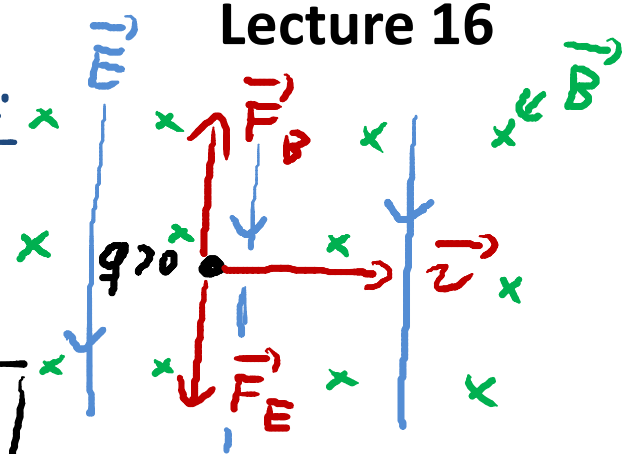
Recap

Lecture 16

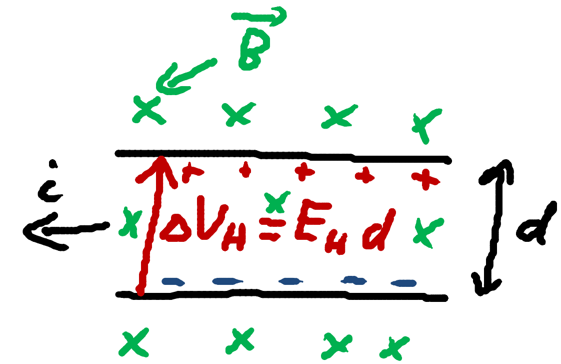
- Crossed Electric and Magnetic Fields:



  - velocity selector:

    charge moves along straight line, if $F_B = F_E \Rightarrow \boxed{v_{\substack{straight\\line}} = \frac{E}{B}}$

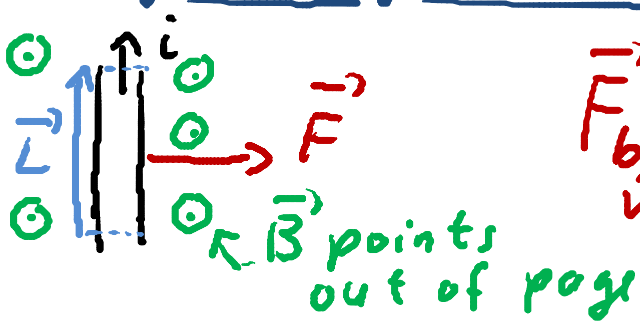
  - Hall effect:



    Production of a potential difference ΔV_H accross an electric conductor by a magnetic field $\perp$ to the current direction

    $$v_{drift} = \frac{E_H}{B} = \frac{\Delta V_H}{B d} \qquad n = \frac{i B d}{e \Delta V_H A}$$

- Magnetic force on a current-carrying wire:



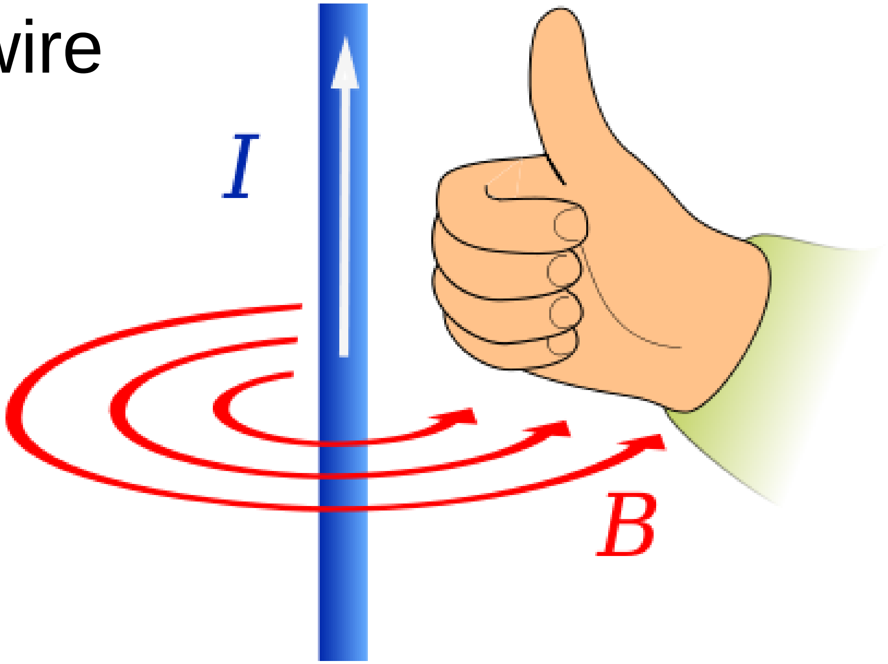
$\vec{F}_{\text{by } \vec{B}\text{-field on wire of length } L} = i \vec{L} \times \vec{B} \Rightarrow |\vec{F}| = i L B \sin\phi$

