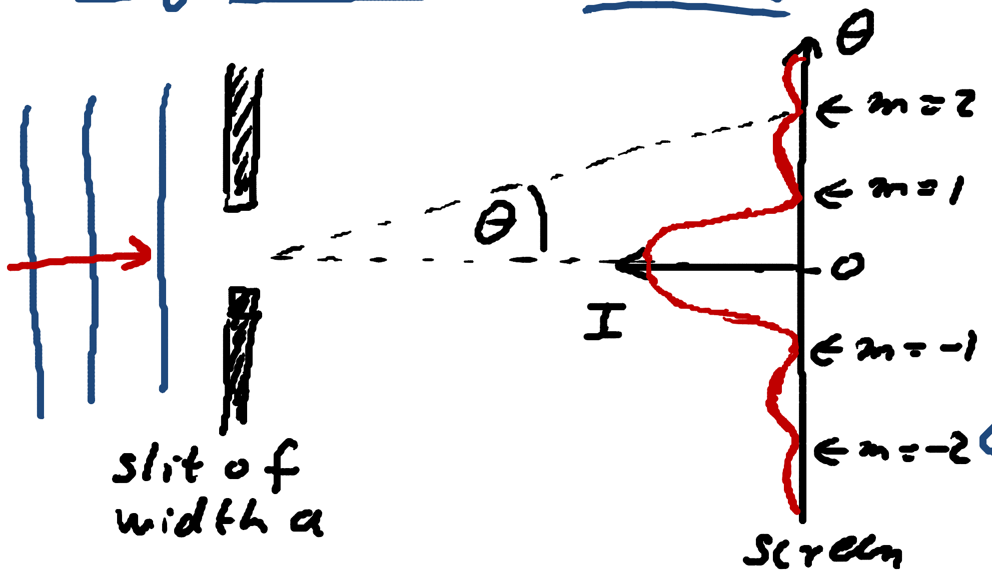
Recap

- **Diffraction**: Apparent "bending" of waves around small obstacles/edges/apertures.
- **Single Slit Diffraction**:



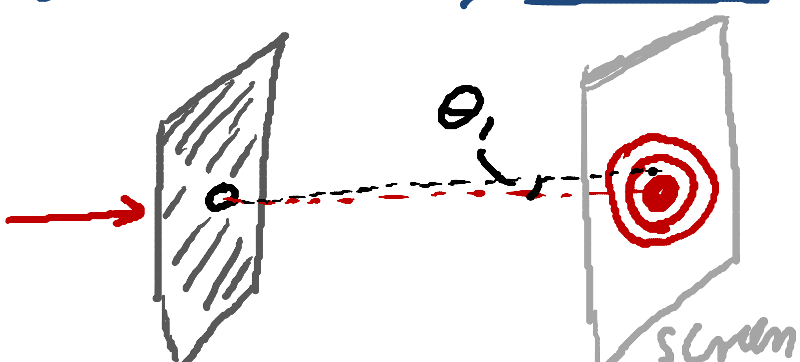
Intensity: $I(\theta) = I_{max}\left(\frac{\sin\alpha}{\alpha}\right)^2$

where $\alpha = \frac{\pi a}{\lambda}\sin\theta$

Minima: $\sin\theta_m = m\frac{\lambda}{a} \leq 1$

$m = \pm 1, \pm 2, \pm 3 \ldots$

- **Diffraction by Circular Aperture**:



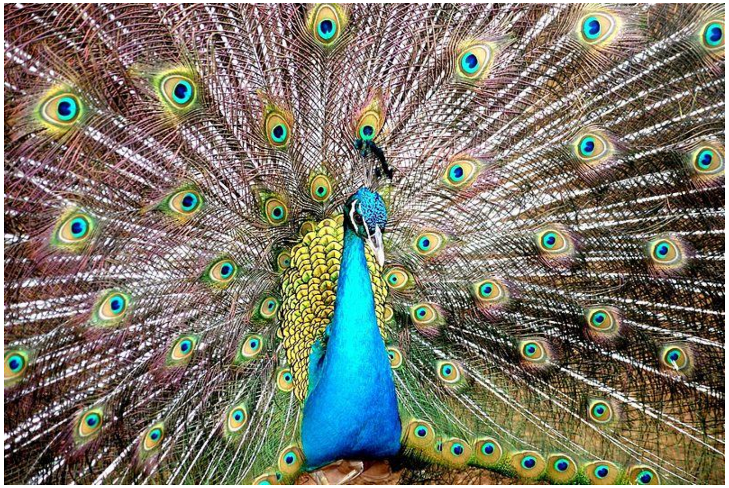
1st intensity minimum at:

$\sin\theta_1 = 1.22\ \lambda/a$

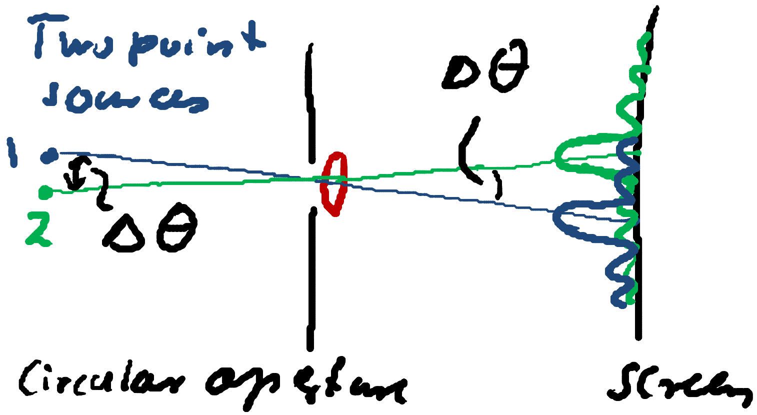
a = diameter of aperture

Today:

- **Diffraction**
  - Diffraction limited resolution
  - Double slit (again)
  - N slits
  - Diffraction gratings
  - Examples



Diffraction – limited Resolution:



$\Delta\theta$: angular separation of the two sources

⇒ For very small $\Delta\theta$:

Maximum of diffraction pattern from one source starts to fall into 1st minimum of diffraction pattern of other source

⇒ Rayleigh's criterion:

Smallest angular separation that can be resolved $\Delta\theta = \theta_1$

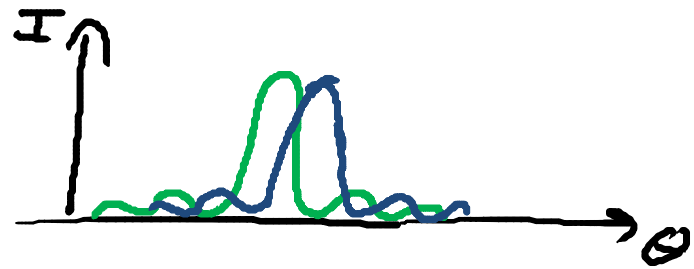
$$\Delta\theta_R = \sin^{-1}\left(\frac{1.22\lambda}{a}\right) \approx 1.22\frac{\lambda}{a}$$

a: diameter of aperture

$\approx$: small angles

⇒ for human eye:

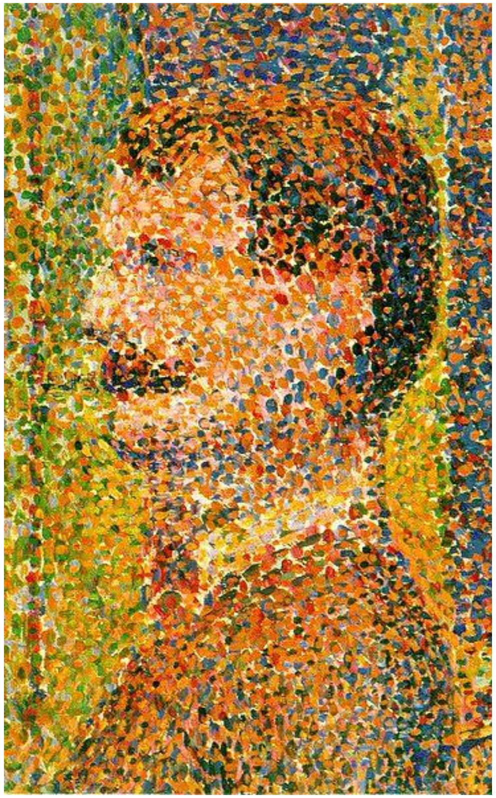
$a = 2.5\,mm \Rightarrow \Delta\theta_R \approx 0.01°$

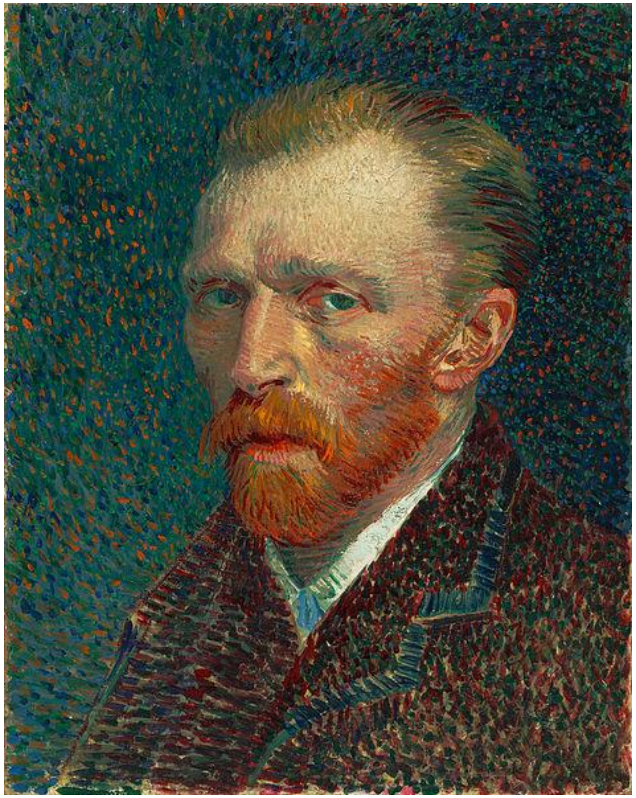


⇒ for small $\Delta\theta$:

Images of the two sources can no longer be resolved on the screen!

