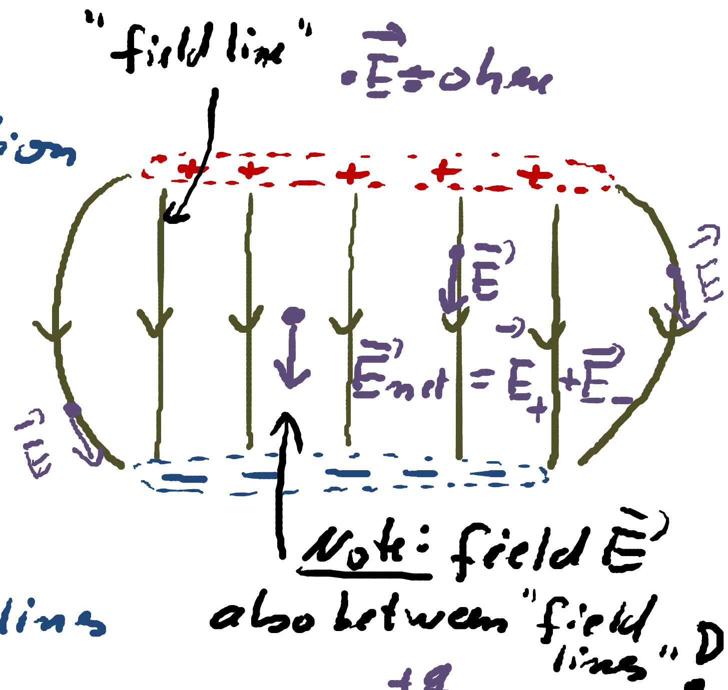
Recap

Lecture 5

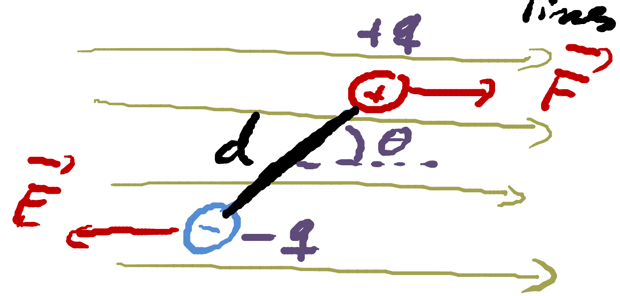
- **Electric Field Lines:**
  - way of visually representing information about the electric field
  - field lines are not real; just a tool (the electric field is real!)
  - start on + charges and end on − charges
  - $|E| \propto$ density of field lines shown
  - direction of $\vec{E}$ is tangent to field lines