angle between $\vec{L}$ and $\vec{B}$

Length vector, points in direction of current

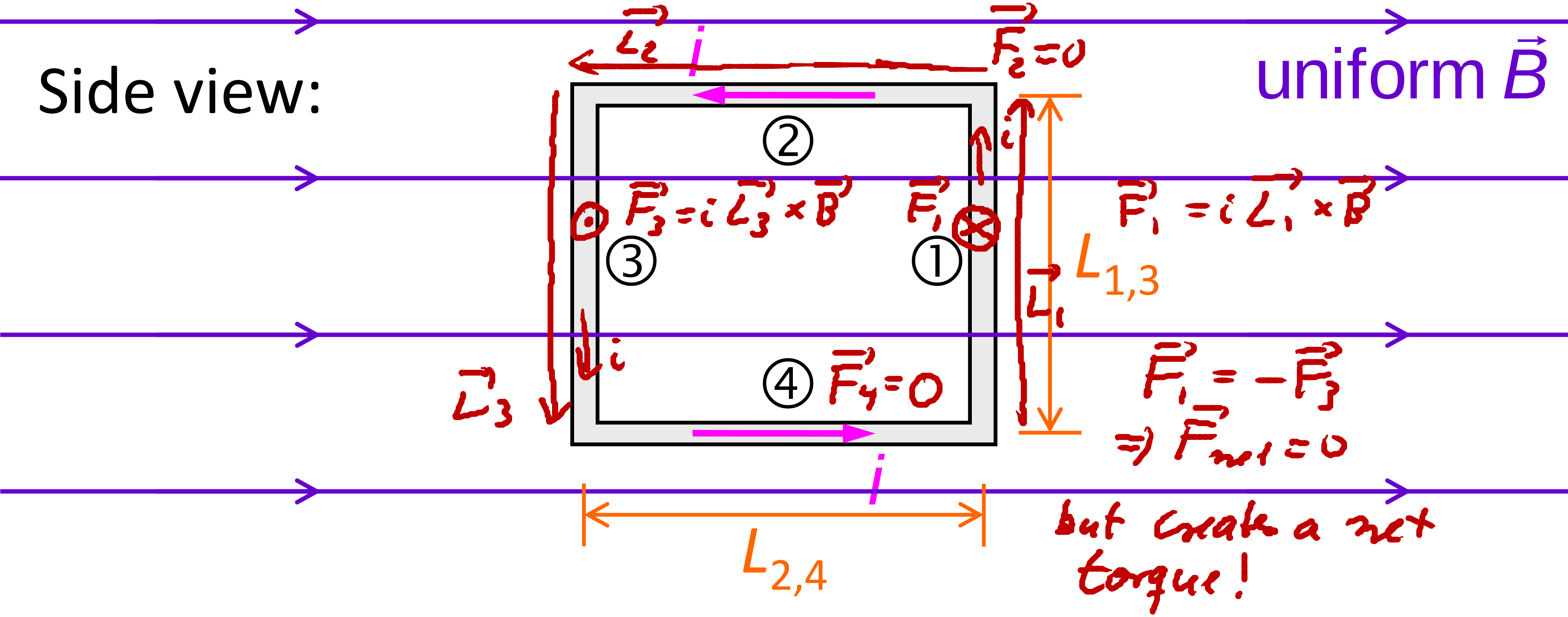
Today:

- **Magnetic force on a current carrying wire**
  - Torque on a current loop
- **Magnetic field due to a current**
  - Field due to a circular arc
  - Field due to a straight wire



Current loop in a uniform magnetic field:

Side view:

