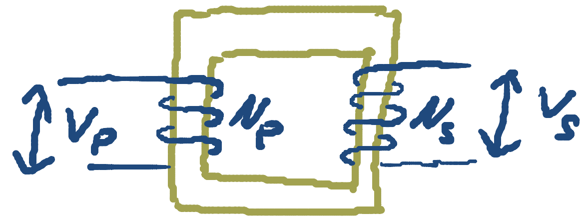
Recap

Lecture 24

- **Transformer:**
  - Transformation of voltage: $V_s = \frac{N_s}{N_p} V_p$
  - Transformation of current: $i_s = \frac{N_p}{N_s} i_p$

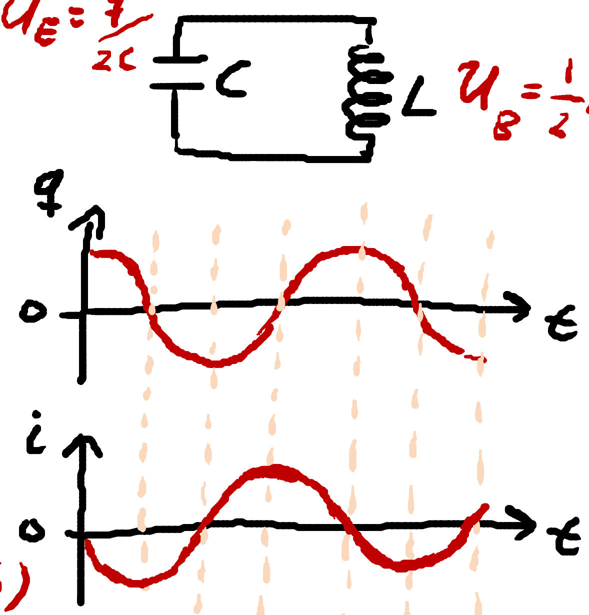


$$\Phi_p = \Phi_s$$

- **Ideal LC-circuit:**

Energy oscillates between electric and magnetic fields, but the total sum of energy remains constant:

$$U_E = \frac{q^2}{2C} \qquad U_B = \frac{1}{2} L i^2$$



$$U_{total} = U_E(t) + U_B(t) = \frac{q(t)^2}{2C} + \frac{L}{2} i(t)^2$$

$$= \frac{1}{2} \frac{Q_{max}^2}{C} = \text{const}$$

oscillation of charge: $q(t) = Q_{max} \cos(\omega t + \phi)$

oscillation of current: $i(t) = \frac{dq}{dt} = -\omega Q_{max} \sin(\omega t + \phi)$

angular frequency: $\omega_0 = \frac{1}{\sqrt{LC}} = 2\pi f_0$

Today:

- **RLC circuit: damping and driven**
- **Another look at Faraday's law**
- **Next time: Maxwell's equations**

$$\oint \mathbf{E}\cdot d\mathbf{A} = \frac{q_{enc}}{\varepsilon_0}$$

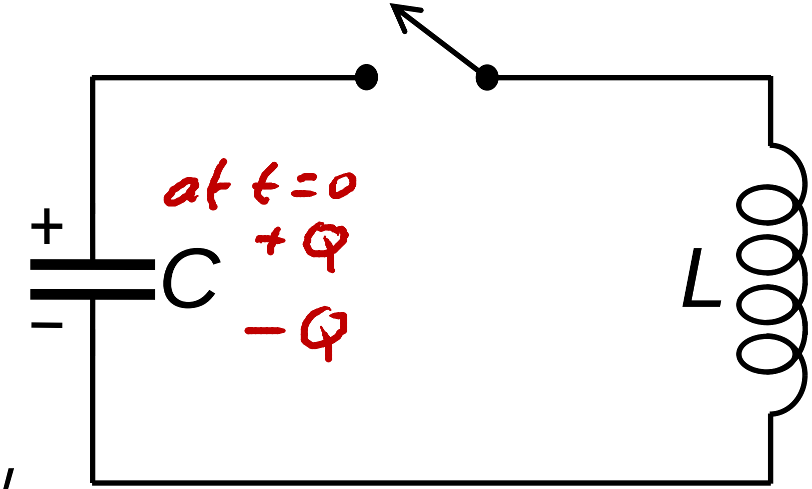
$$\oint \mathbf{B}\cdot d\mathbf{A} = 0$$

