$$y(x, t_0) = y_m \sin[kx - \omega t_0]$$

$$= y_m \sin[kx + \phi_0]$$

with $\phi_0 = -\omega t_0 = -\frac{2\pi}{T} t_0$

→ for $\Delta t = T/4$: crest moves by $\Delta x = \lambda/4$ in $\underline{+x}$ direction

$$\Rightarrow \boxed{\text{wave speed} = \text{"speed of crest"} = \frac{\Delta x}{\Delta t} = \frac{\lambda}{T} = \lambda f = \frac{\omega}{k}}$$

indep. of time!

A transverse wave on a string traveling in the **+x** direction is described by

$$\mathbf{y(x,t) = y_m \sin [(2\pi(x/\lambda - t/T)].}$$

If a **wave crest** (i.e., $y = y_m$) is located at **x=0 at some time t**, at what time t' will it have **moved to x=λ**?

- **A.** $t' = t + T$
- **B.** $t' = t - T$
- **C.** $t' = t + \lambda / v$
- **D.** $t' = t - \lambda / v$

$\Delta x = \lambda$

$\Rightarrow \Delta t = ?$

$\Rightarrow$ need wave speed:

$v_{wave} = \frac{\Delta x}{\Delta t} = \frac{\lambda}{T}$

$\Rightarrow$ for crest to "move" $\Delta x = \lambda$ takes $\Delta t = T =$ period

$\Rightarrow t' = t + \Delta t = t + T = t + \lambda / v_{wave}$

A transverse wave on a string traveling in the **+x** direction is described by

$$y(x,t) = y_m \sin[(\underbrace{2\pi x/1m - 4\pi t/1s}))] = y_m \sin\left[2\pi\left(\frac{x}{\lambda} - \frac{t}{T}\right)\right]$$

where *x* is in m and *t* is in s. $2\pi\left(\frac{x}{1m} - \frac{t}{0.5\,sec}\right)$

(1) What is the **wavelength** of the wave?

$\lambda =$ 1m

(2) What is the **period** of the local oscillations?

T = ½ sec

(3) What is the **wave speed**?

$v = \lambda/T = \frac{1m}{0.5s} = 2\ m/s$

The **speed v of a wave** on a **stretched string** or wire depends on the **wire tension F_T** and the **wire's mass per unit length μ**.

From **dimensional analysis**, what must be the relation **between v, F_T, and μ**?

V_{wave} | F_T | $\mu = \frac{mass}{length}$

$\frac{m}{s}$ | $N = \frac{kg\,m}{s^2}$ | $\frac{kg}{m}$

$\Rightarrow V_{wave} \propto \sqrt{\frac{F_T}{\mu}}$

A. $v \propto F_T / \mu$

B. $v \propto (F_T / \mu)^{1/2}$

C. $v \propto \mu / F_T$

D. $v \propto (\mu / F_T)^{1/2}$

→ wave speed on a stretched string / wire:

$$\boxed{V_{wave} = \sqrt{\frac{F_T}{\mu}}}$$

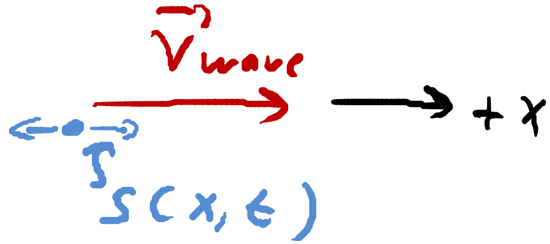
Tension of wire

$\mu = \frac{\text{mass}}{\text{length}}$ of wire

⇒ change tension to tune violin via changing V_{wave}!