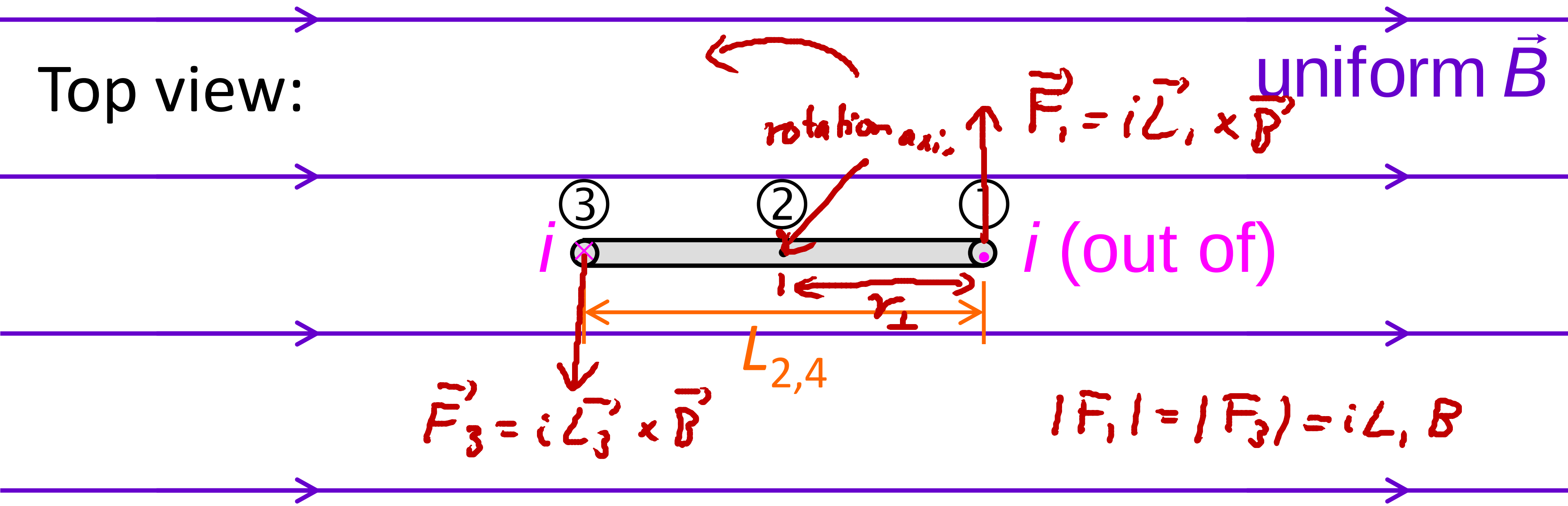


What is the direction of the net magnetic force on the current loop? $\vec{F}_{wire} = i\vec{L} \times \vec{B}$ for straight wire section

A. ⊙ (out of) B. ⊗ (into) C. ↑ D. ↓

E. The net magnetic force on the loop is zero.

Current loop in a uniform magnetic field:



⇒ net torque:

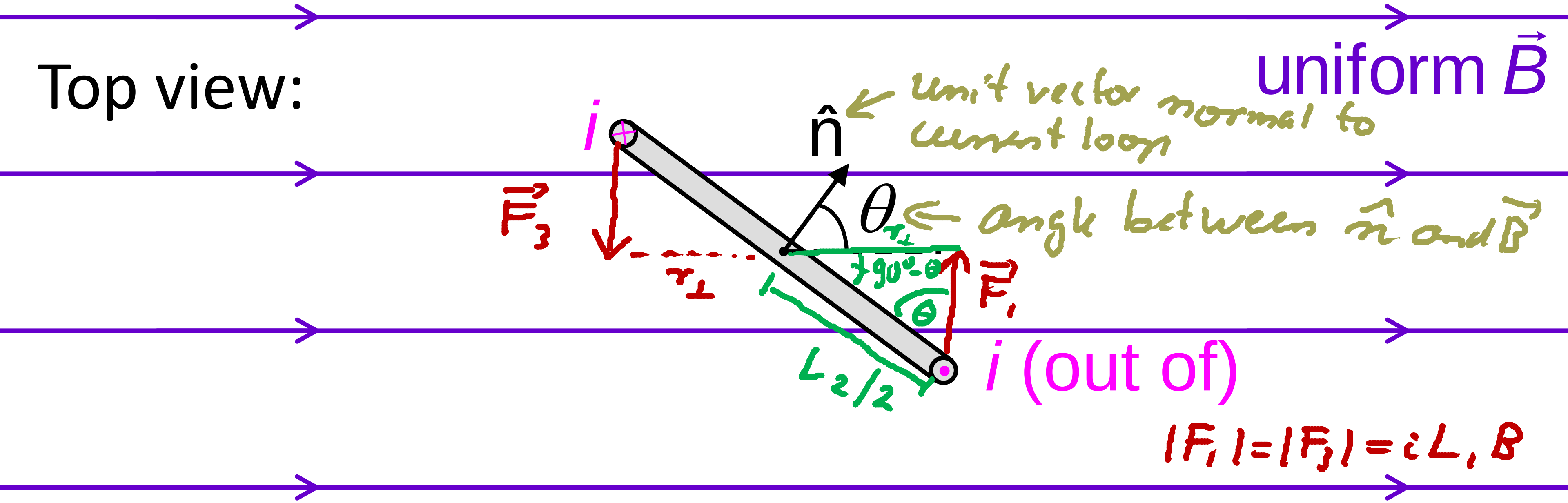
$$|\tau| = |\tau_1| + |\tau_3| = \overbrace{\frac{L_2}{2}}^{r_\perp} F_1 + \frac{L_2}{2} F_3$$

$$\Rightarrow \quad \tau_{net} = L_1 L_2 i B = \underbrace{A}\, i B$$

$A = L_1 \cdot L_2 =$ area enclosed by the loop

For general orientation of the loop relative to $\vec{B}$:

Top view:



$|F_1| = |F_3| = i L_1 B$

$\Rightarrow$ net torque:

$$|\tau| = |\tau_1| + |\tau_3| = \frac{L_2}{2} \sin\theta \, F_1 + \frac{L_2}{2} \sin\theta \, F_3$$

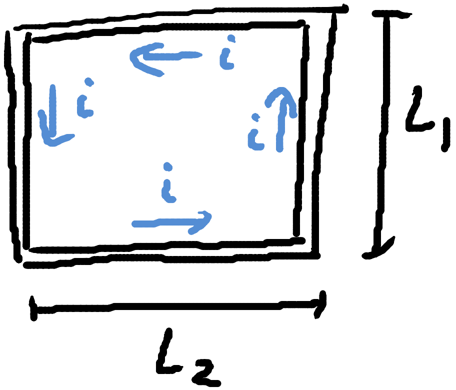
$$\Rightarrow |\tau| = L_2 \sin\theta \, i L_1 B = \underset{\rightarrow}{A} \, i B \, \underline{\underline{\sin\theta}}$$

$A = L_1 \cdot L_2$

Note: in this case, there are also forces on wire sections #2 and #4, but these create no net force and no net torque!