$$\oint \mathbf{E}\cdot d\mathbf{s} = -\frac{d\Phi_{\mathrm{B}}}{d\mathrm{t}}$$

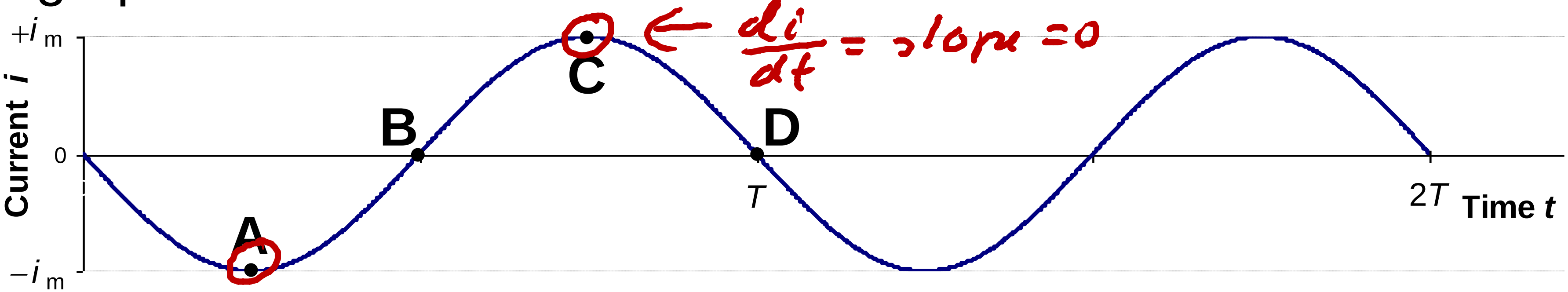
$$\oint \mathbf{B}\cdot d\mathbf{s} = \mu_0\varepsilon_0\frac{d\Phi_{\mathrm{E}}}{d\mathrm{t}} + \mu_0 i_{enc}$$

Ideal LC circuit (no resistance):

The capacitor starts with charge $q = Q > 0$ with the polarity shown. At time $t = 0$ the switch is closed and current $i \equiv dq/dt$ flows in the circuit.



A graph of i versus t is shown below.



$\Delta V_L = L \frac{di}{dt}$ $\Rightarrow$ $\Delta V_L = 0$ when $\frac{di}{dt} = \text{slope} = 0$

Which of the labeled points correspond(s) to **no voltage across the inductor**?

A. **A** B. **B** C. **C** D. **D** E. Both **A** & **C**

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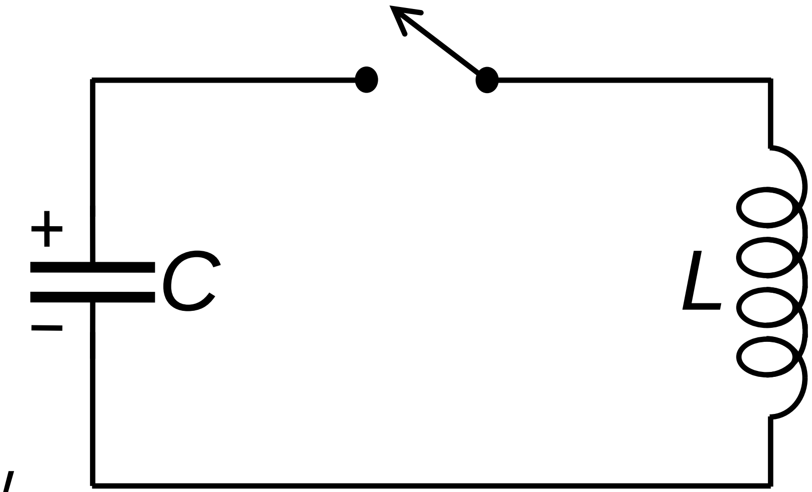

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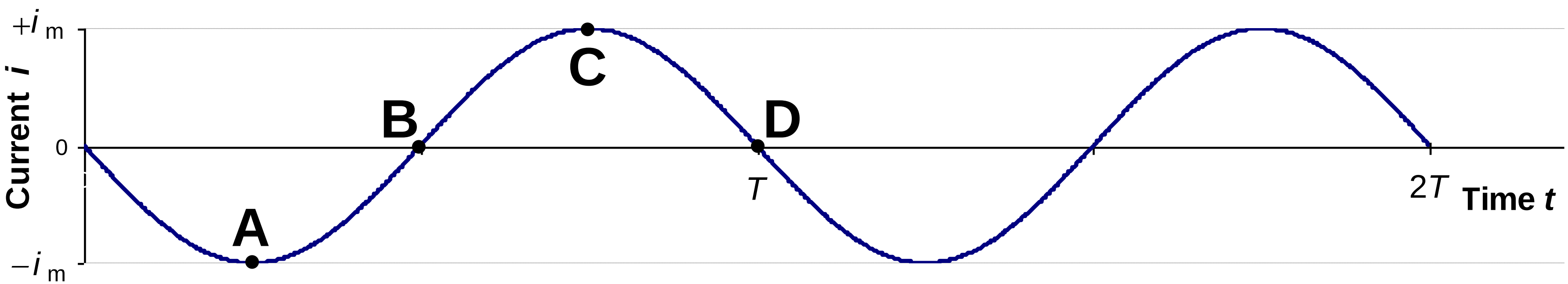
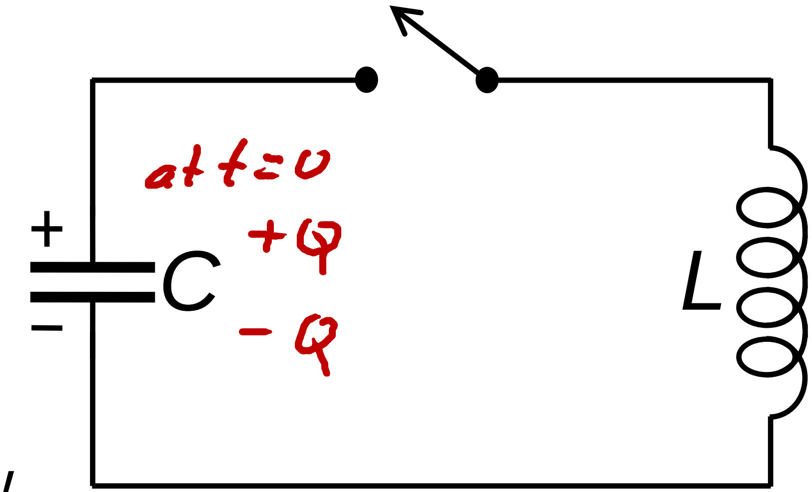