→ Sound Waves (traveling)

- Longitudinal waves (in gas or liquid)



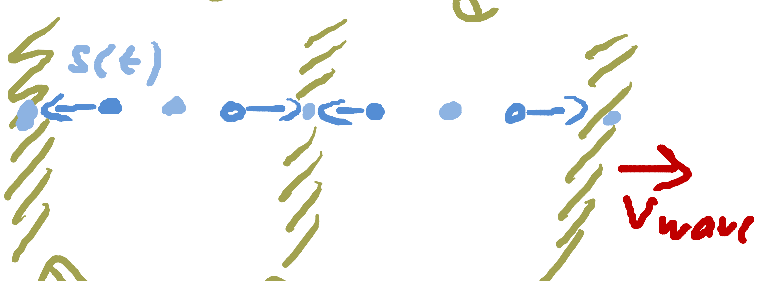
Displacement of molecule back and forth along x ($\parallel$ to $\vec{v}_{wave}$)

$$s(x,t) = s_m \cos[kx \mp \omega t] \Big\} \text{displ. wave}$$

displacement of molecule at position x and time t

$\Rightarrow$ Displacement wave causes compression and expansion of gas / liquid $\Rightarrow$ **pressure wave**

expansion: $\Delta p < 0$



high compression $\Rightarrow \Delta p > 0$

$$\underbrace{\Delta p}(x,t) = \Delta p_m \sin[kx \mp \omega t]$$

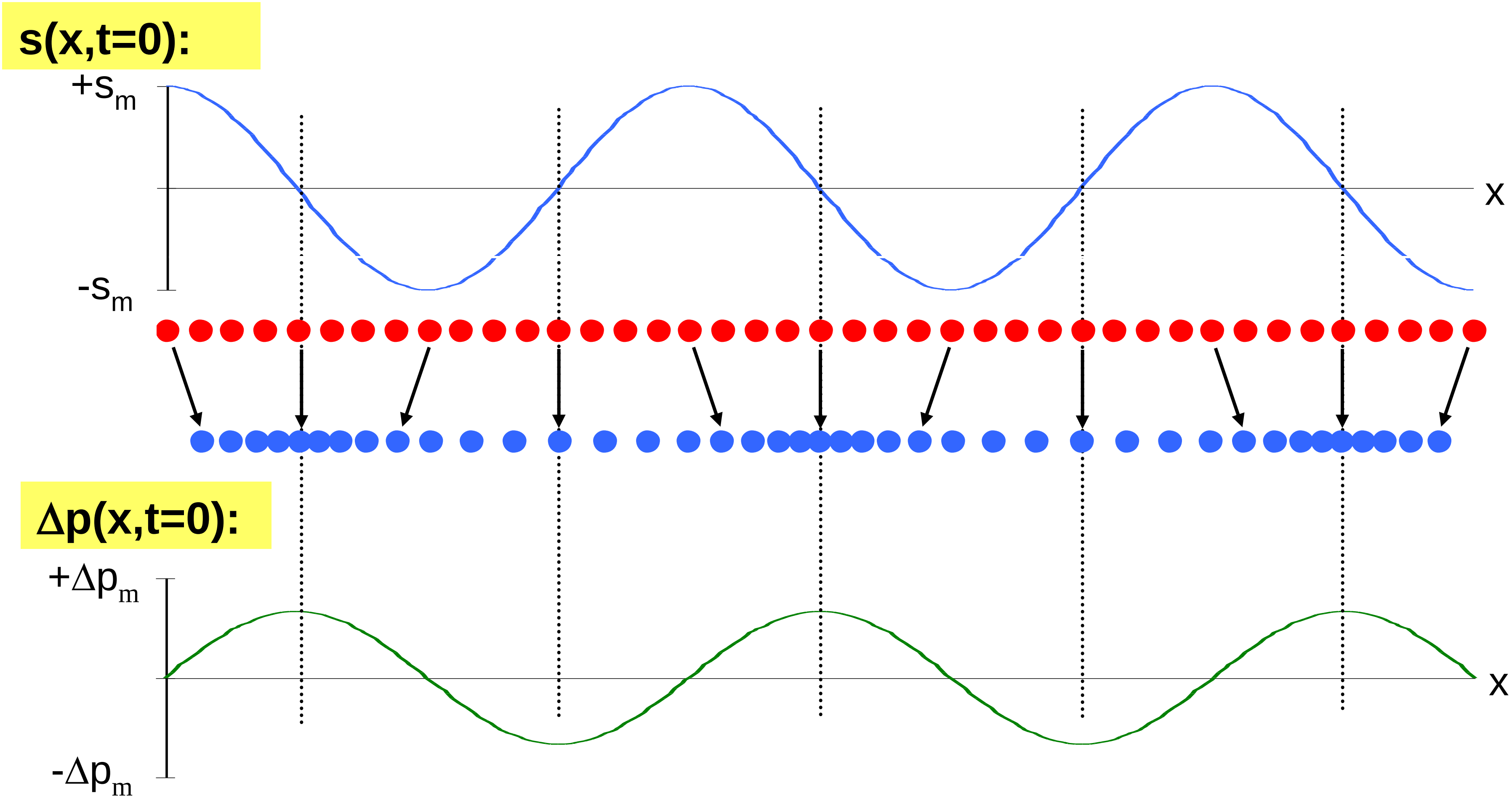
relative to p_0, i.e. change in pressure due to wave