Conclusion: Torque on current Loop:

Side view:



$$|\vec{\tau}| = A\, i\, B \sin\theta$$

current in loop

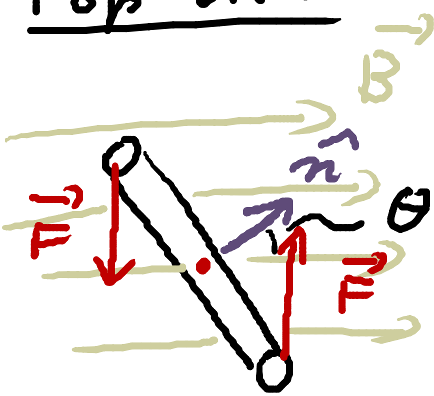
area enclosed by the loop

$|\vec{B}|$

angle between $\vec{B}$ and unit vector $\hat{n}$ normal to the loop

⇒ This equation is valid for all flat current loops, no matter what the shape; e.g. (A) i, (A) i

Top view:



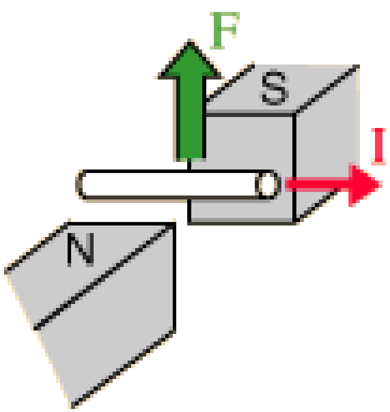
⇒ for coil with N loops, or turns:

$$\boxed{|\vec{\tau}| = (A N i)\, B \sin\theta = \mu\, B \sin\theta}$$

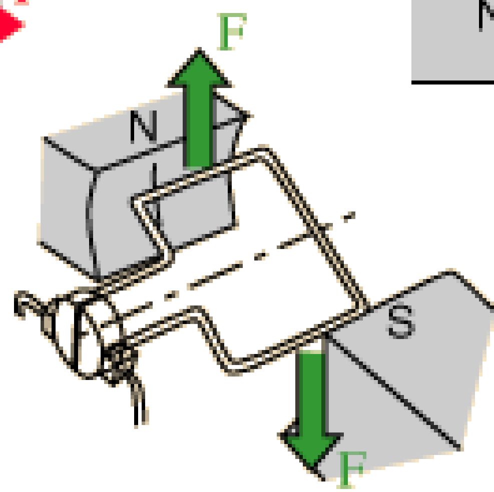
with "magnetic dipole moment"

$\boxed{\mu = A N i}$ of coil.

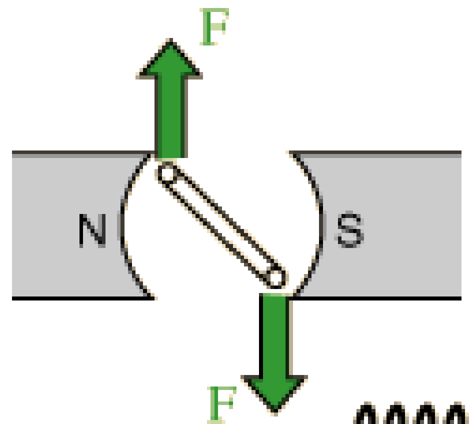
Electric Motor: How it works



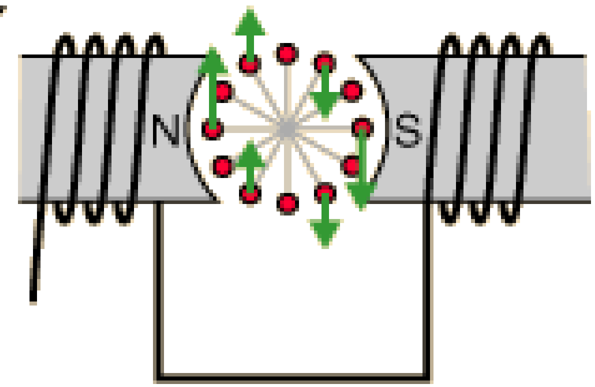
An electric current in a magnetic field will experience a force.



If the current-carrying wire is bent into a loop, then the two sides of the loop which are at right angles to the magnetic field will experience forces in opposite directions.

