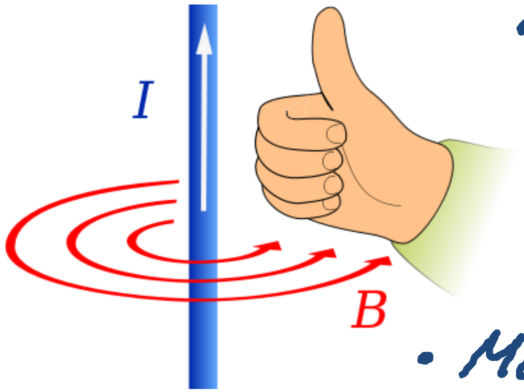
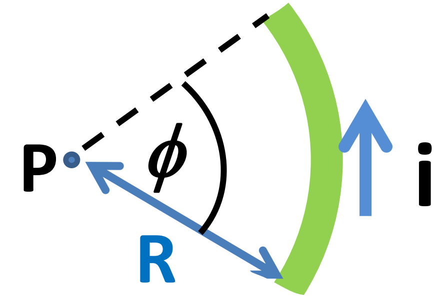
Recap



- Magnetic field at center of circular arc of wire:

$$B_p = \frac{\mu_0 i}{4\pi R}\,\phi \leftarrow \text{angle in rad!}$$

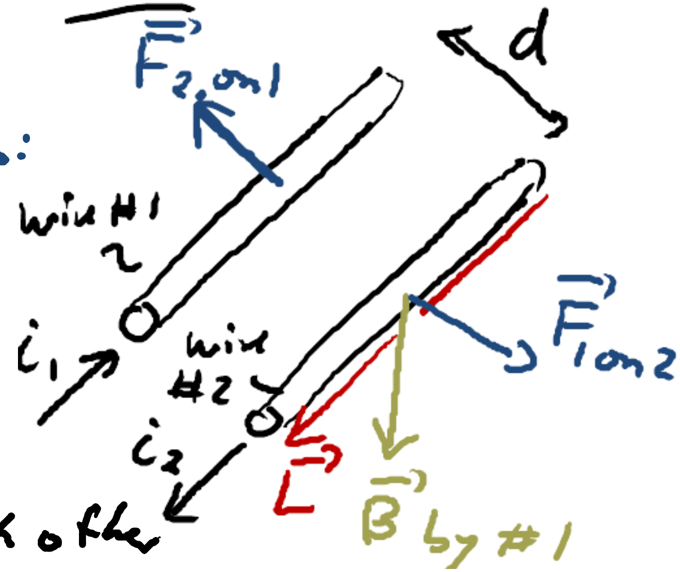


Arc length = ϕR

- Magnetic field by an infinite straight wire:

$$B_p = \frac{\mu_0 i}{2\pi R}$$

3D:



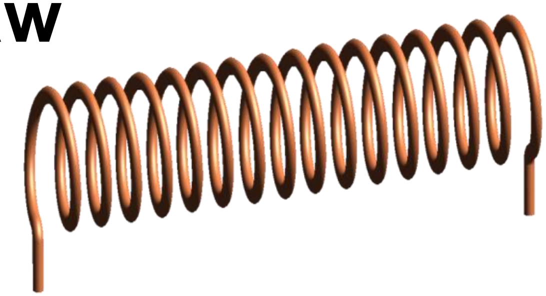
- Forces between two parallel wires:

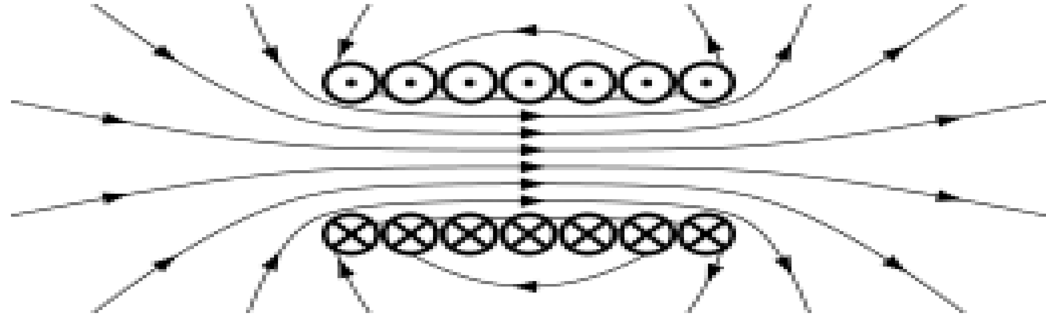
$$|F_{1on2}| = |F_{2on1}| = |i\vec{L}\times\vec{B}| = \frac{\mu_0 L i_1 i_2}{2\pi d}$$

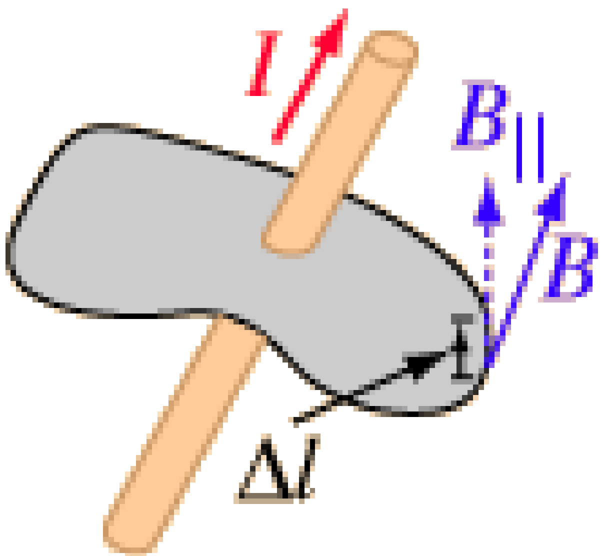
- Parallel currents attract each other
- Anti parallel currents repel each other