Pointillism

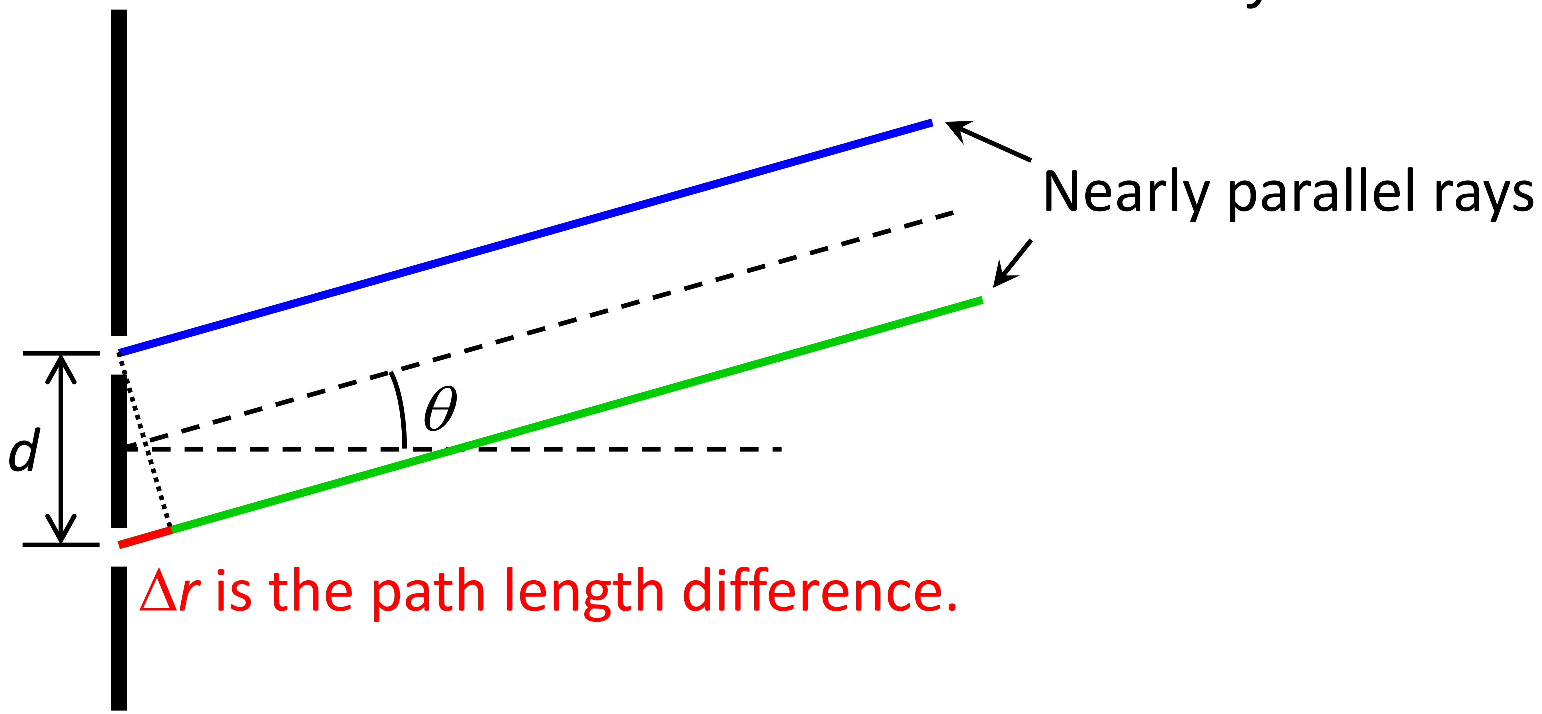




- **Technique of painting in which small, distinct dots of pure color are applied in patterns to form an image.**
- **At normal viewing distance, the dots are irresolvable, and thus blend.**

Revisit: **2-slit Interference**

Look at the case where the screen is far away: $D >> d$ & $D >> \lambda$.



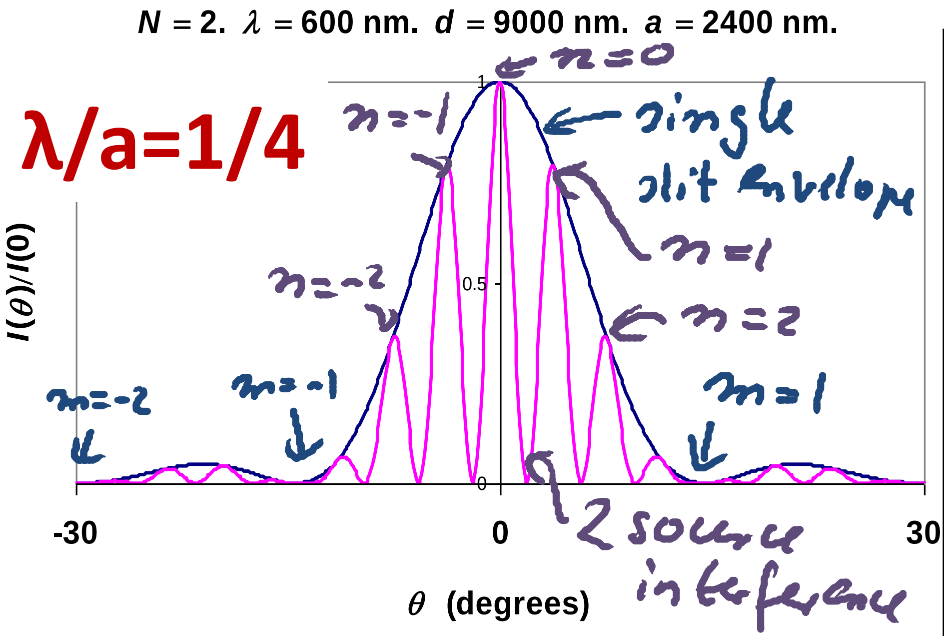
From before: **Interference maxima** where the path length difference is:

$$\Delta r = d\sin(\theta_m) = m\lambda, \;\; m = 0, \pm 1, \pm 2, \ldots$$

But, the slits have some finite width *a*!

$\Rightarrow$ The intensities of these interference maxima are modulated by an 'envelope' single-slit diffraction function.

λ/a=1/4



Actual patterns are the pink curves.

2-source interference:

Maxima:

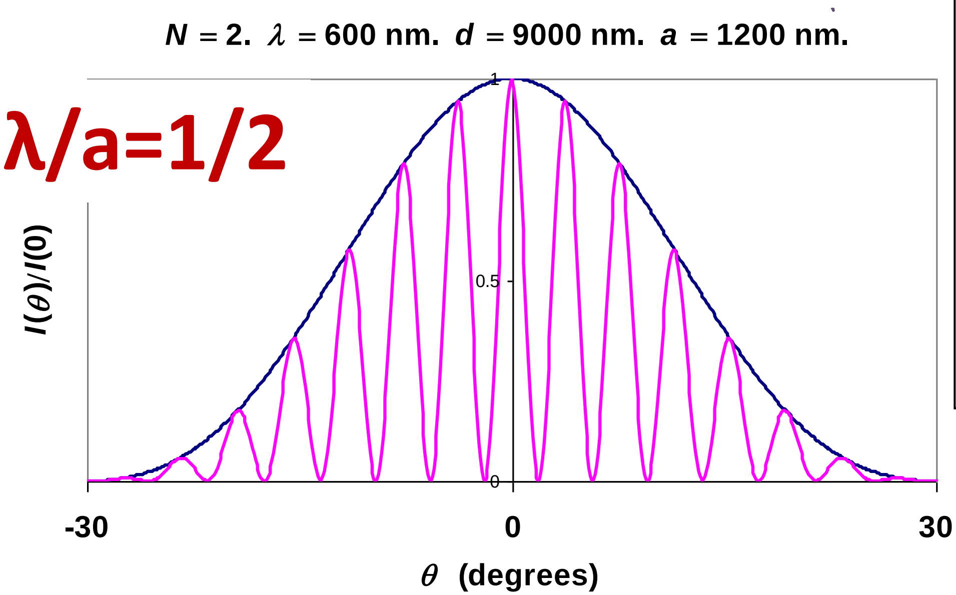
$$\sin(\theta_n) = n\lambda/d, \quad n = 0, \pm 1, \pm 2, \ldots$$

Minima:

$$\sin(\theta_n) = (n + ½)\lambda/d, \quad n = 0, \pm 1, \pm 2, \ldots$$

Here ***d* is the spacing between slit centers**.