loop rule: $\Delta V_C + \underbrace{\Delta V_L}_{=0 \text{ at A and C}} = 0$

Which of the labeled points correspond(s) to **no voltage across the capacitor**?

A. **A**	~~B. **B**~~	C. **C**	~~D. **D**~~	E. Both **A** & **C**

The capacitor starts with charge $q = Q > 0$ with the polarity shown. At time $t = 0$ the switch is closed and current $i \equiv dq/dt$ flows in the circuit.



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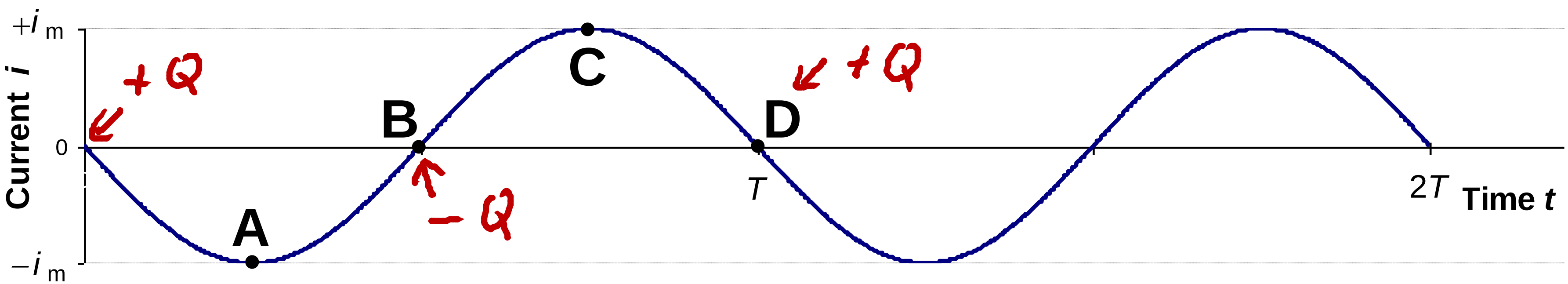

Which of the labeled points correspond(s) to **charge +Q on the capacitor**?

A. **A**	B. **B**	C. **C**	D. **D**	E. Both **A** & **C**

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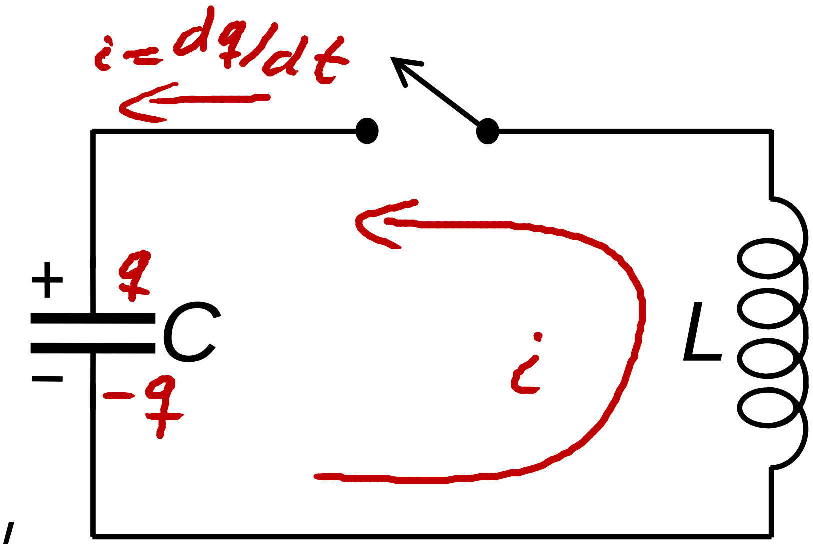