Today:

- **Ampere's law**
- **Applications of Ampere's law**
  - **Straight wire**
  - **Solenoid**



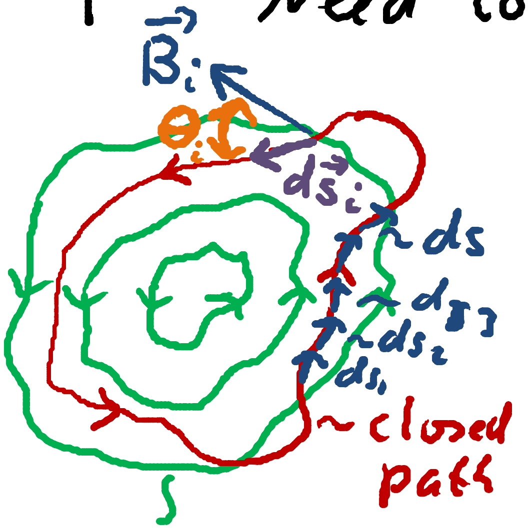




$$\sum B_{||} \Delta l = \mu_0 I$$

Next: Ampere's Law:

• 1st: Need to define circulation Γ of a $\vec{B}$-field:



Some magnetic field (not necessarily uniform)

~> consider some imaginary closed path in a given magnetic field

~> Then "walk" along the closed path and integrate over (sum up) the magnetic field component $B_{\parallel}$ pointing along the direction of the path, for one full turn.

~> Break path into small path length elements ds_i, with $\vec{B}_i \approx$ const over given path section

define:

angle between $\vec{B}_i$ and $\vec{ds}_i$

$$\Gamma \equiv \sum_{\substack{i \\ \text{closed} \\ \text{path}}} B_{\parallel\,\text{to path}} \cdot ds_i = \sum_{\substack{\text{closed} \\ \text{path}}} B_i \cos\theta_i \, ds_i$$

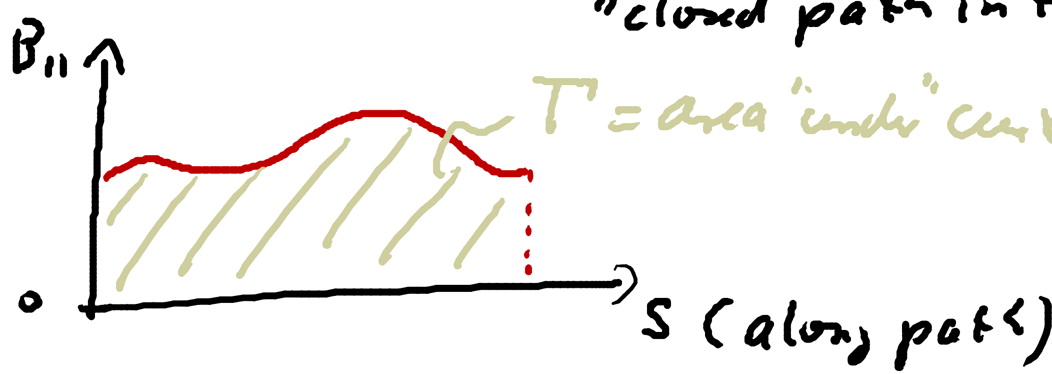
$$= \sum_i \vec{B}_i \cdot \vec{ds}_i$$

$\Rightarrow$ get integral around closed path

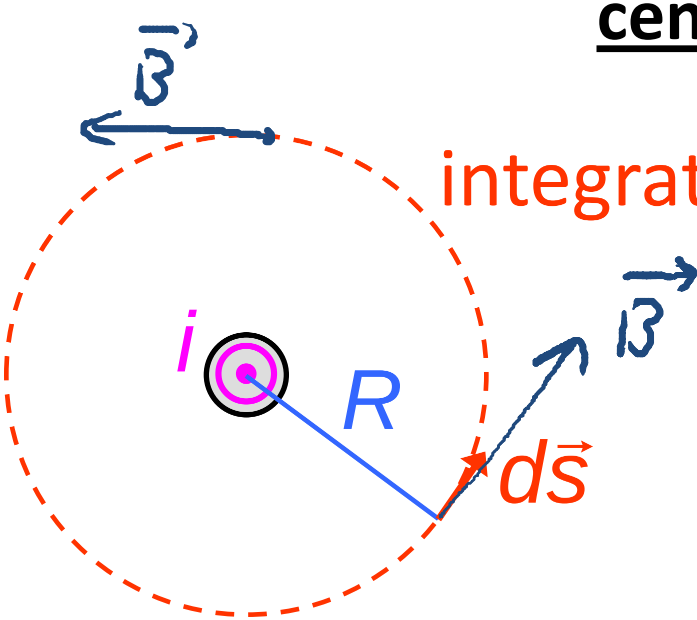
$$\boxed{\Gamma = \oint \vec{B} \cdot \vec{ds} = \oint B \cos\theta \, ds = \oint B_{\parallel} \, ds}$$

"closed path integral"

component $\parallel$ to path



Ex.: Calculate Γ for a circular path centered around a long straight wire:



$\vec{B}$ points along integration path at each point on the path!