90° out of phase with displacement wave!

Sound Waves:

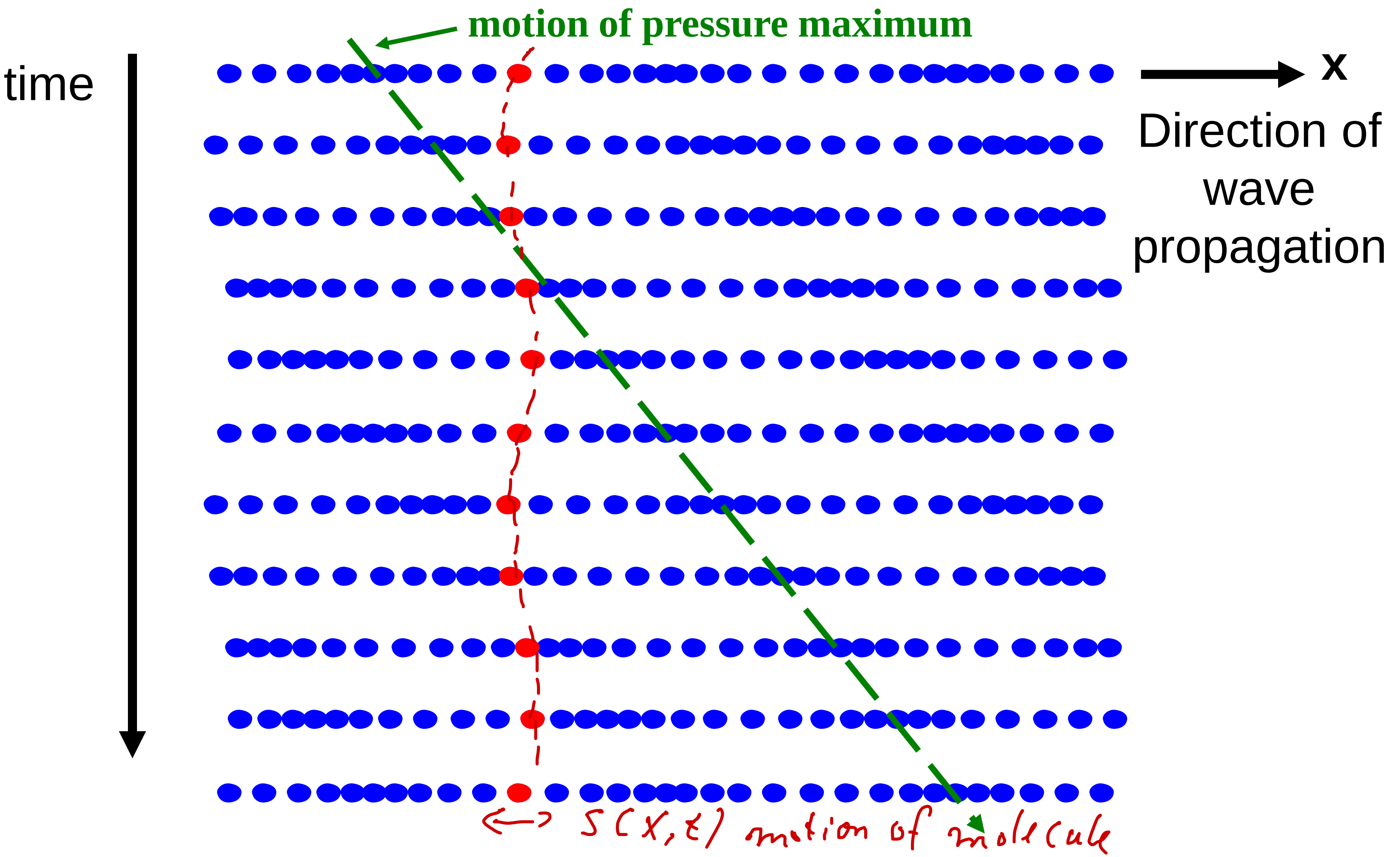
longitudinal molecular displacement: $s(x,t) = s_m \cos[kx - \omega t]$