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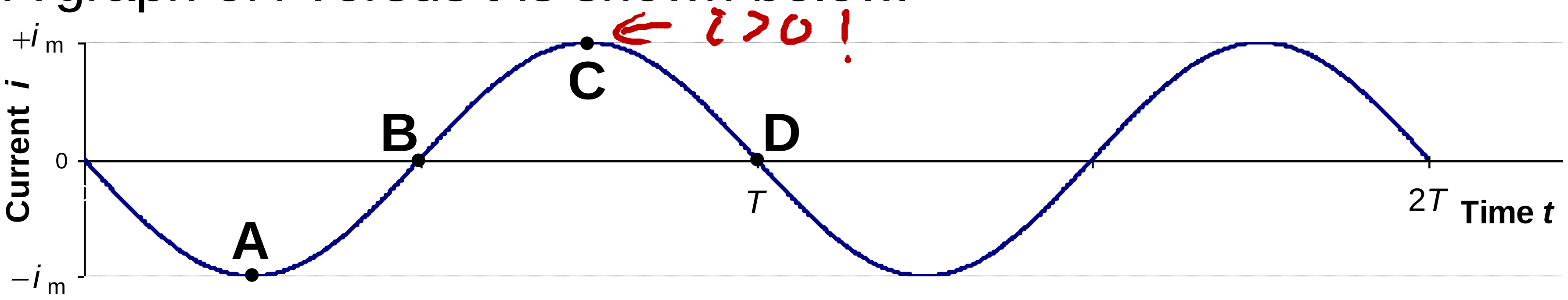
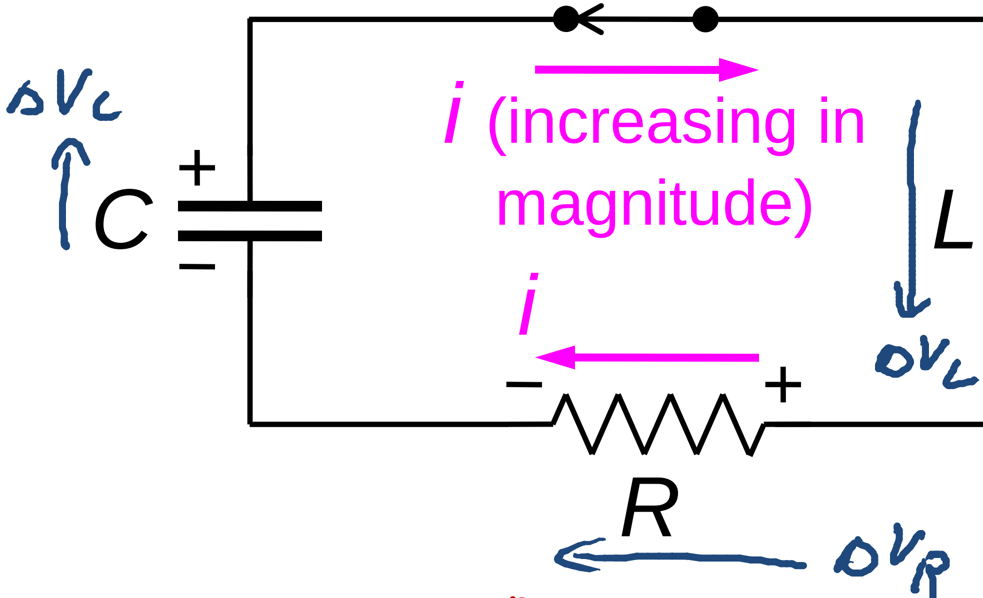


Which of the labeled points correspond(s) to **counterclockwise current flow in the circuit**?

~~A. **A**~~ | ~~B. **B**~~ | C. **C** | ~~D. **D**~~ | ~~E. Both **A** & **C**~~

RLC circuit:



– Resistance R

⇒ dissipate power

⇒ damping of oscillation!