$B_{\parallel} \text{ to } d\vec{s} = |B| = \frac{\mu_0 i}{2\pi R}$ } from before

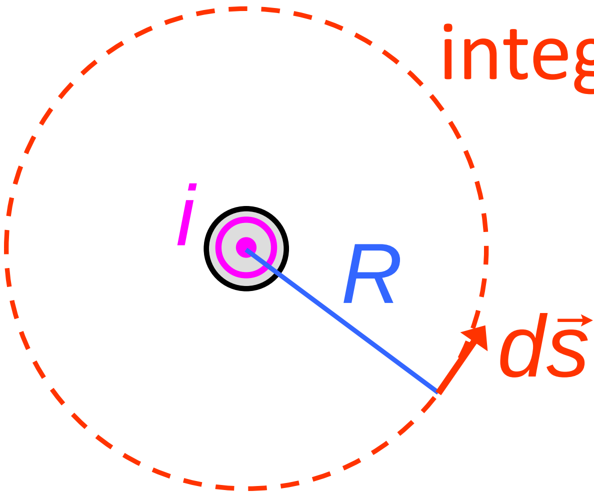
What is the component of $\vec{B}$ along the direction of $d\vec{s}$?

A. $B_s = \mu_0 i/(2\pi R)$. B. $B_s = -\mu_0 i/(2\pi R)$. C. 0.

D. It depends on where $d\vec{s}$ is along the path.

E. Not enough information.

Ex.: Calculate Γ for a circular path centered around a long straight wire:



$$\Gamma = \oint \vec{B} \cdot d\vec{s} = \oint B_{\parallel \text{ to path}} \, ds$$

$$= \oint \frac{\mu_0 i}{2\pi R} \, ds = \underbrace{\frac{\mu_0 i}{2\pi R}}_{\text{const. along closed path here}} \overbrace{\oint ds}^{\text{length of path}}$$

$$\Rightarrow \underline{\Gamma} = \left(\frac{\mu_0 i}{2\pi R}\right)(2\pi R) = \underline{\mu_0 i}$$

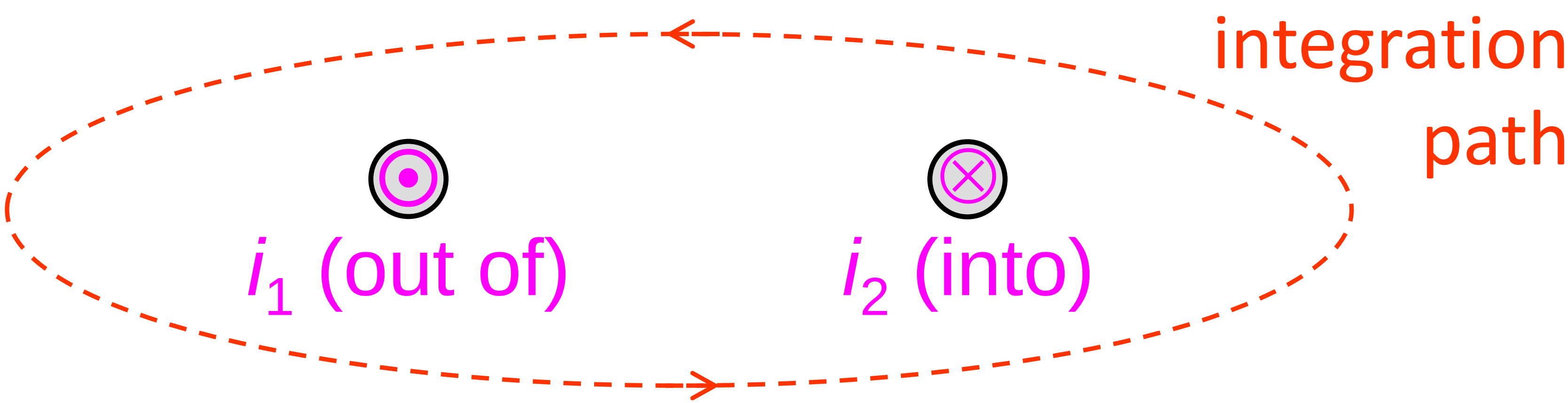
$\Gamma = \mu_0 i$ turns out to be true for <u>any</u> given magnetic field and <u>any</u> closed path!

$\Rightarrow$ Ampere's Law!

Ampere's law:

$$\boxed{\oint \vec{\mathbf{B}} \cdot d\vec{\mathbf{s}} = \oint B\cos\theta ds = \oint B_{\parallel} ds = \mu_0 i_{\text{enc, net}}}$$

where $i_{\text{enc,net}}$ is the net current enclosed by the closed path of integration and θ is the angle between $\vec{\mathrm{B}}$ and $\vec{\mathrm{ds}}$.