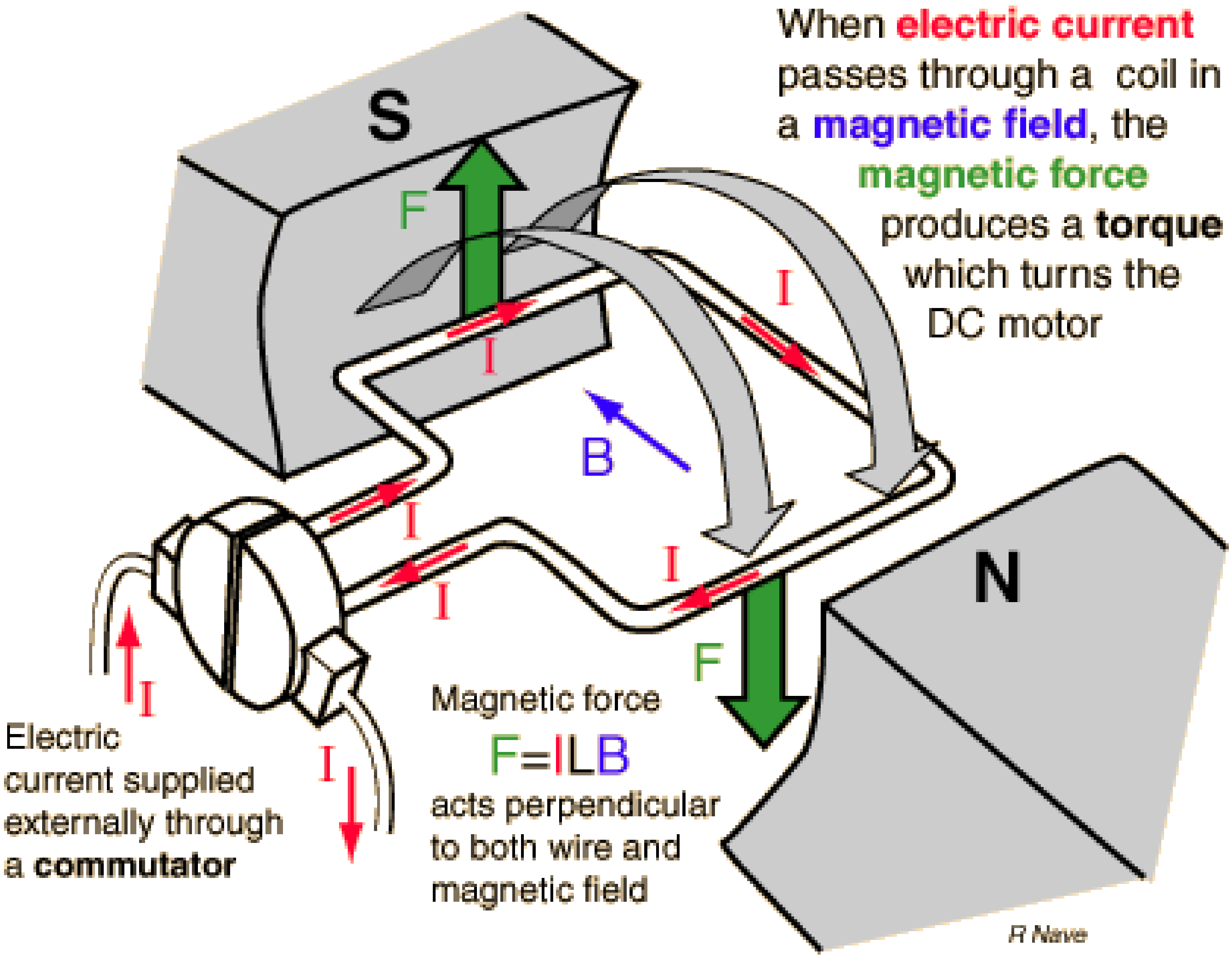


The pair of forces creates a turning influence or torque to rotate the coil.