– loop rule now gives:

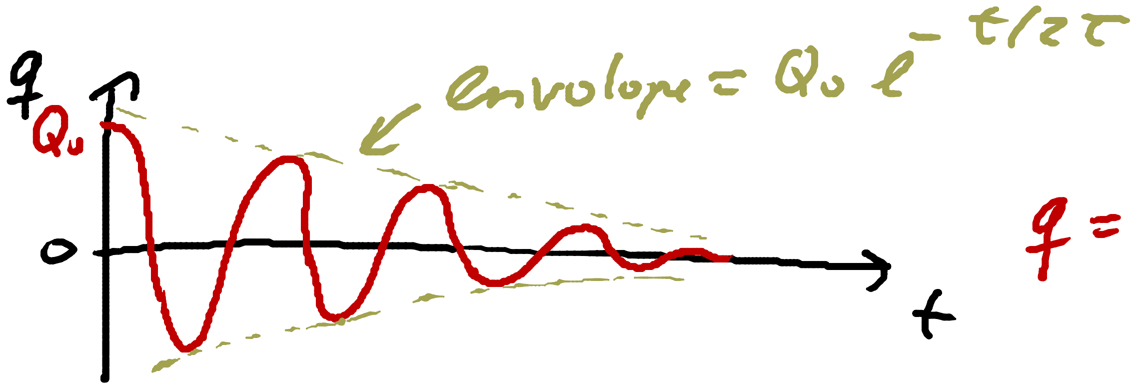
$$\underbrace{\frac{q}{C}}_{\Delta V_C} + \underbrace{L\frac{d^2q}{dt^2}}_{\Delta V_L} + \underbrace{\frac{dq}{dt}R}_{\Delta V_R} = 0$$

⇒ solution: $\boxed{q(t) = Q_0\, e^{-t/2\tau} \cos(\omega' t + \phi)}$

with energy decay time constant $\tau_{RLC} = L/R$

and $\omega' = \sqrt{\omega_0^2 - \left(R/2L\right)^2}$

⇒ for small R : $\omega' \approx \omega_0 = \frac{1}{\sqrt{LC}}$



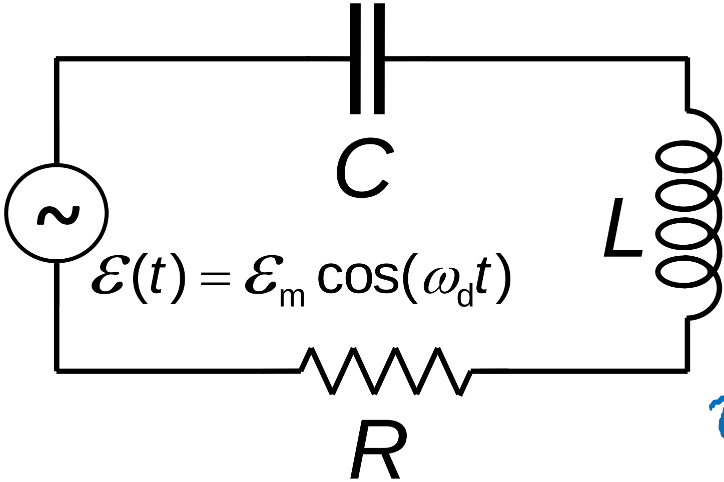
$$q = \underbrace{Q_0 e^{-t/2\tau}}_{Q_{max}(t)} \cos(\omega' t + \phi)$$

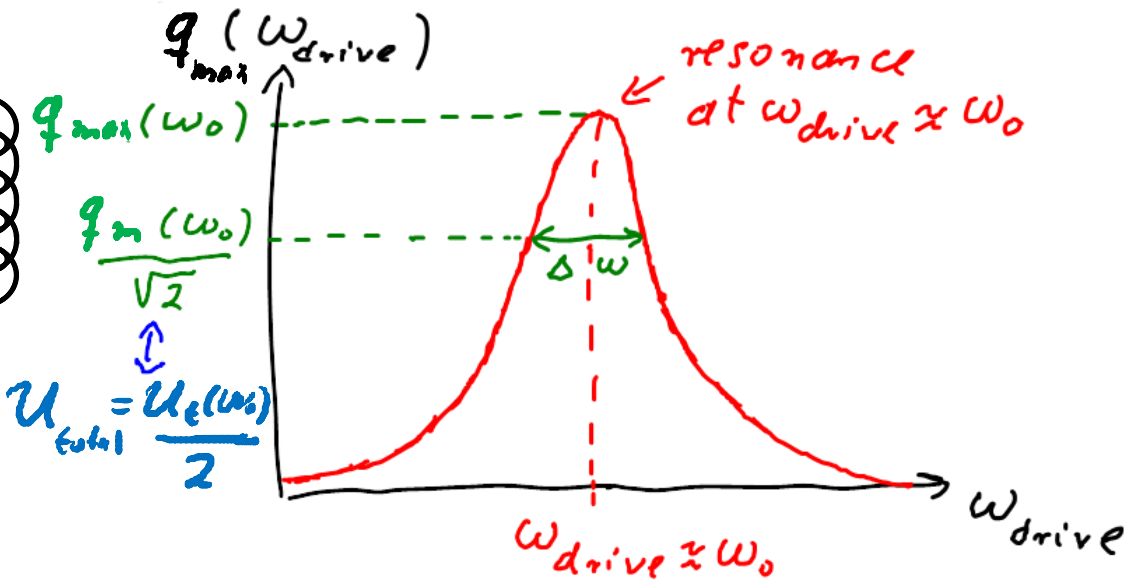
$\Rightarrow$ U_{total} (total energy stored in circuit) also decreases exponentially:

$$U_{total}(t) = \frac{1}{2C} Q_{max}(t)^2 = \frac{1}{2C} Q_0^2 e^{-t/\tau}$$ ← "energy decay time constant"