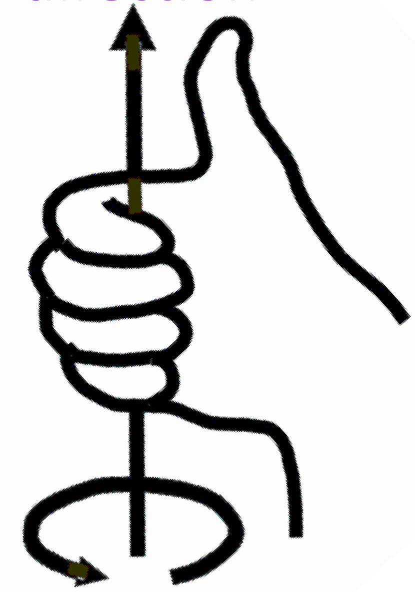


$$i_{\text{enc,net}} = i_1 - i_2.$$

Use a right-hand rule to assign + or – signs to enclosed currents.

- “current enclosed by the closed path”:
  - current must pierce through imaginary surface that is completely bounded by the closed integration path

- right-hand rule to find sign of current:
  - Curl fingers of your right hand along the direction of the closed integration path. Then a positive current will run in the general direction of your thumb, while a current which runs in the opposite direction is negative.

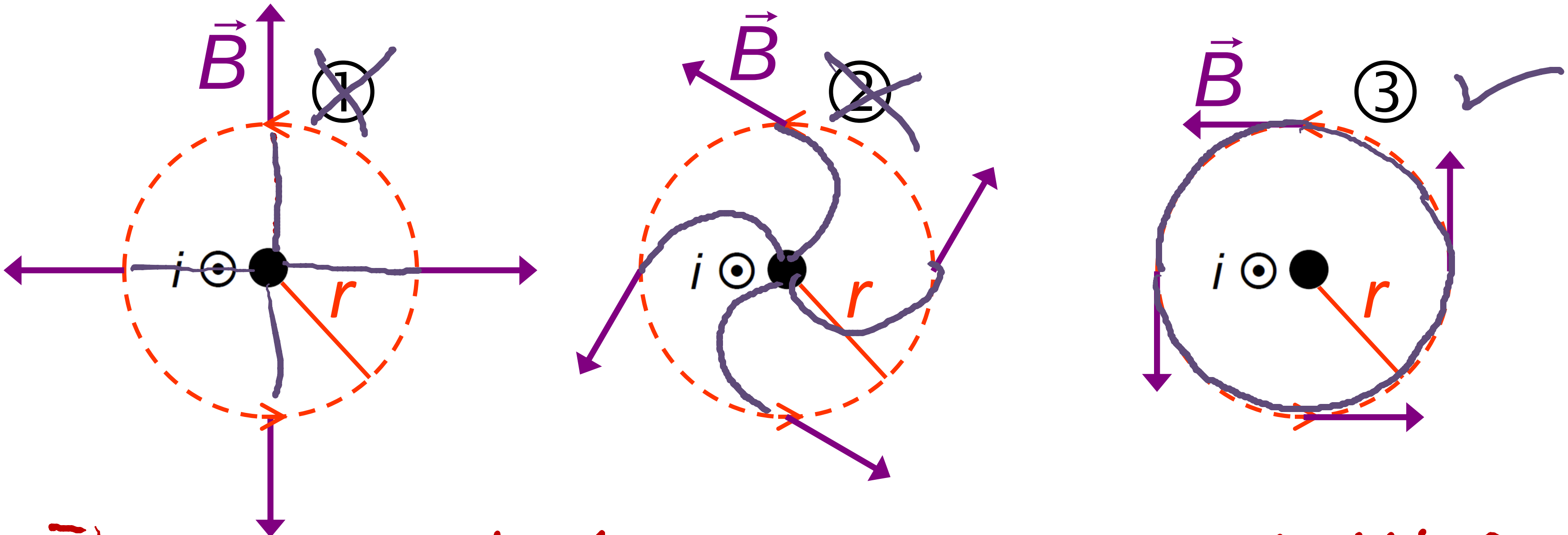


Applications of Ampere’s law:

In certain cases, Ampere’s law can be used together with symmetry arguments to find an unknown magnetic field.

- Magnetic field by a long, straight wire
- Magnetic field by a long solenoid

Consider a long, straight Wire:



– $\vec{B}$ must be cylindrically symmetric here ⇒ could be ①, ②, or ③

– but also: magnetic field lines must be closed loops ⇒ ③

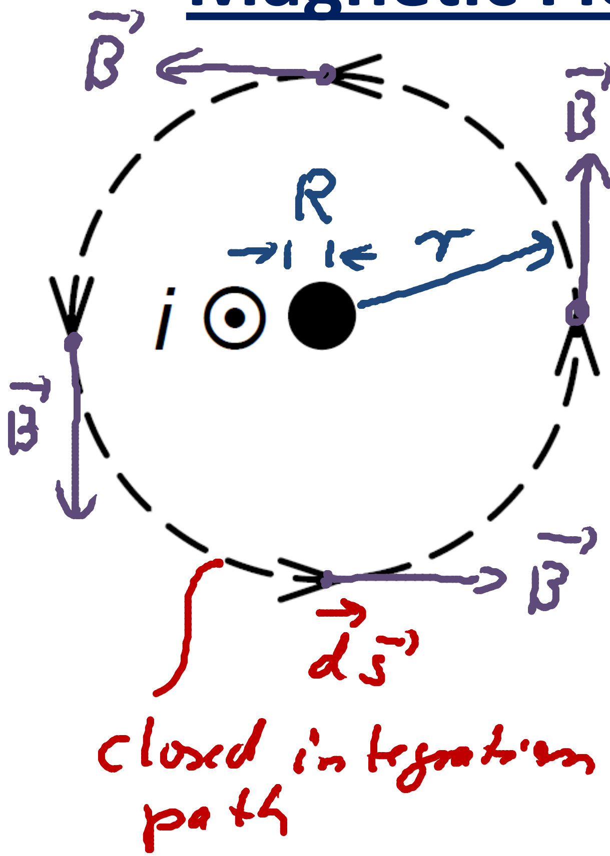
Which configuration of magnetic field along the integration path can be correct (use symmetry arguments)?