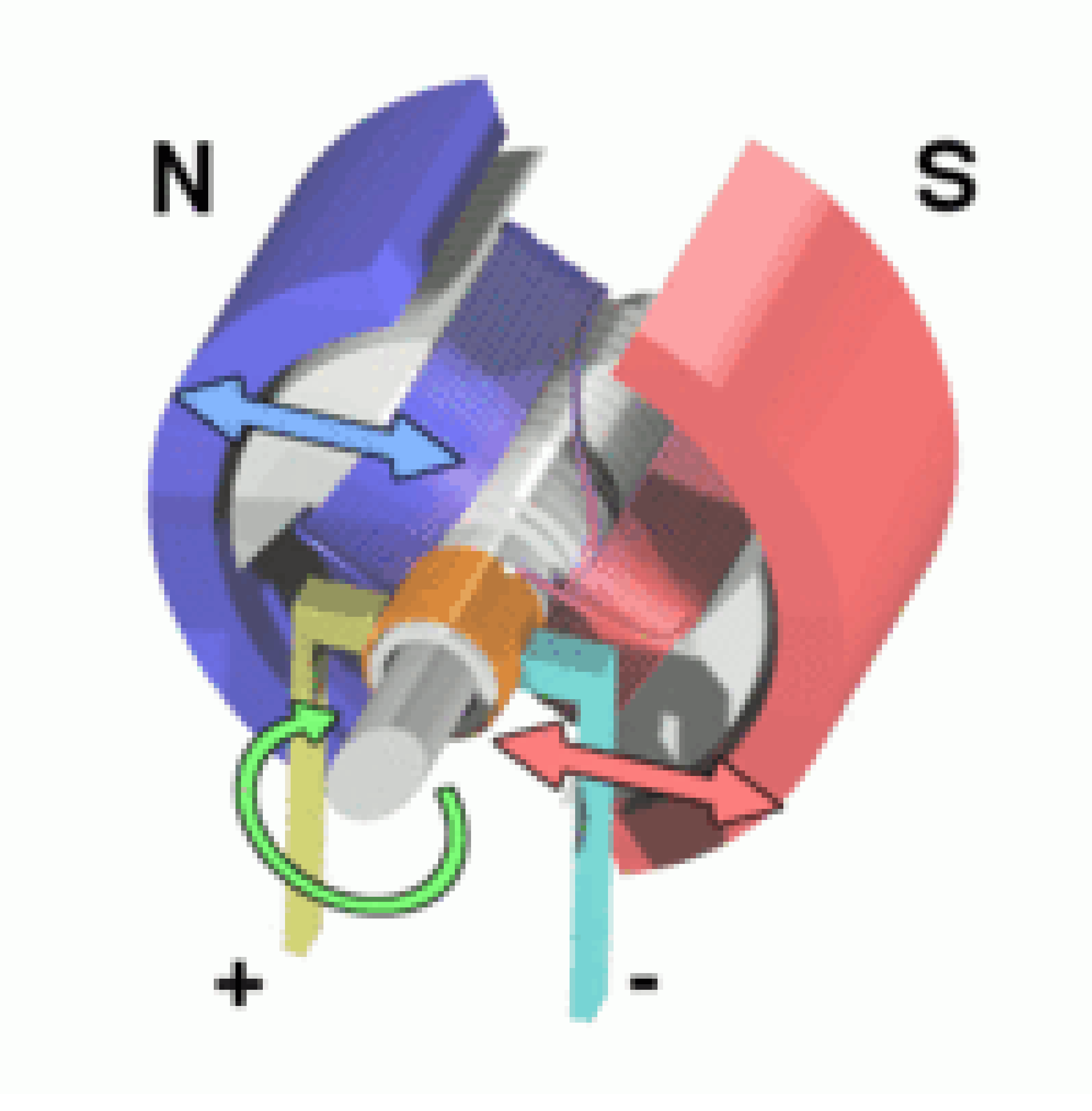


Practical motors have several loops on an armature to provide a more uniform torque and the magnetic field is produced by an electromagnet arrangement called the field coils.

Electric Motor: How it works

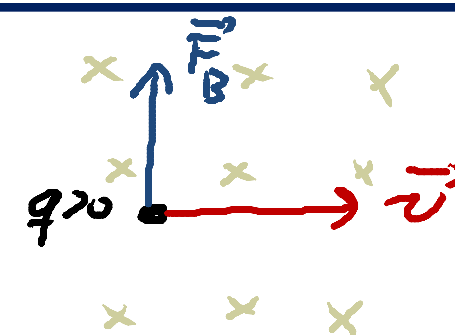


Electric Motor: How it works



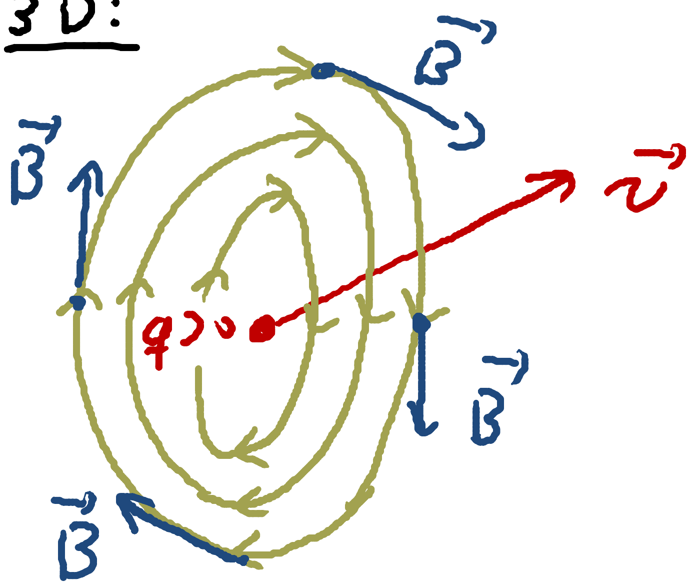
Magnetic Fields due to Currents:

- **so far:** electric charge moving in a magnetic field
  $\Rightarrow$ force: $\vec{F} = q\,\vec{v} \times \vec{B}$



- **now:** moving electric charge produces a magnetic field around itself!