$\tau = L/R$

Driven *RLC* circuit:





Current will oscillate at the driving frequency: $f_d = \omega_d/(2\pi)$

- **Maximum current amplitude when driving frequency matches natural frequency of circuit:**

$$\boxed{f_d = f_0 \frac{1}{2\pi\sqrt{LC}}} \quad \text{(resonance)}$$

- resonance width

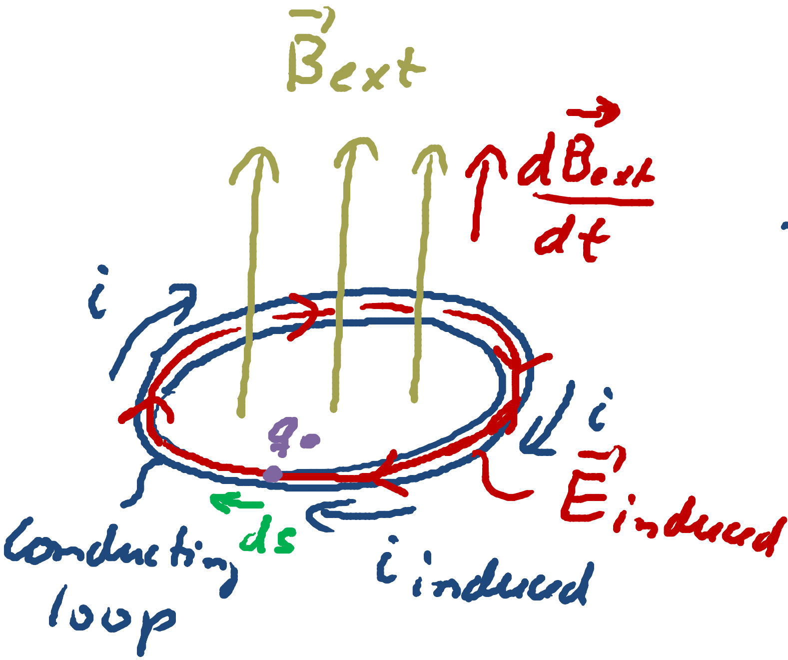
$$\boxed{\Delta\omega = \frac{1}{\tau}} \leftarrow \text{energy decay time}$$

- define quality factor:

$$\boxed{Q \equiv \frac{\omega_0}{\Delta\omega} = \omega_0 \tau = \frac{2\pi\tau}{T_0}}$$

Another look at Faraday's Law:

A changing magnetic field induces an electric field



- Induced electric field drives the current in the conducting loop $(i = \mathcal{E}/R)$
- work done by the electric field on a charge q_0

$$dW_{\text{on } q_0 \text{ by } \vec{E}} = \vec{F} \cdot d\vec{s} = q_0 \vec{E} \cdot d\vec{s}$$

$\Rightarrow$ total work done on charge q_0 while it moves around the loop:

$$\underline{W_{\text{on } q_0 \text{ by } \vec{E}}} = \oint_{\text{around loop}} dW = \underline{\oint_{\text{around loop}} q_0 \vec{E} \cdot ds = \oint q_0 E_{\parallel} ds}$$

But: induced emf in loop $= \mathcal{E} \equiv \dfrac{W_{\text{on } q_0}}{q_0} = \dfrac{\text{work done on charge}}{\text{charge}}$

so: $\mathcal{E} = \dfrac{W_{\text{on } q_0}}{q_0} = \underset{\text{closed path}}{\oint} \vec{E} \cdot d\vec{s}$

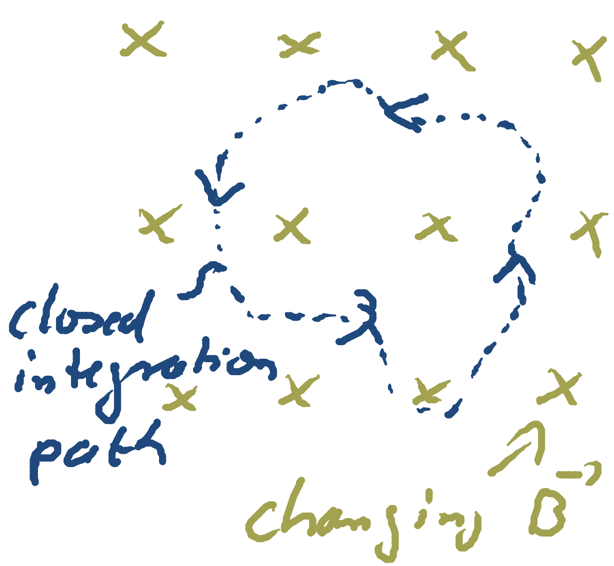
$\Rightarrow$ Can write Faraday's Law $\mathcal{E}_{ind} = -\dfrac{d\Phi_B}{dt}$ in the following way:

induced $\vec{E}$

$$\boxed{\underset{\text{closed path}}{\oint} \vec{E} \cdot d\vec{s} = -\frac{d\Phi_B}{dt}}$$