A. ①　　B. ②　　C. ③

D. None of the above.

Applications of Ampere's Law:

Magnetic Field *outside* of a Long, straight Wire



$\vec{B}$ points along integration path:

$|B| = |B_{\parallel}| =$ const along path

$\Rightarrow \quad \vec{B} \cdot d\vec{s} = B\,ds$

use Ampere's Law:

$$\oint \vec{B} \cdot d\vec{s} = \mu_0 i_{enc}$$

$$\Downarrow$$

$$\oint B\,ds = B \underbrace{\oint ds}_{2\pi r} = \mu_0 i$$

positive here

$$\Rightarrow \boxed{B = \frac{\mu_0 i}{2\pi r}}$$

for long wire,

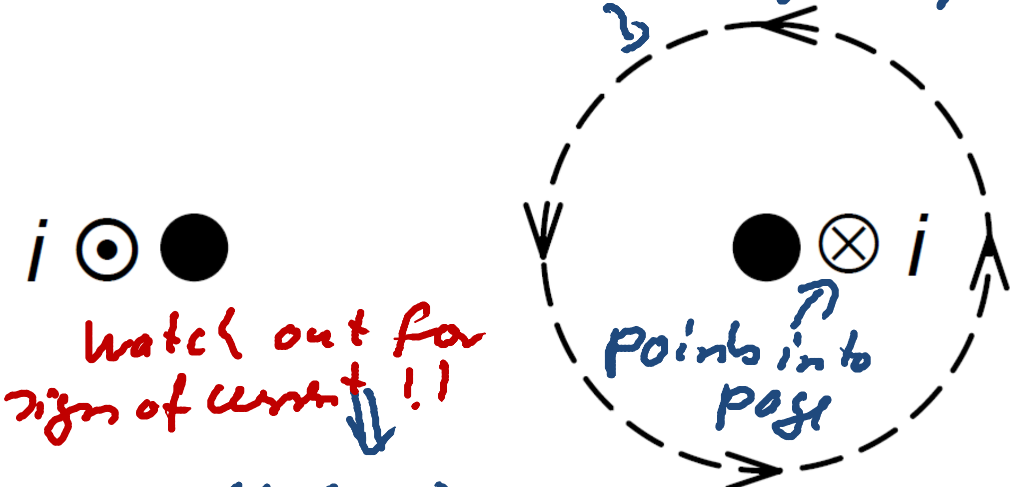
for $r > R$ (outside of wire)

Consider two long straight current-carrying wires as shown below:

closed integration path

What is the value of

$$\oint \vec{B} \cdot d\vec{s}$$



points into page

watch out for sign of current!!

for the path shown?

$$\oint \vec{B} \cdot d\vec{s} = \mu_0 \, \underbrace{i_{enc,net}}_{\text{only current enclosed by path counts!}} = \mu_0 (-i)$$

for sign use right hand rule!

A. $2\mu_0 i$ B. $\mu_0 i$ C. 0

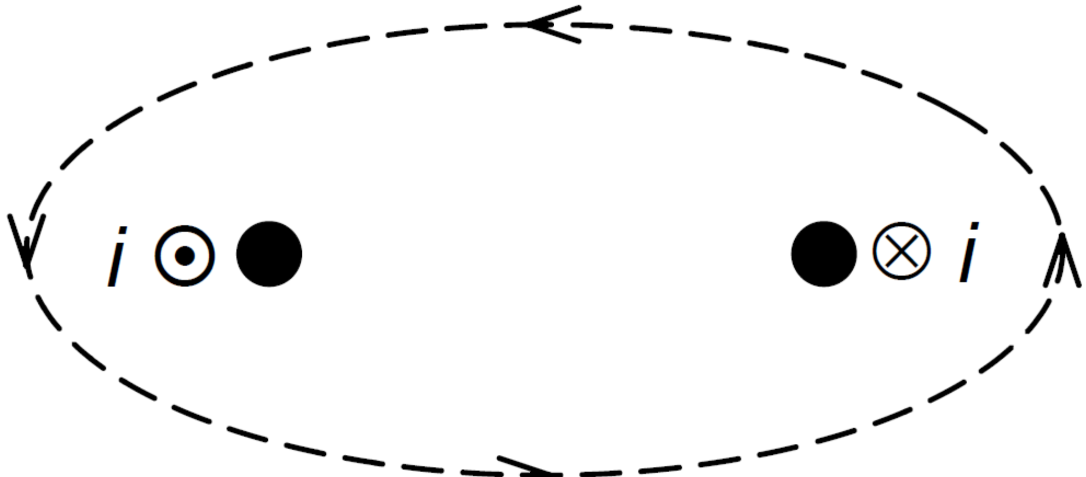
D. $-\mu_0 i$ E. Can't tell.

Consider two long straight current-carrying wires as shown below:

What is the value of

$$\oint \vec{B} \cdot d\vec{s}$$

for the path shown?