λ/a=1/2



Single-slit diffraction envelope:

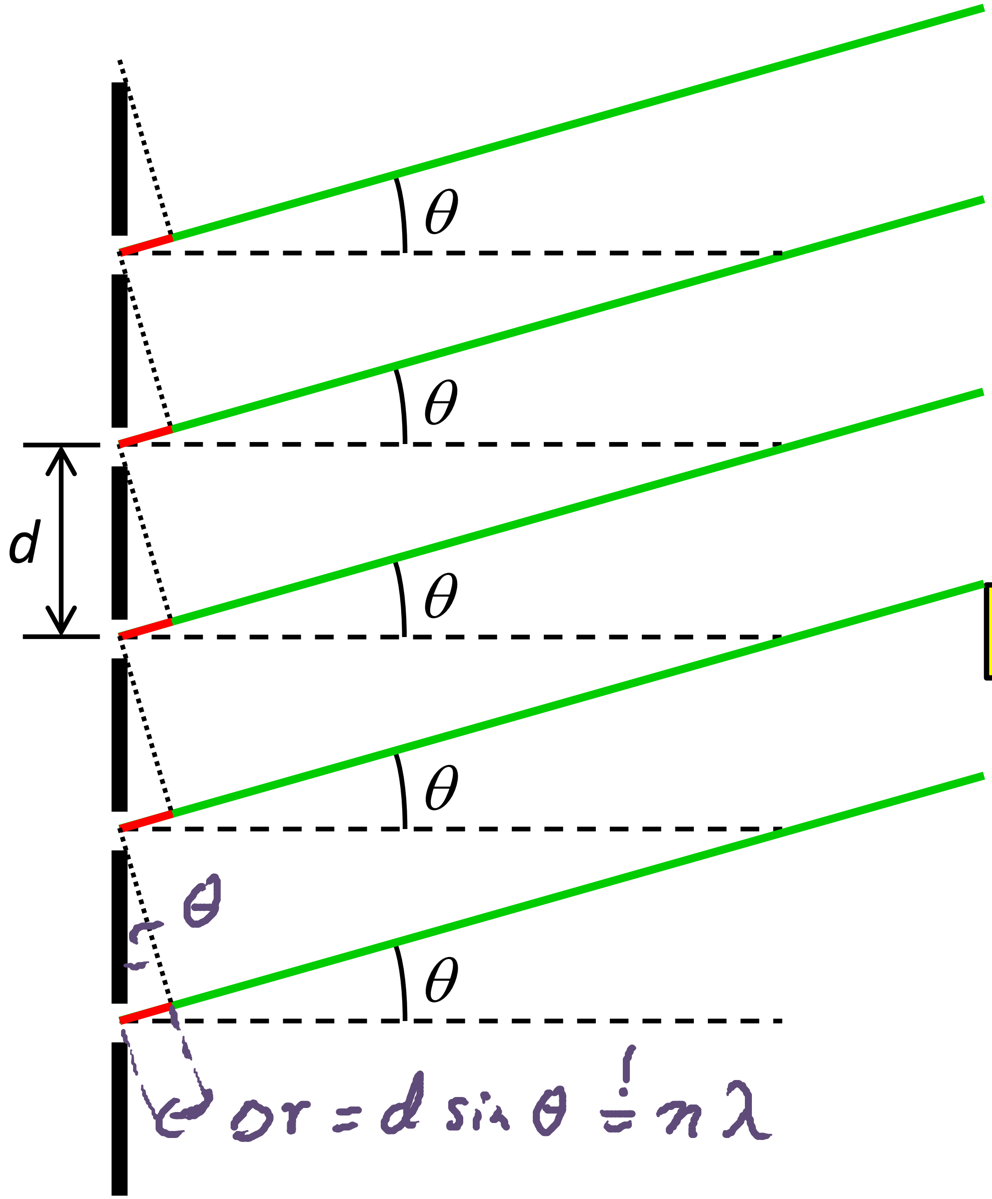
Minima:

$$\sin(\theta_m) = m\lambda/a, \quad m \neq 0, m = \pm 1, \pm 2, \ldots$$

Here ***a* is the slit width**.

$d > a$

N-slit Interference



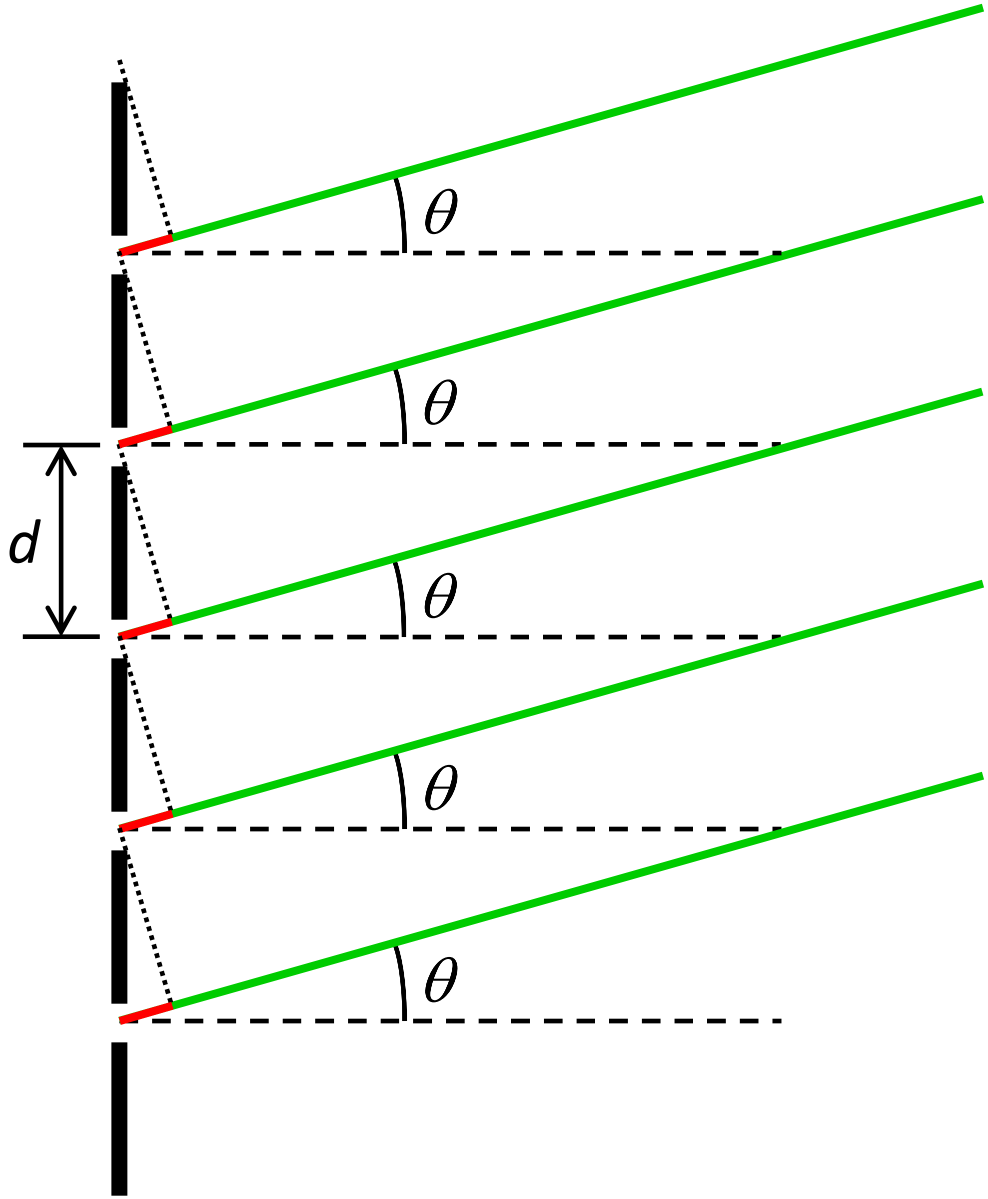
Principal maxima:

occur where the path difference between rays from adjacent slits is an integer # of wavelengths.

$$\sin(\theta_n) = n\lambda/d,\ n = 0, \pm 1, \pm 2, \ldots$$

Here ***d*** **is the spacing between slit centers**.