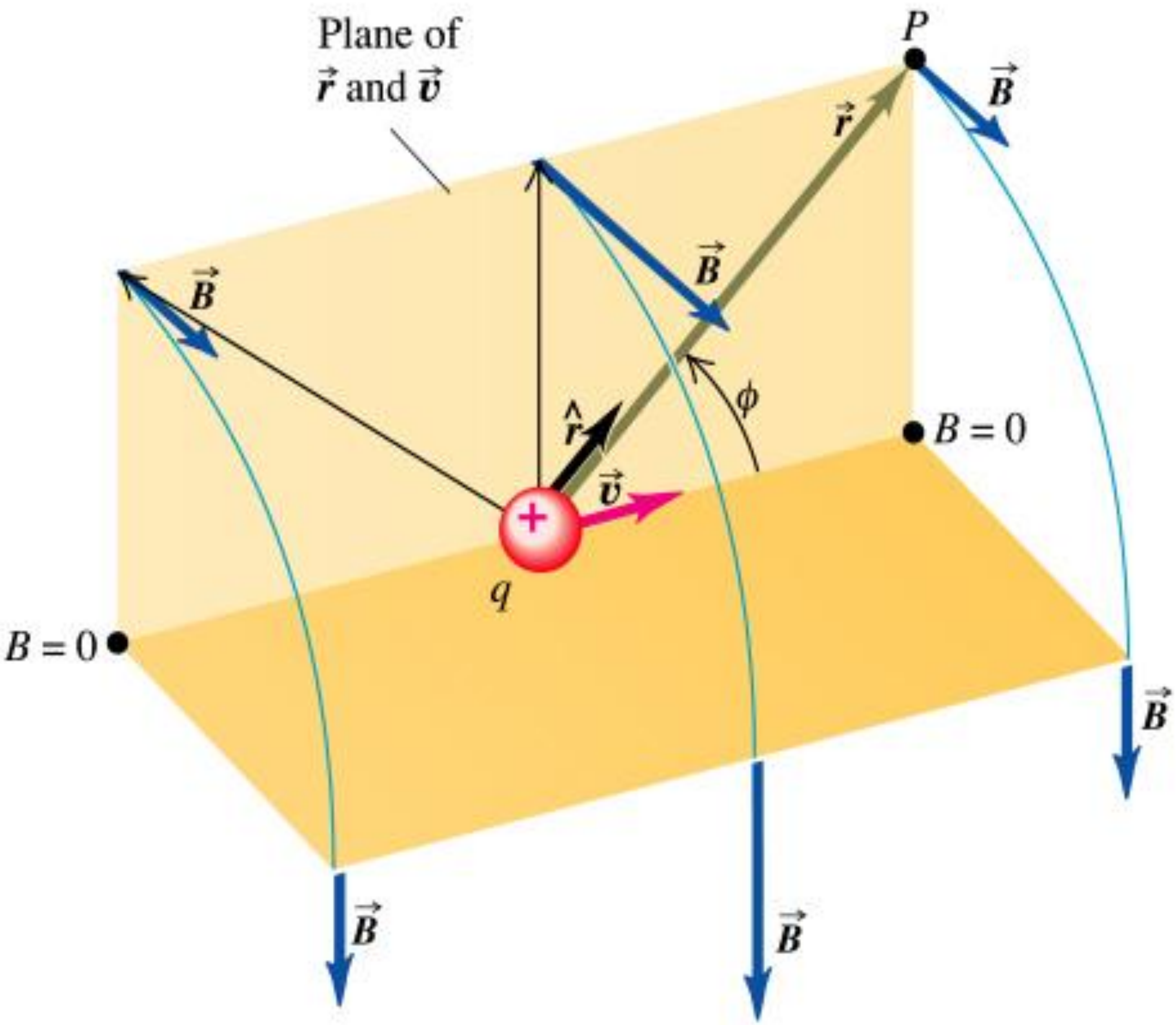
3D:

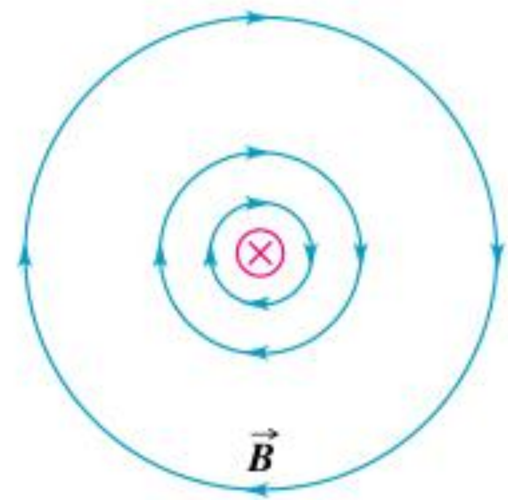


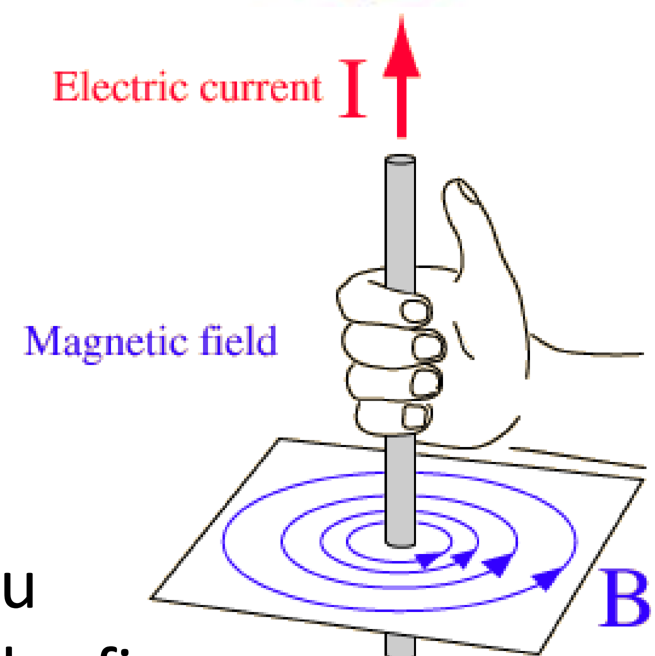
$\vec{B}$ produced by moving charge

$$\boxed{|B| \propto (\text{charge } q) \cdot (\text{speed } v)}$$

use Right-Hand-Rule to find direction of magnetic field around moving charge.