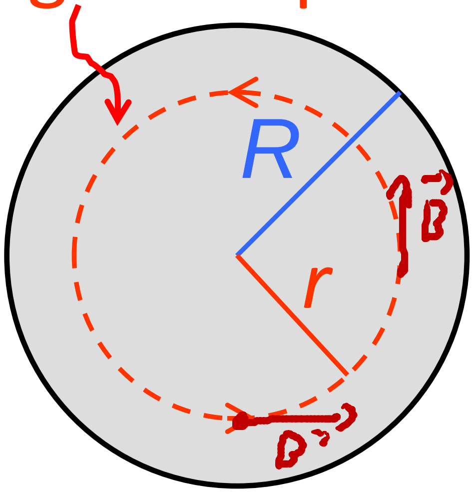


$\oint \vec{B} \cdot d\vec{s} = \mu_0 i_{enc,net} = \mu_0 (i - i) = \underline{\underline{0}}$

A. $2\mu_0 i$
B. $\mu_0 i$
C. 0
D. $-\mu_0 i$
E. Can't tell.

Applications of Ampere's Law:

Magnetic Field *inside* of a Long, straight Wire



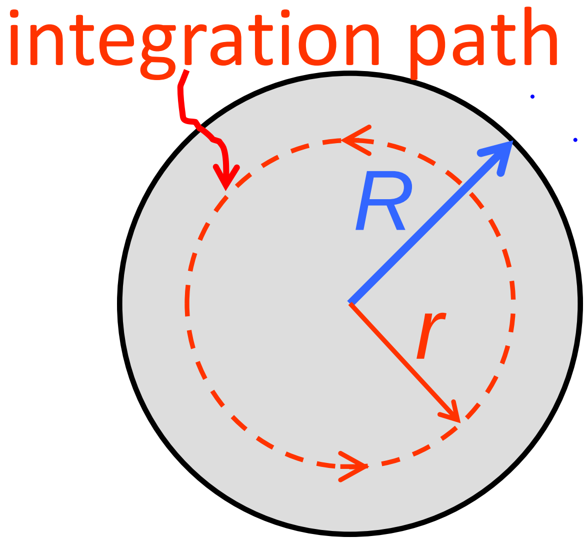
Wire, shown in cross section, carries a current *i* out of (⊙) the screen. Assume that the magnitude of the current density is constant across the wire.

Because of the cylindrical symmetry, the only coordinate that *B* can depend on is *r*. $\Rightarrow B = B(r) = \text{const}$ along path

also: $\vec{B}$ must point along circular integration path

$\Rightarrow \oint \vec{B} \cdot d\vec{s} = \oint B\,ds = B \oint ds = B(2\pi r) = \mu_0 i_{enc}$

Magnetic Field *inside* of a Long, straight Wire



Wire, shown in cross section, carries a current *i* out of (⦿) the screen. Assume that the magnitude of the current density is constant across the wire.

What is the current enclosed by the integration path?

have: $\oint \vec{B}\cdot d\vec{s} = \underline{B}\, 2\pi r = \mu_0 i_{enc} = \mu_0 i \frac{r^2}{R^2}$