excess pressure: $\Delta p(x,t) = \Delta p_m \sin[kx - \omega t]$



Sound Waves:

longitudinal molecular displacement: $s(x,t) = s_m \cos[kx - \omega t]$



⇒ pressure amplitude ΔP_m and displacement amplitude S_m are related:

$$\Delta P_m = (V_{\text{sound wave}}\, \rho\, \omega)\, S_m$$

density of medium

⇒ <u>Speed of sound wave:</u>

$$V_{\text{sound wave}} = \sqrt{B/\rho}$$

B: bulk modulus:

$$\frac{F}{A} = -B\,\frac{\Delta V}{V}$$

$B > 0$

<u>Examples:</u>

$V_{\text{sound}} = 343\ \text{m/s}$ in air ($\approx 1/5$ mile/s)

$= 1482\ \text{m/s}$ in water

$= 6420\ \text{m/s}$ in aluminum

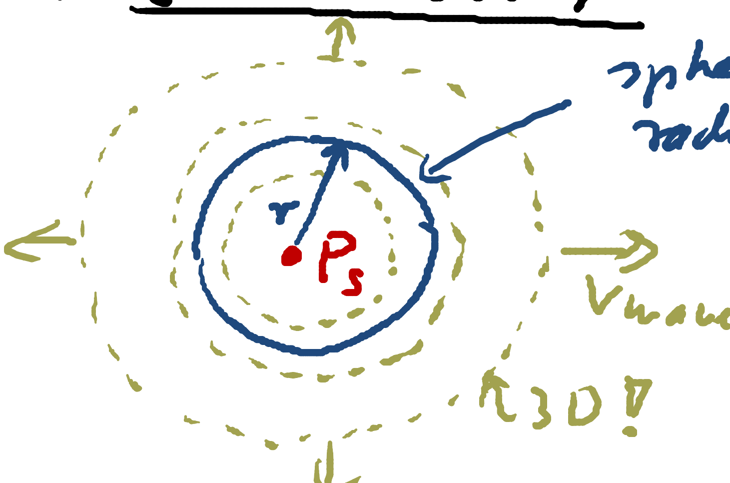
→ Intensity I of a sound wave:

$$I = \frac{\text{energy transferred by a sound wave per unit time}}{\text{area through which it is transferred}}$$

$$= \frac{\text{acoustic power}}{\text{area}} = \frac{P}{A} \qquad [I] = \frac{W}{m^2}$$

$$I = \Big(\overbrace{\tfrac{1}{2}\,\underbrace{\rho}_{\text{density of medium}}\,\underbrace{\omega^2 s_m^2}_{V_{max}^2}}^{K/\text{volume}}\Big)\, v_{wave} = \frac{\Delta p_m^2}{2\, v_{wave}\, \rho}$$

→ for an isotropic point source:



sphere of radius r

power of point source

$$I = \frac{P_s}{A} = \frac{P_s}{\underbrace{4\pi r^2}_{\text{area of sphere}}}$$

A **point source** emits sound waves isotropically. The **intensity** of the wave **1 m** from the source is **1 W/m²**.