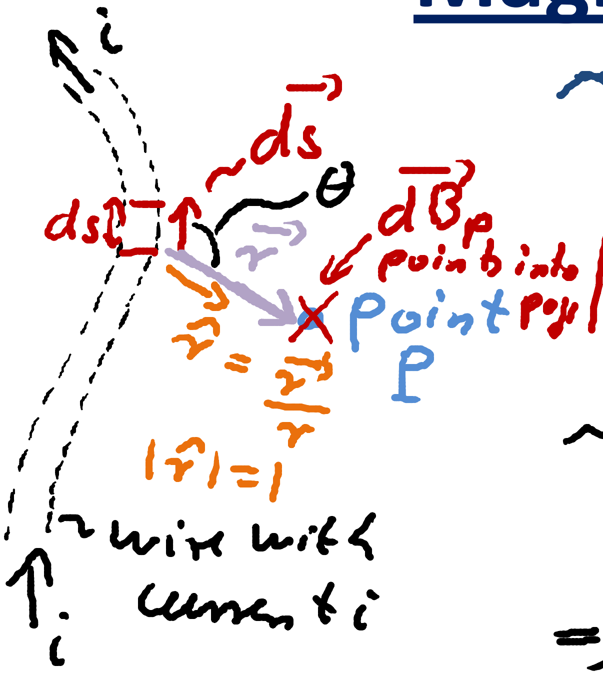






Right-hand rule: Point the thumb of you right hand in direction of the current. The fingers then reveal the B-field vector's direction.

Magnetic Fields due to a Current:



→ Break current path into small sections of length ds

⇒ define length vector : $\vec{ds}$; points in direction of current

→ each section produces some magnetic field $d\vec{B}_p$ at point P

⇒ total field at point P is sum of contributions $d\vec{B}_p$ from all sections of wire:

$$\boxed{\vec{B}_p = \underbrace{\sum d\vec{B}_p}_{\text{all contributions from all wire sections}} = \underbrace{\int d\vec{B}_p}_{\text{along current path}}}$$

What is the magnetic field $\vec{dB}_P$ at point P produced by the current in a very short section $\vec{ds}$ of the current path?

from above:

$$dB_P \propto dQ_{\text{in section}} \cdot v = i\,dt\,v = i\,ds$$

current

length of current path section

turns out to be:

$$\boxed{dB_P = \frac{\mu_0}{4\pi}\frac{i\,ds}{r^2}\sin\theta}$$

angle between $d\vec{s}$ and $\vec{r}$

r: distance from current path section to point P

μ_0: permeability constant

$\mu_0 = 4\pi \cdot 10^{-7}$ T m/A

include information about direction of magnetic field:

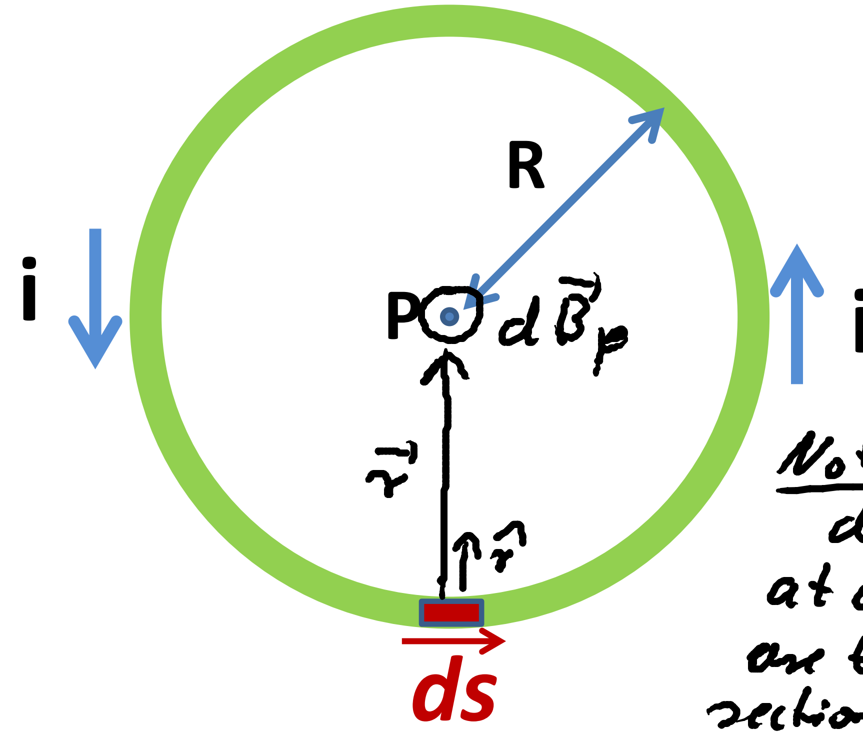
Law of Biot and Savart

$$\boxed{d\vec{B}_P = \frac{\mu_0}{4\pi} i \frac{d\vec{s} \times \hat{r}}{r^2}}$$

with $\hat{r} = \frac{\vec{r}}{r}$ unit vector; points from current path element ds to point P

Note: $d\vec{B}_P$ points $\perp$ to $d\vec{s}$ and $\vec{r}$ always!

Consider a current carrying circular wire loop:



use right hand rule

Note: magnitude and direction of $d\vec{B}_P$ at center of loop are the same for each section ds along the loop!

What is the **direction of the magnetic field at the center of the loop due to the current at ds**?

A. ↑   B. ↓   C. ←   D. ⊙ (out of)   E. ⊗ (into)