$i_{enc} = J A_{\text{enclosed by path}} = J \pi r^2$

$i_{\text{total in wire}} = i = J \pi R^2$

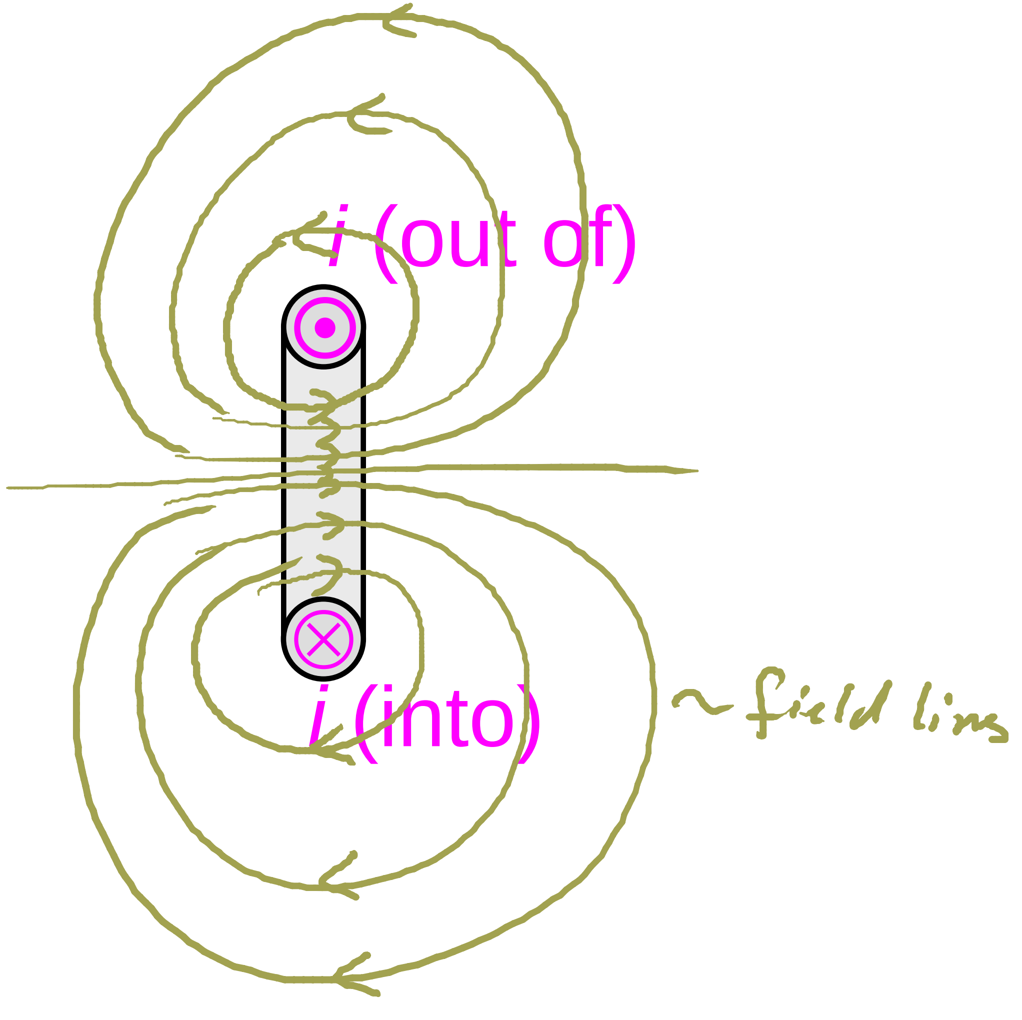
$i_{enc} = i \frac{r^2}{R^2}$

$B = \frac{\mu_0 i}{2\pi R^2}\, r \propto r$

for $r < R$ (inside)

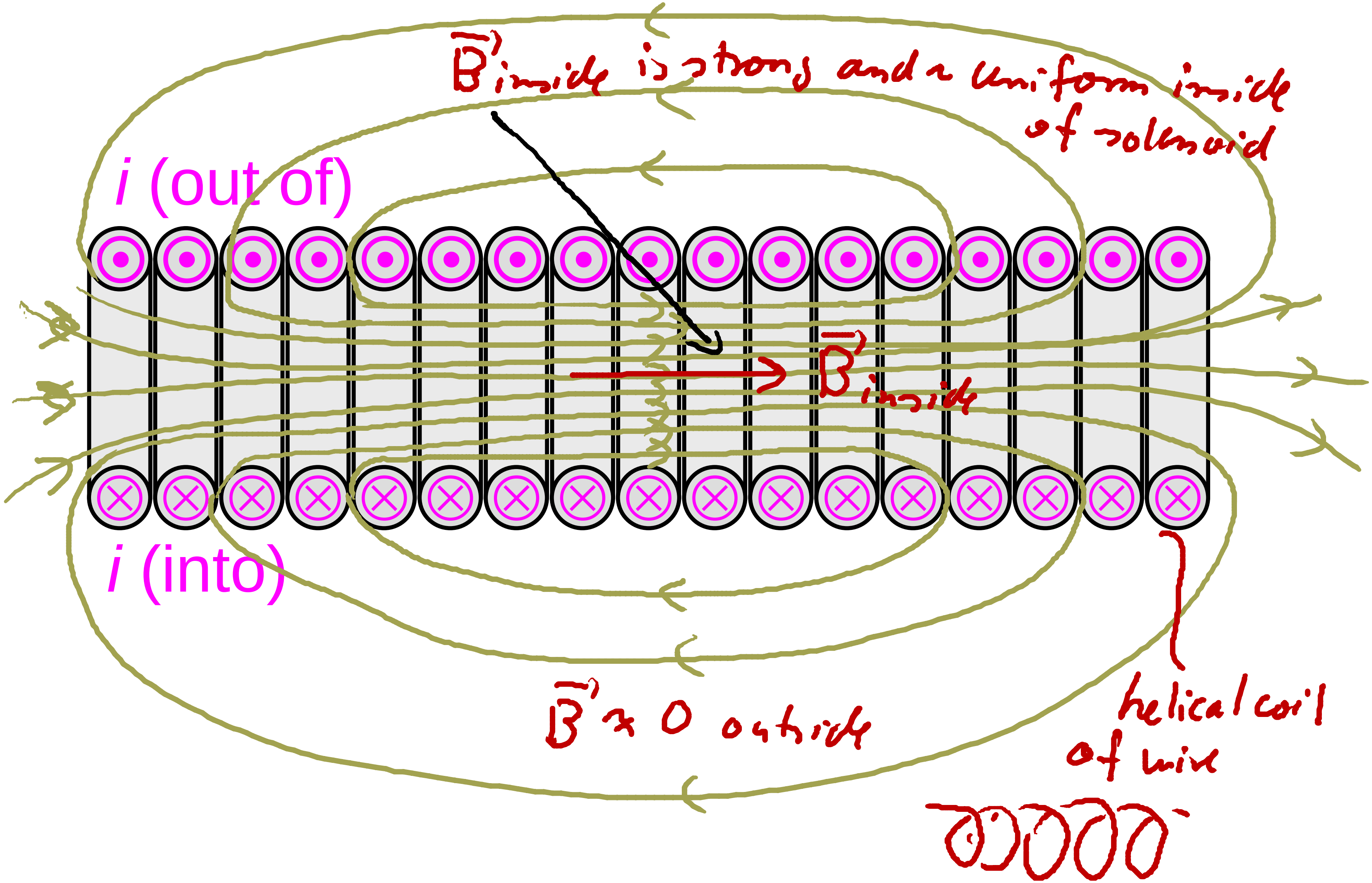
A. i B. $-i$ C. ir^2/R^2 D. $-ir^2/R^2$ E. ir/R