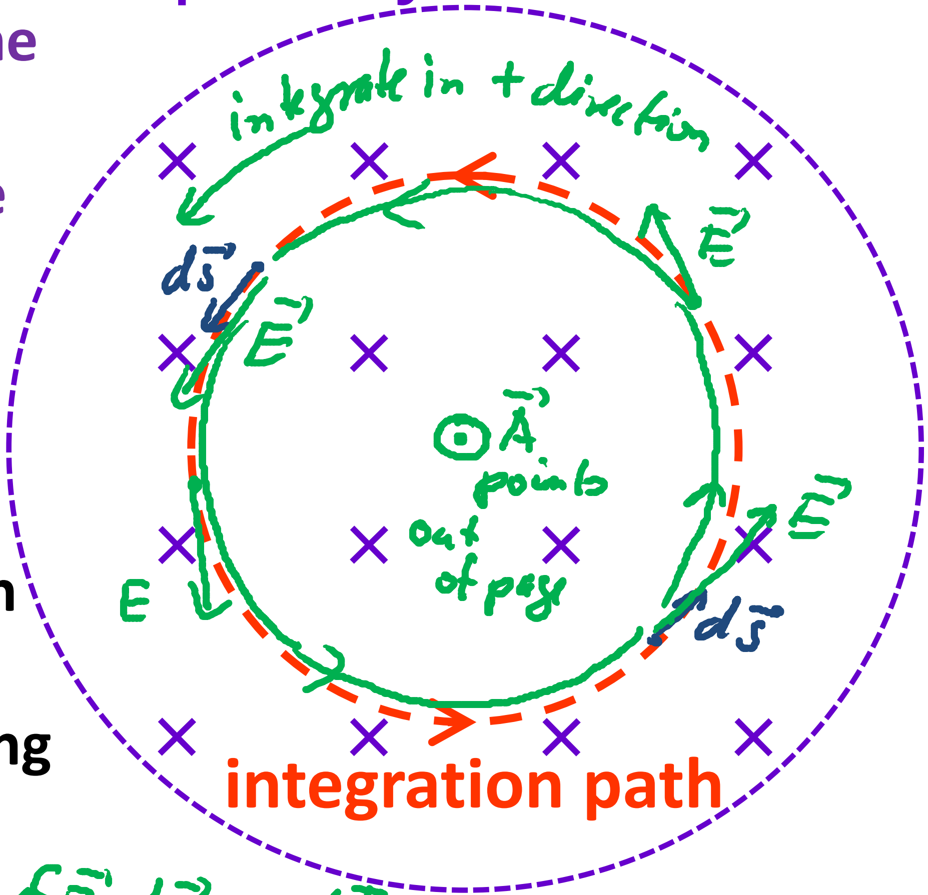
rate of change of magnetic flux through interior of closed path

This is true whether or not the conducting loop is present!



spatially uniform $\vec{B}$

The magnetic field is confined to the cylindrical region shown and is spatially uniform but its magnitude is increasing with time.