N-slit Interference



Interference minima:

Interference minima occur where

total width of N-slit setup

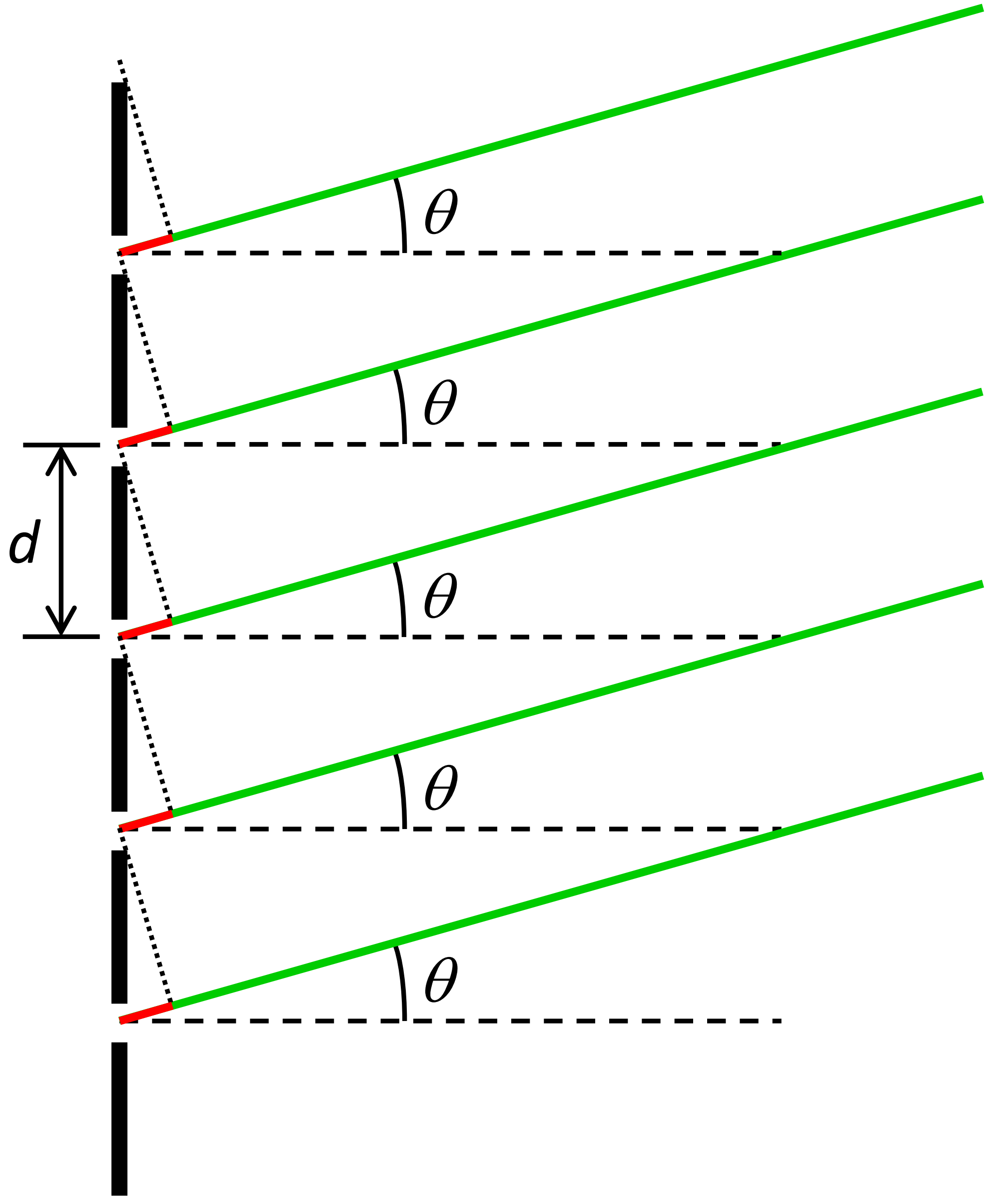
$$\sin(\theta_s) = s\lambda/(Nd),$$

$s \neq 0$, $s = \pm1, \pm2, \ldots$, except when s/N is an integer (position of principal maxima).

Here ***d* is the spacing between slit centers**, **and N is the number of slits.**

-> $(N-1)$ minima between any two consecutive principal maxima.

N-slit Interference

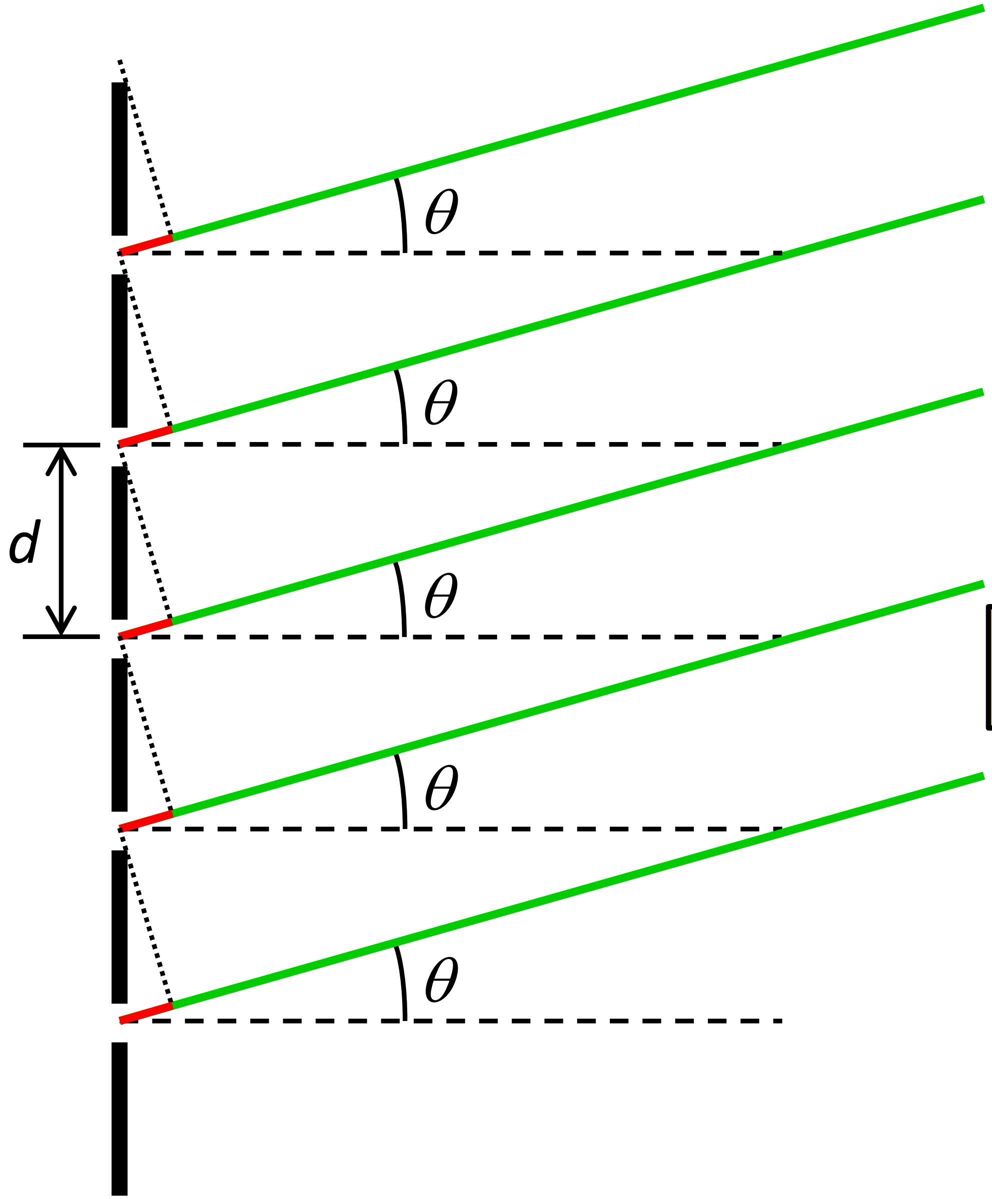


Subsidiary maxima:

There are also ($N-2$) <u>subsidiary maxima</u> between any two consecutive principal maxima.

These are much dimmer than the principal maxima.

N-slit Interference



Single-slit diffraction envelope:

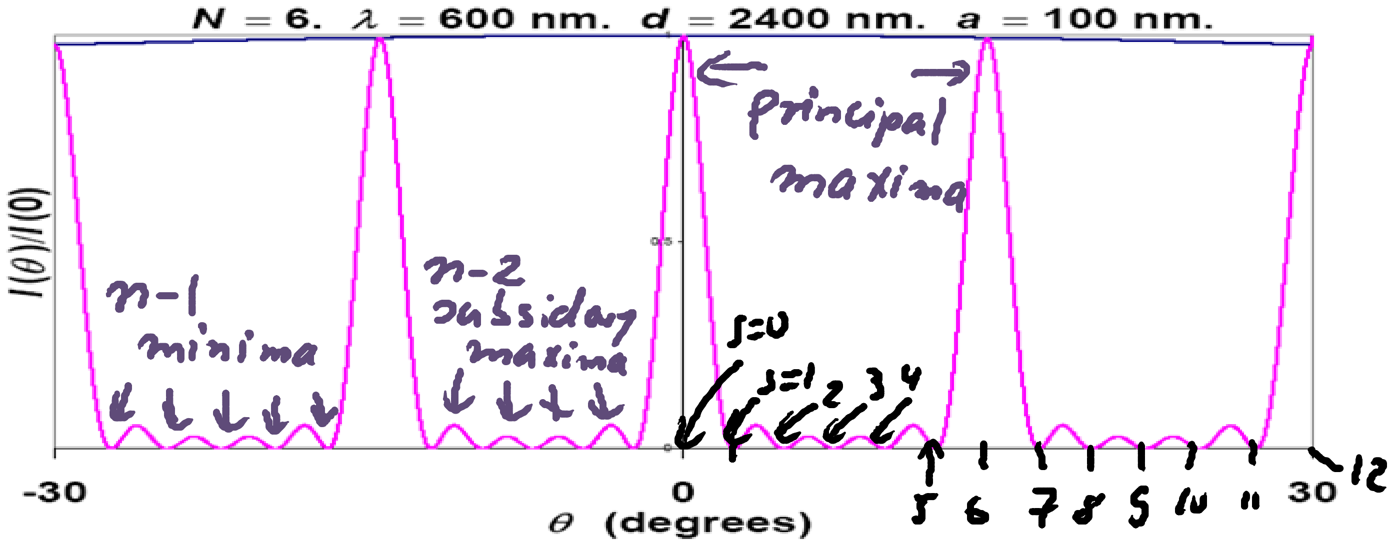
Finally, if each slit has width *a*, then these interference patterns are modulated by a single-slit envelope function with minima at:

$$\sin(\theta_m) = m\lambda/a, \quad m = \pm 1, \pm 2, \ldots$$

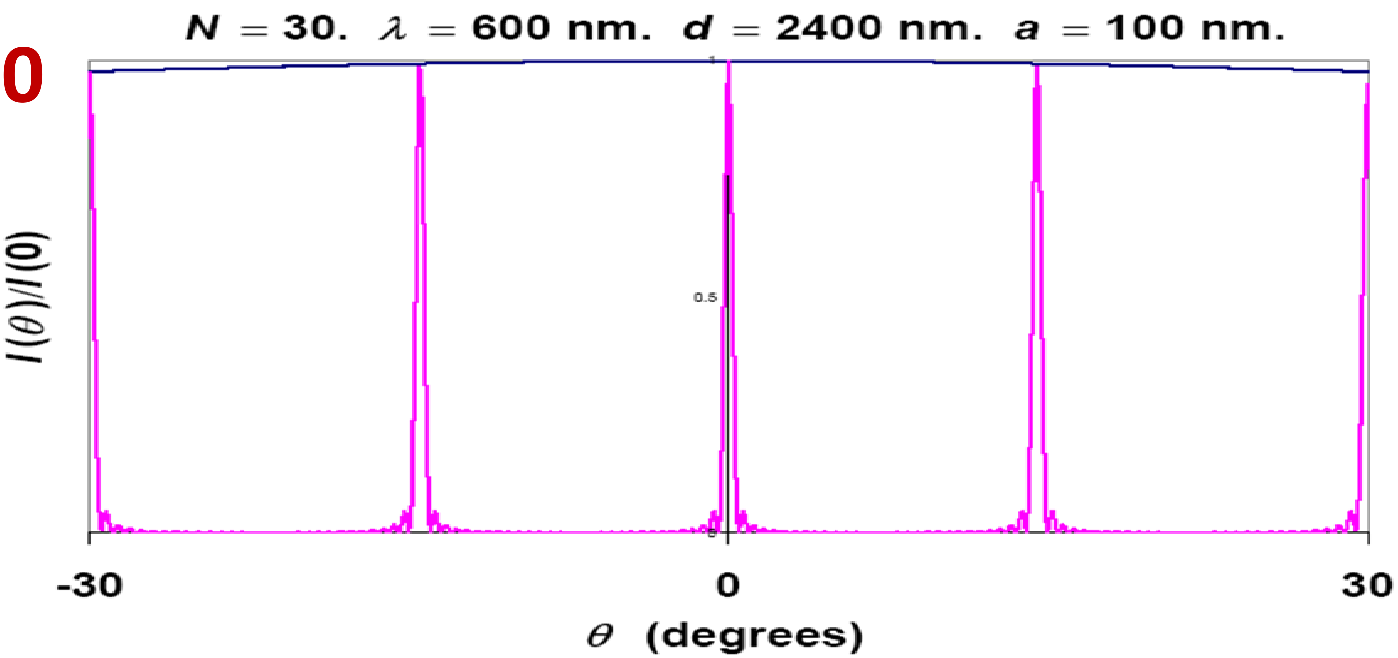
$$m \neq 0$$

N-slit: Effect of increasing N

N=6

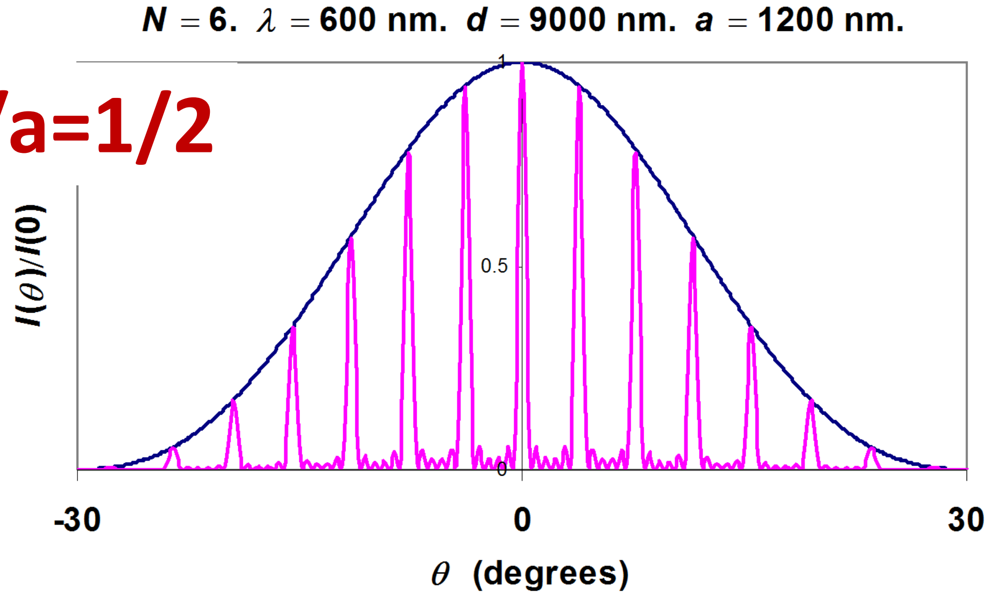


N=30

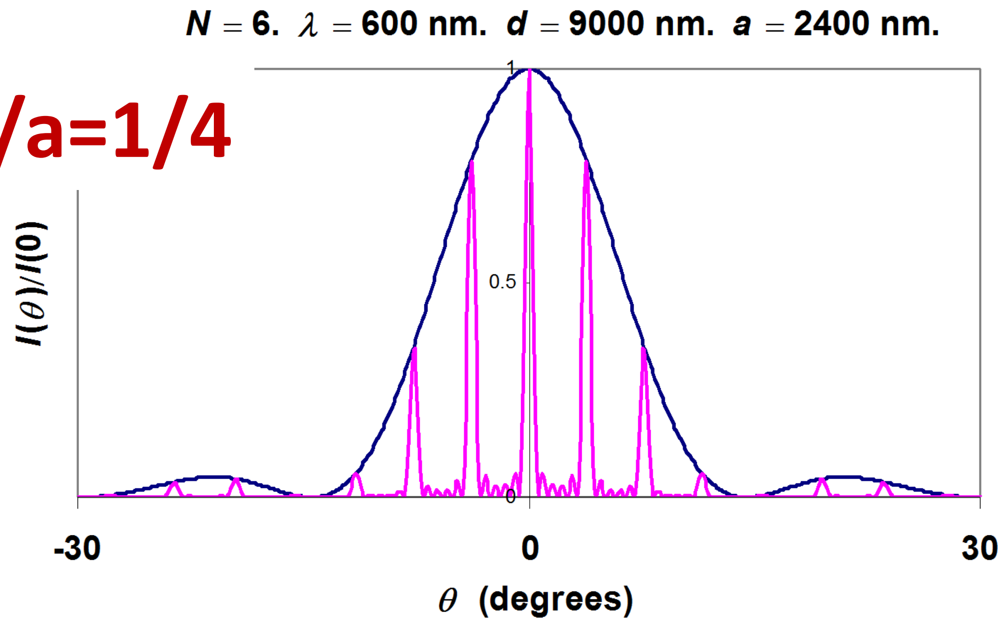


N-slit: Effect of increasing slit width *a*

λ/a=1/2



λ/a=1/4



Actual patterns are the pink curves.

Single slit envelope functions are the blue curves.

In the following equations, *d* represents center-to-center slit spacing, *a* represents slit width, *λ* represents the wavelength of normally incident plane waves, and *N* represents the # of slits.

① $d\sin\theta_m = m\lambda \qquad m = 0, \pm1, \pm2, \ldots$

② $d\sin\theta_m = (m + \tfrac{1}{2})\lambda \qquad m = 0, \pm1, \pm2, \ldots$

③ $a\sin\theta_n = n\lambda, n = \pm1, \pm2, \ldots$

④ $Nd\sin\theta_s = s\lambda, s = \pm1, \pm2, \ldots$ except when *s/N* is an integer

Which of the above gives angles of intensity principal maxima?

A. ①.	B. ②.
C. ③.	D. ④.
E. None of the above.	

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③ $a\sin\theta_n = n\lambda, n = \pm 1, \pm 2, \ldots$

④ $Nd\sin\theta_s = s\lambda, s = \pm 1, \pm 2, \ldots$ except when s/N is an integer

Which of the above gives angles of intensity subsidiary maxima?