<u>Magnetic field due to a circular current-carrying loop:</u>

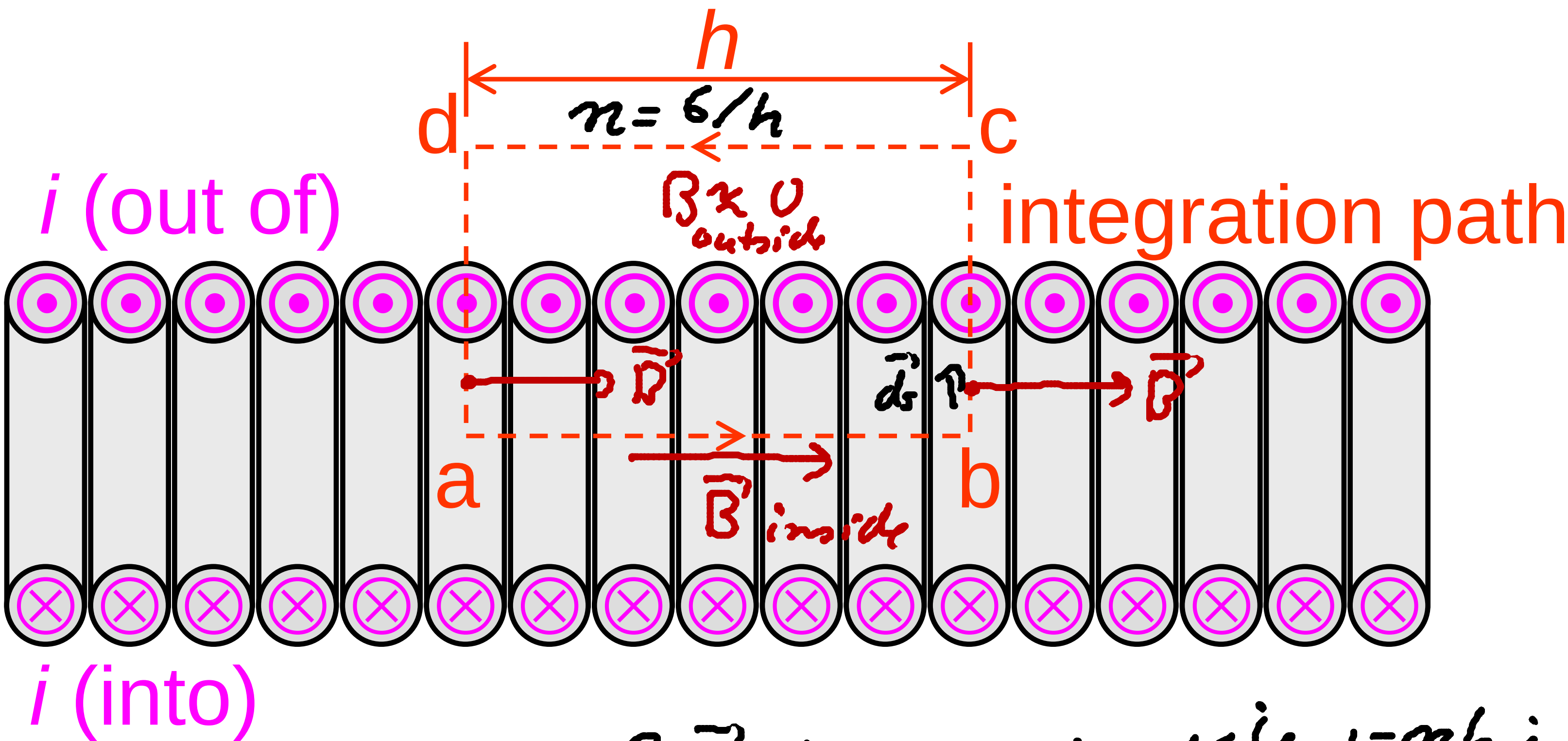


Applications of Ampere's Law:

Magnetic Field inside a Solenoid



Magnetic Field inside a Solenoid



apply Ampere's Law: $\oint \vec{B}\cdot d\vec{s} = \mu_0 i_{enc}$ ← $i_{encl} = n h i$

↑ # of turns per length of solenoid

$$\oint \vec{B}\cdot d\vec{s} = \underbrace{\int_a^b \vec{B}\cdot d\vec{s}}_{= B\cdot h} + \underbrace{\int_b^c \vec{B}\cdot d\vec{s}}_{=0 \text{ since } \vec{B}\perp d\vec{s}} + \underbrace{\int_c^d \vec{B}\cdot d\vec{s}}_{=0} + \underbrace{\int_d^a \vec{B}\cdot d\vec{s}}_{=0 \text{ since } \vec{B}\perp d\vec{s}}$$

$\Rightarrow$ <u>for solenoid:</u>

$$\oint \vec{B}\cdot d\vec{s} = \underbrace{B\int_a^b ds}_{\substack{\text{since } B \text{ inside is const, and} \\ \vec{B} \text{ points along } d\vec{s}}} = \underline{\underline{Bh}} = \mu_0 i_{enc} = \mu_0 n h i$$

$$\Rightarrow \boxed{B_{\substack{\text{inside of} \\ \text{solenoid}}} = \mu_0 \, n \, i}$$

i: current in wire of solenoid

$$n = \frac{\#\text{ of turns}}{\text{unit length of solenoid}}$$