What is the **intensity** of the wave **2 m** from the source?

$$I = \frac{P_s}{4\pi r^2} \propto \frac{1}{r^2}$$

$$\Rightarrow \frac{I(2m)}{I(1m)} = \left(\frac{1m}{2m}\right)^2 = \frac{1}{4}$$

***I*(2m) = ?**

A. 0.25 W/m²

B. 0.5 W/m²

C. 1 W/m²

D. 2 W/m²

E. 4 W/m²

→ Sound level β:

range of human hearing : $I = \underbrace{10^{-12}\,\frac{W}{m^2}}_{\text{theshold of hearing}} \ldots \underbrace{1\,\frac{W}{m^2}}_{\text{theshold of pain}}$

⇒ use logarithmic scale : decibel scale

$$\boxed{\beta = (10\,dB)\cdot \log_{10}\left(\frac{I}{I_0}\right)}$$

$I_0 = 10^{-12}\,\frac{W}{m^2}$ (reference intensity)

"units" = decibel (dB) = dimensionless

"dB" means that we have taken $10 \cdot \log_{10}$(quantity)

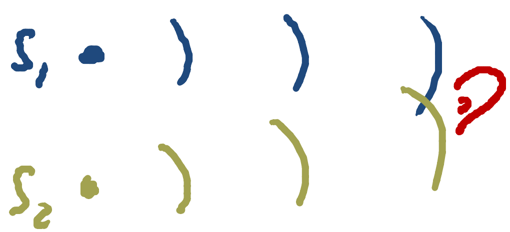
$I = 10^{-12}\ W/m^2 \longrightarrow \beta = 0\,dB$

$I = 1\ W/m^2 \longrightarrow \beta = 120\,dB$

} factor of 10 in intensity every +10 dB

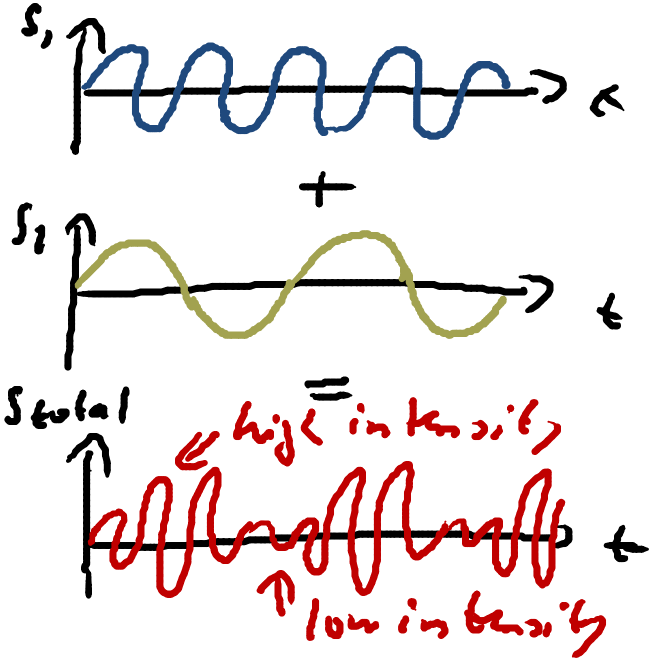
→ Beats:

Two sound waves of equal amplitude but different frequencies, detected at some position $x=0$



$$s_1(t) = s_m \cos(\omega_1 t)$$

$$\underbrace{s_2(t)}_{\text{displacements due to waves at position } x=0} = s_m \cos(\omega_2 t)$$



⇒ total displacement:

(principle of superposition)

$$s_{total}(t) = s_1(t) + s_2(t)$$

$$= s_m[\cos(\omega_1 t) + \cos(\omega_2 t)]$$

$$= 2 s_m[\cos(\omega' t) \cdot \cos(\omega_{avg} t)]$$

with $\omega' = \frac{|\omega_1 - \omega_2|}{2} = \frac{\text{difference}}{2}$

$\omega_{avg} = \frac{\omega_1 + \omega_2}{2}$ = average of ω's

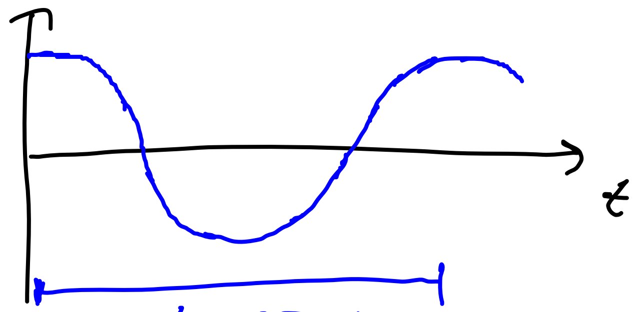
$$\cos(A) + \cos(B) = 2\cos\left(\frac{A-B}{2}\right) \cdot \cos\left(\frac{A+B}{2}\right)$$

$\Rightarrow$ for ω_1 close to (but not identical to) ω_2:

$$\omega_1 \sim \omega_2 \quad \Rightarrow \quad \omega_{avg} \gg \omega'$$

$$S(t) = 2 S_m \cos(\omega' t) \cos(\omega_{avg} t)$$

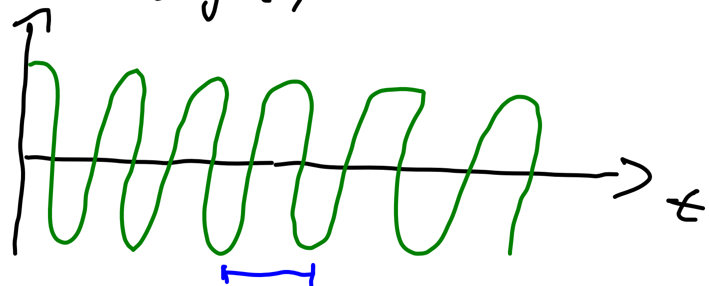
$\cos(\omega' \cdot t)$



$T' = 2\pi/\omega'$

$\times$

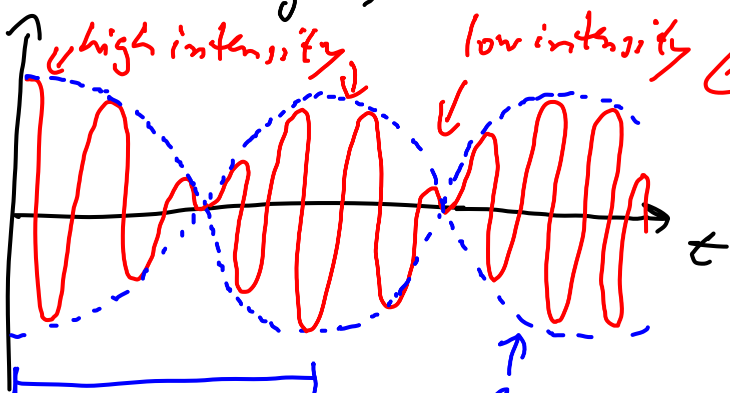
$\cos(\omega_{avg} \cdot t)$



$T_{avg} = 2\pi/\omega_{avg}$

$\cos(\omega' t) \cdot \cos(\omega_{avg} \cdot t)$

$\Rightarrow$



envelope function

$T_{beat} = T'/2$

(high intensity to high intensity)

beats! Intensity oscillates in time!

$$\boxed{f_{beat}} = \frac{1}{T_{beat}} = \frac{1}{T'/2} = 2f' = \frac{2\omega'}{2\pi}$$

$$= \frac{|\omega_1 - \omega_2|}{2\pi} \boxed{= |f_1 - f_2|}$$

$= |$difference between the two frequencies$|$