A. ①.	B. ②.
C. ③.	D. ④.
E. None of the above.	

In the following equations, *d* represents center-to-center slit spacing, *a* represents slit width, *λ* represents the wavelength of normally incident plane waves, and *N* represents the # of slits.

① $d\sin\theta_m = m\lambda \qquad m = 0, \pm 1, \pm 2, \ldots$

② $d\sin\theta_m = (m + \frac{1}{2})\lambda \qquad m = 0, \pm 1, \pm 2, \ldots$ } for 2-slit

③ $a\sin\theta_n = n\lambda, n = \pm 1, \pm 2, \ldots$ } for single slit diffraction

④ $Nd\sin\theta_s = s\lambda, s = \pm 1, \pm 2, \ldots$ except when *s/N* is an integer } for N-slits

Which of the above give(s) angles of intensity minima?

A. ①.	B. ②.
C. ③.	D. ④.
E. ②, ③, and ④.	

Diffraction gratings:

Have a very large number *N* of equally spaced slits.

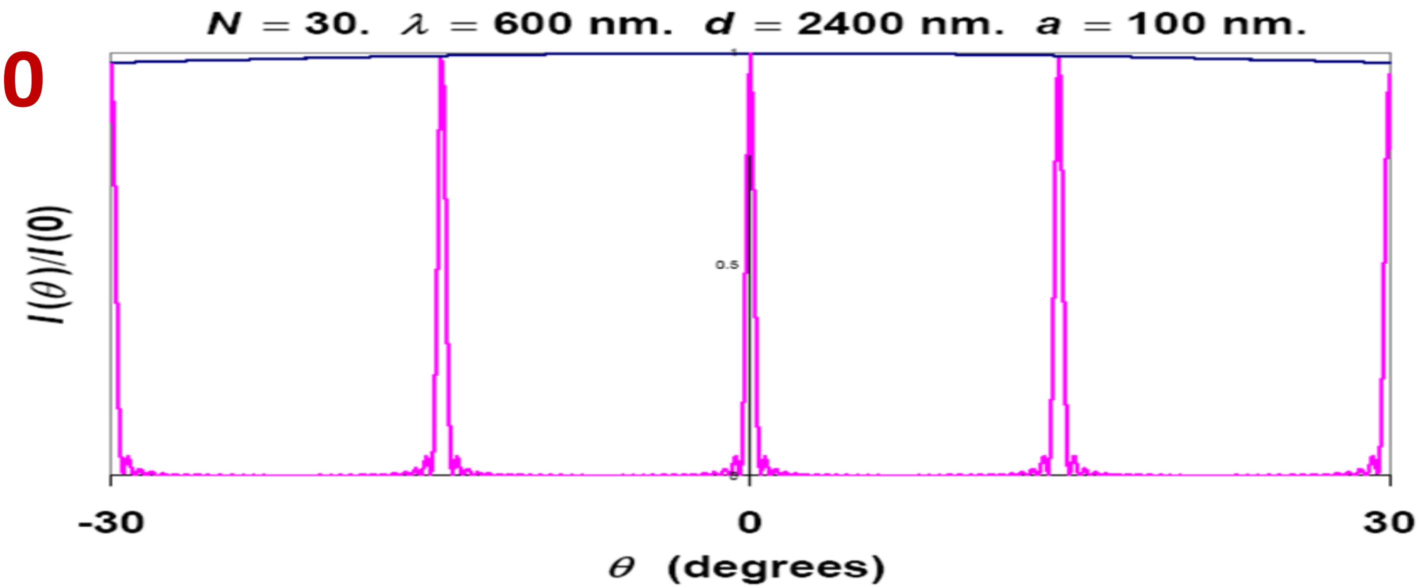
Interference maxima are very narrow and occur where

$$\sin(\theta_n) = n\lambda/d, \qquad n = 0, \pm1, \pm2, \ldots,$$

where *d* is the distance between slit centers.

For a given value of *n*, different wavelengths will diffract at different angles and, because the maxima are very narrow, **gratings can be used to analyze the wavelength composition of light**.

N=30



CD as Diffraction Grating: Interference

- The tracks of a compact disc act as a diffraction grating
- Nominal track separation on a CD is 1.6 micrometers, corresponding to about 625 tracks per millimeter.
  - This is in the range of ordinary laboratory diffraction gratings.
  - For red light of wavelength 600 nm, this would give a first order diffraction maximum at about 22°

In the following equations, *d* represents center-to-center slit spacing, *a* represents slit width, *λ* represents the wavelength of normally incident plane waves, and *N* represents the # of slits.

① $d\sin\theta_m = m\lambda \qquad m = 0, \pm1, \pm2, \ldots$

② $d\sin\theta_m = (m + \tfrac{1}{2})\lambda \qquad m = 0, \pm1, \pm2, \ldots$

③ $a\sin\theta_n = n\lambda, n = \pm1, \pm2, \ldots$

④ $Nd\sin\theta_s = s\lambda, s = \pm1, \pm2, \ldots$ except when *s/N* is an integer

Which of the above could be used to derive an expression for the angular width of a principal maximum of a diffraction grating?

A. ①.	B. ②.
C. ③.	D. ④.
E. None of the above.	

Diffraction Grating: Width of Lines:

- For N-slits: Interference minima at

$$\sin\theta_s = \frac{s\lambda}{Nd}$$

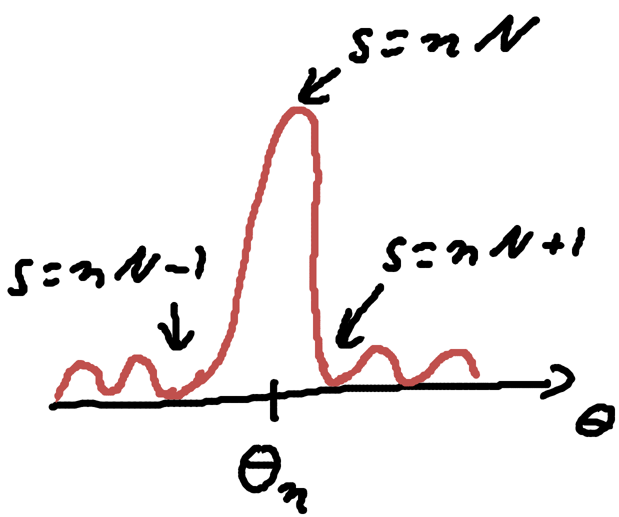
$s = \pm 1, \pm 2 \ldots$ except $s = 0$
and except $s/N = n = \text{integer}$

- At $s_n = nN$: get n^{th} principle maximum:

$$\sin\theta_{nN} = \frac{nN\lambda}{Nd} = n\frac{\lambda}{d}$$

$\Rightarrow$ maxima! $n = 0, \pm 1, \pm 2 \ldots$

- Minima that border the n^{th} principle maximum are at:



$$\sin\theta_{nN\pm 1} = \frac{(nN\pm 1)\lambda}{Nd}$$

⇒ find:

$$\underbrace{\sin\theta_{nN+1} - \sin\theta_{nN-1}}_{\Delta \sin\theta} = [nN+1-(nN-1)]\frac{\lambda}{Nd} = \frac{2\lambda}{Nd}$$

- finally, use:

$$\cos\theta = \frac{d(\sin\theta)}{d\theta} \simeq \frac{\Delta \sin\theta}{\Delta\theta} \quad \text{for small } \Delta\theta$$

- this gives:

$$\sin\theta_{nN+1} - \sin\theta_{nN-1} = \frac{2\lambda}{Nd} \simeq \overbrace{(\theta_{nN+1} - \theta_{nN-1})}^{\Delta\theta}\cos\theta_n$$

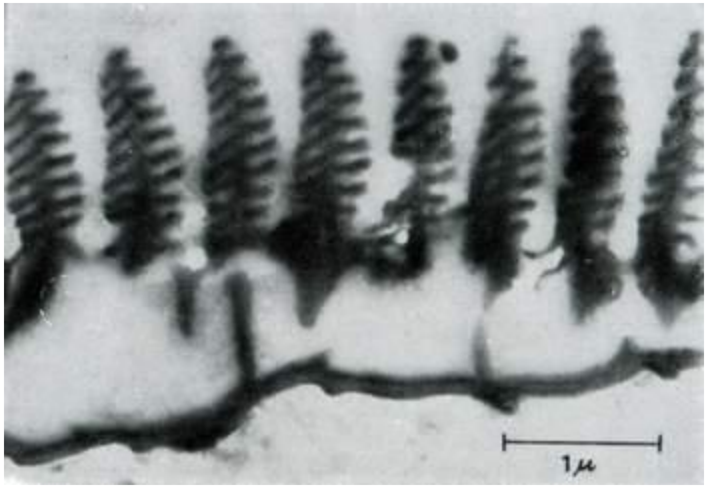
⇒ full width of n^{th} maximum:

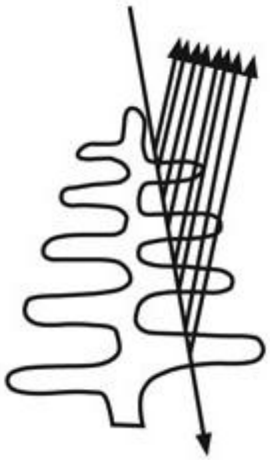
$$\text{width} = \Delta\theta = \theta_{nN+1} - \theta_{nN-1} \simeq \frac{2\lambda}{Nd\cos\theta_n}$$