When Faraday's law,

$$\oint \vec{E}\cdot d\vec{s} = -\frac{d\Phi_B}{dt},$$

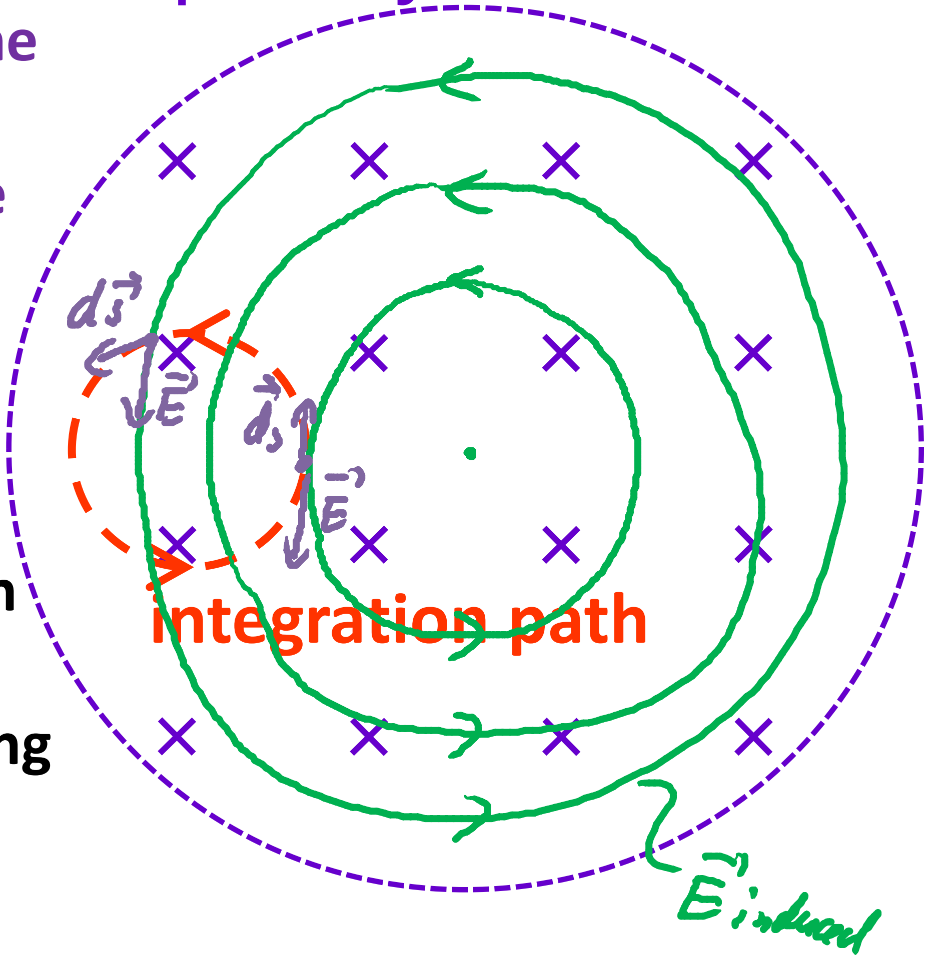
is applied to the circular integration path, which best describes E_s, the component of the electric field along the direction of $d\vec{s}$ **?**

$\Phi_B = \vec{B}\cdot\vec{A} < 0 \Rightarrow \frac{d\Phi_B}{dt} < 0$ here $\Rightarrow \oint \vec{E}\cdot d\vec{s} = -d\Phi_B/dt > 0$

A. $E_s > 0$ B. $E_s < 0$ C. $E_s = 0$

D. E_s depends on where $d\vec{s}$ is along the integration path.

spatially uniform $\vec{B}$



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~~A.~~ $E_s > 0$ B. $E_s < 0$ C. $E_s = 0$

D. E_s depends on where $d\vec{s}$ is along the integration path.

Ampère-Maxwell Law:

symmetry is powerful in physics!

$\boxed{\text{changing magnetic field produces an electric field}}$

$\Downarrow$ symmetry

$\boxed{\text{changing electric field produces a magnetic field}}$

Faraday's Law for magnetic induction

$$\oint_{\text{closed path}} \vec{E} \cdot d\vec{s} = -\frac{d\Phi_B}{dt}$$

$\Longleftrightarrow$

Maxwell's Law for electric induction

$$\oint_{\text{closed path}} \vec{B} \cdot d\vec{s} = \mu_0 \epsilon_0 \underbrace{\frac{d\Phi_E}{dt}}_{\text{rate of change of electric flux through interior of closed path}}$$

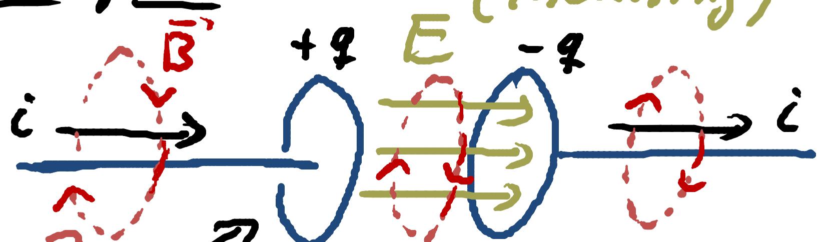
also have Ampère's Law: $\oint \vec{B}\cdot d\vec{s} = \mu_0 \, i_{enclosed}$ for B produced by a current

closed path

$\Rightarrow$ <u>combine</u>: <u>Ampère - Maxwell Law:</u>

$$\boxed{\underset{\text{closed path}}{\oint} \vec{B}\cdot d\vec{s} = \mu_0 \, i_{enclosed} + \mu_0 \varepsilon_0 \underbrace{\frac{d\Phi_E}{dt}}_{\text{"displacement current"}}}$$

<u>Example:</u>



$\oint \vec{B}\cdot d\vec{s} = \mu_0 i$ here

capacitor gets charged

$\oint \vec{B}\cdot d\vec{s} = \mu_0 \left(\varepsilon_0 \cdot \frac{d\Phi_E}{dt}\right)$ here

<u>Note</u>: Magnetic field lines must surround either currents or changing electric fields (or both).