← angle of n^{th} principal maximum

large N ⇒ narrow lines

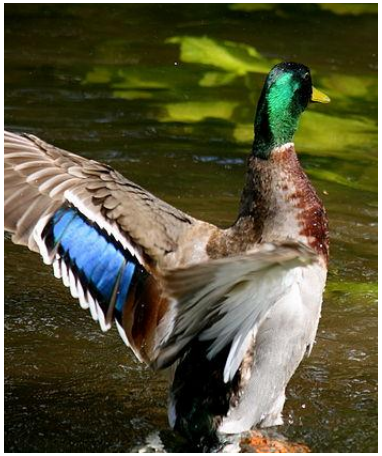
Giant Blue Morpho

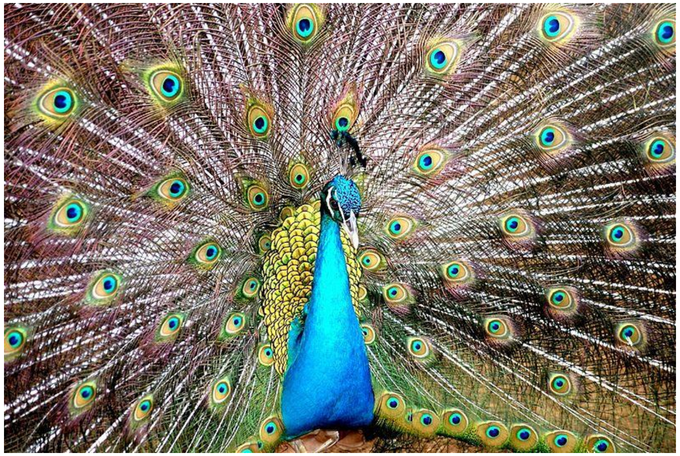


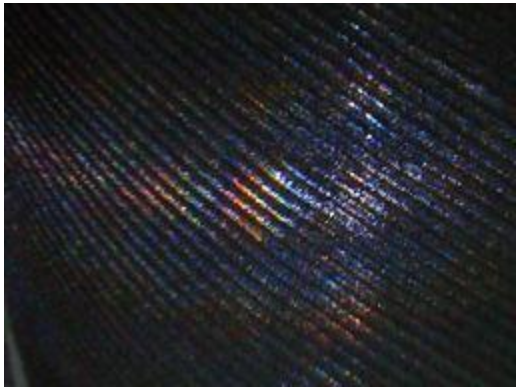


- **Some butterflies have the most striking iridescent blue wings, such as the blue morpho of South America**
- **Blueness in butterflies is caused by optical interference.**
- **The scales have multilayering that reflects light waves so that they travel different distance**

Iridescence







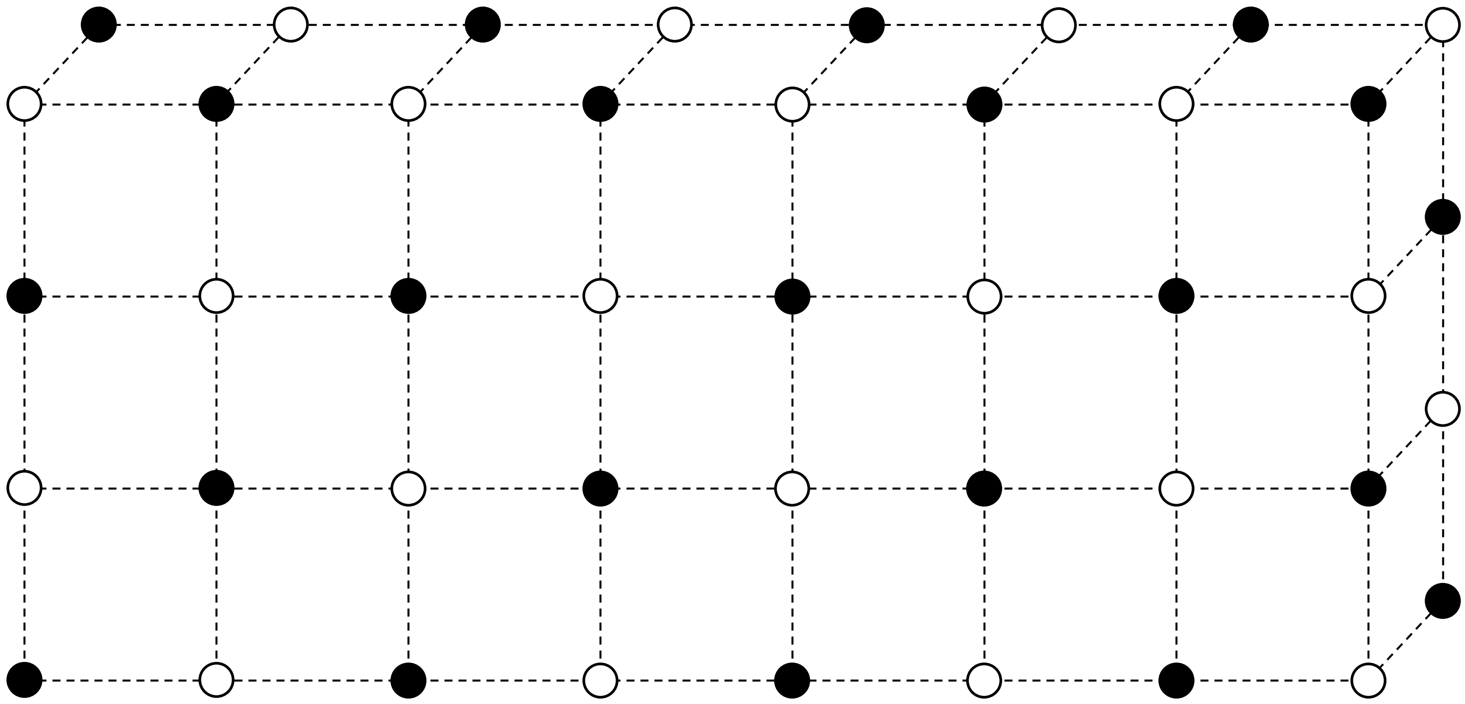
- Iridescence is an optical phenomenon of surfaces in which hue changes in correspondence with the angle from which a surface is viewed
- Caused by multiple reflections from two or more surfaces in which phase shift and interference of the reflections modulates the incidental light.

X-ray (Bragg) Diffraction:

- **X rays** are EM waves whose wavelengths are $\lambda \sim 1$ Å $= 10^{-10}$ m.

  -> $\lambda \sim$ atomic diameters.

- In a crystalline solid the regular **array of atoms forms a 3-dimensional** "**diffraction grating**" for x rays.



X-ray (Bragg) Diffraction (cont.):

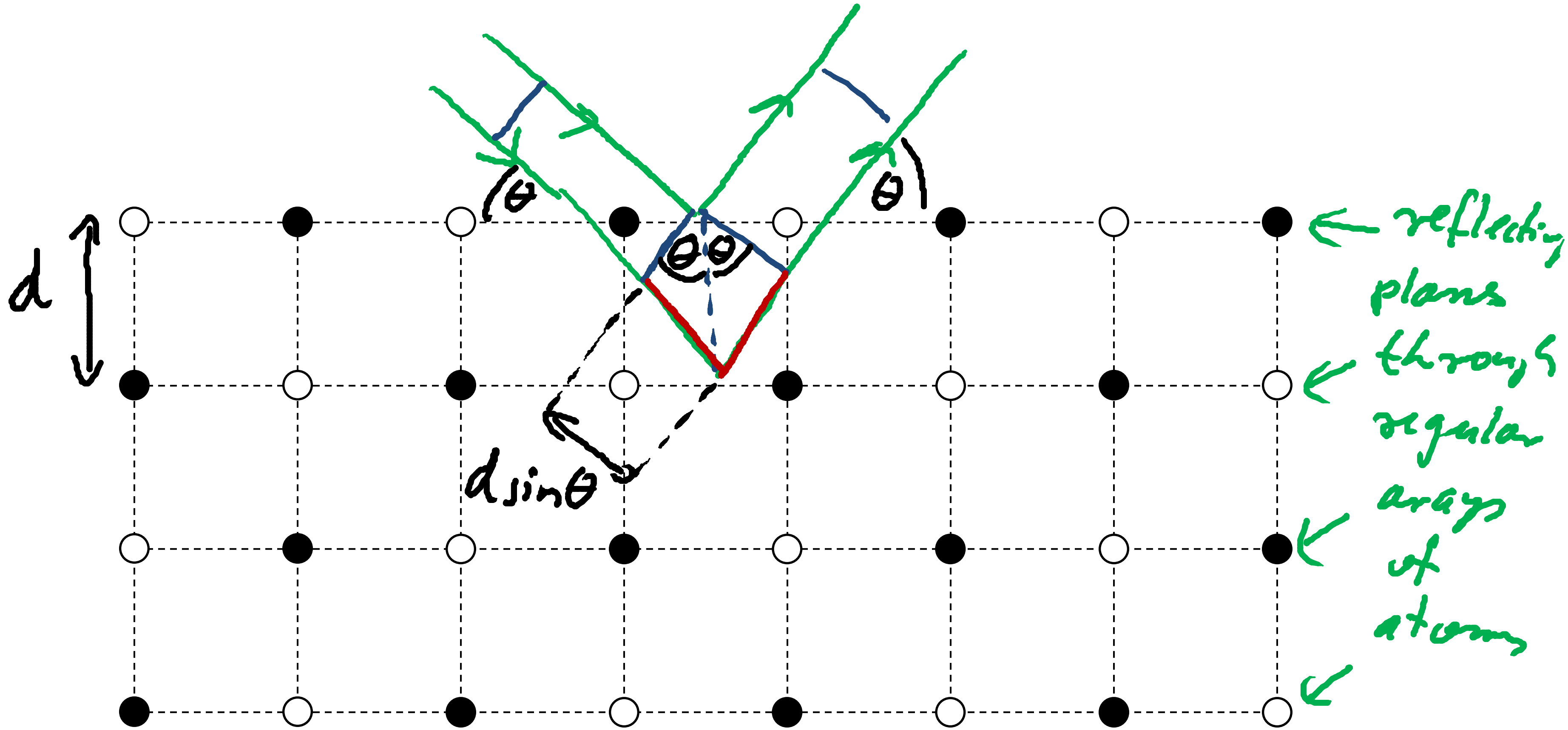
- If an **x-ray beam** is sent into a crystal it **is scattered** (redirected) by the crystal structure.

- In some directions scattered waves undergo **destructive interference** resulting in intensity minima.

- In other directions scattered waves undergo **constructive interference** resulting in **intensity maxima**.

- This scattering process is complicated but **intensity maxima turn out to occur in directions as if the incoming x rays were reflected by a family of parallel reflecting planes** that extend through the atoms within the crystal & that contain regular arrays of the atoms.

X-ray (Bragg) Diffraction:

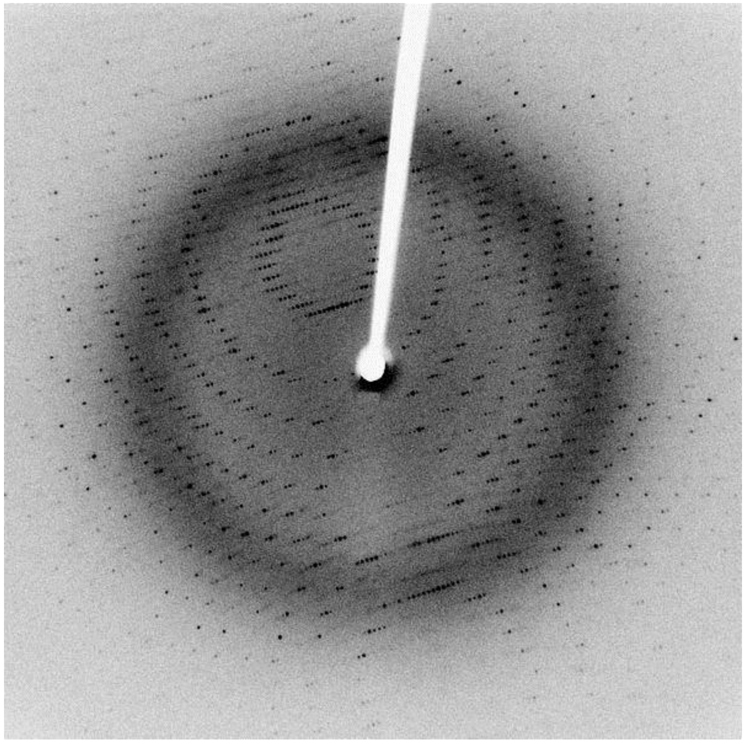


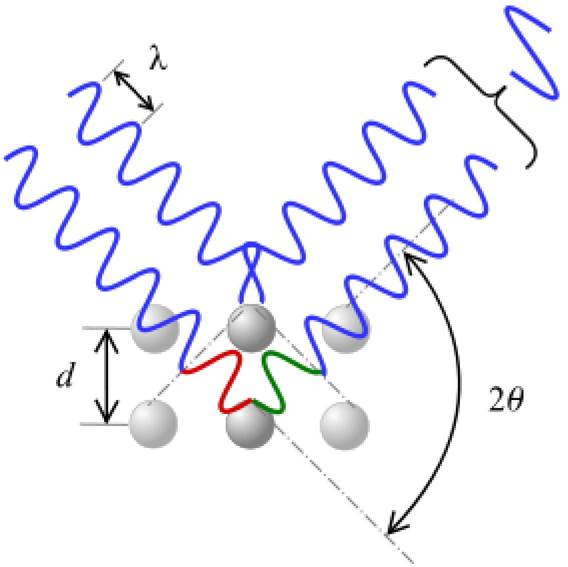
⇒ for constructive interference: $2d\sin\theta = n\lambda$

⇒ maxima at: $$\sin\theta_{maxima} = \frac{n\lambda}{2d}$$ $n = 1, 2, 3 \ldots$

Bragg's Law

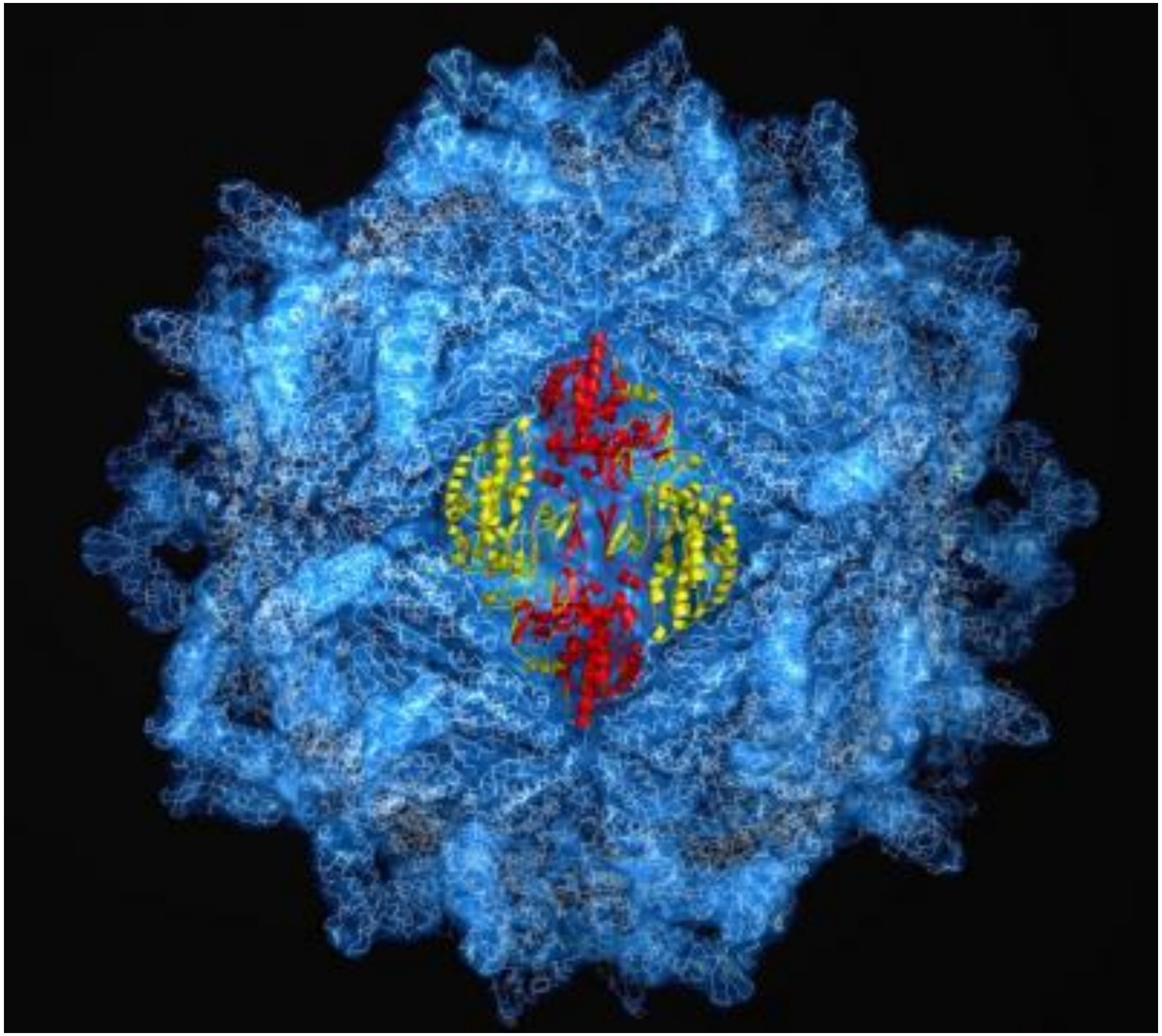
Bragg Diffraction





- **Diffraction from a three dimensional periodic structure such as atoms in a crystal** is called Bragg diffraction.
- Each dot in this diffraction pattern forms from the constructive interference of X-rays passing through a crystal.
- The data can be used to determine the crystal's atomic structure.

X-Ray Diffraction at Cornell: CESR/CHESS



High-energy X-ray diffraction was used to pinpoint some **5 million atoms** in the protective protein coat used by hundreds of viruses.

Credit: J. Pan & Y.J. Tao