- **Electric Dipole:**
  - electric dipole moment

$$p = q\,d$$

  - torque in uniform electric field on dipole: $\tau = pE \sin\theta$

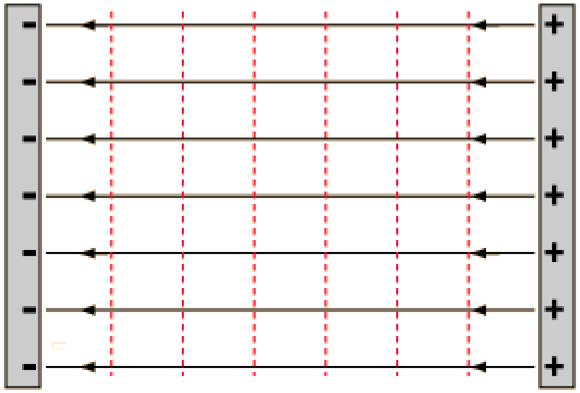


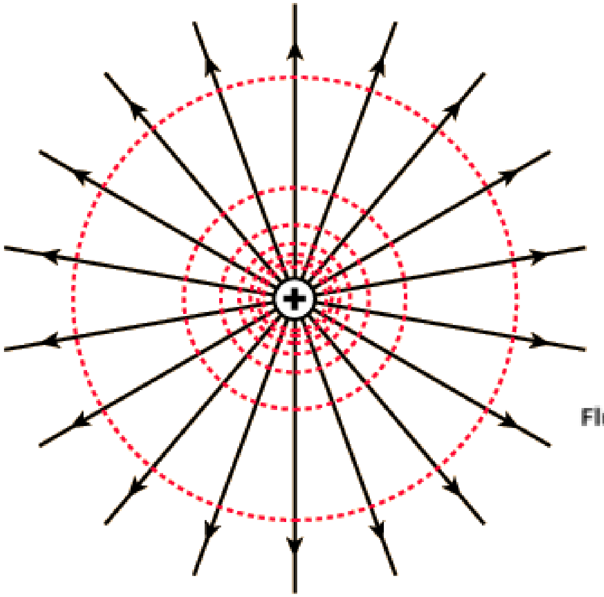
- **Motion of charge in ext. field:**

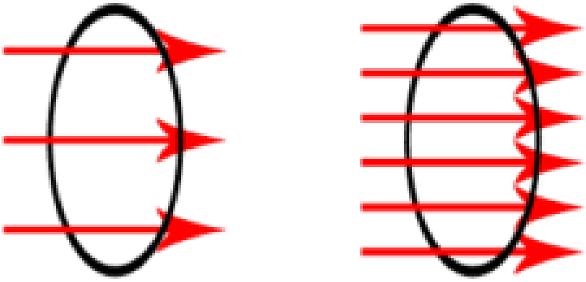
$$\vec{a} = \frac{\vec{F}}{m} = q/m \cdot \vec{E}_{ext}$$

Today:

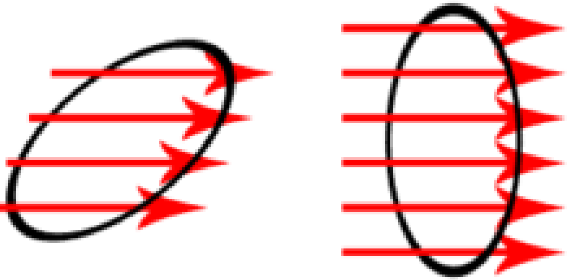
- Gauss' Law



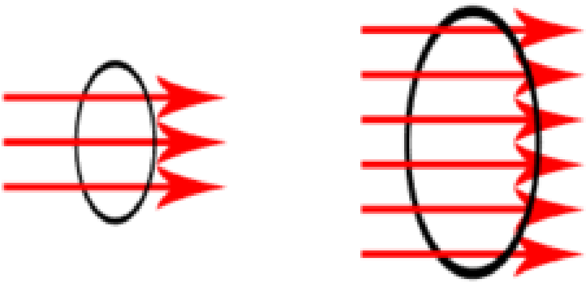




Flux is proportional to the density of flow.



Flux varies by how the boundary faces the direction of flow.



Flux is proportional to the area within the boundary.

Conductors in Electrostatic Equilibrium

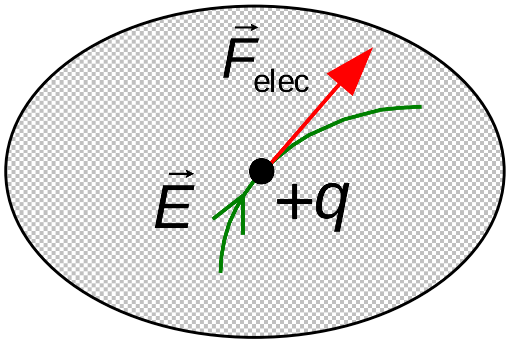
An **electric conductor** has some **mobile charges** that are free to move in the conductor and along its surfaces.

Electrostatic equilibrium means that charges are in **static equilibrium**. This means that there must be no net electric force on any mobile charge.

1. $\vec{E} = 0$ inside a conductor in electrostatic equilibrium.

If was in the conductor, then a mobile charge would be acted on by a net electric force , and would therefore have a nonzero acceleration and would therefore not be in equilibrium

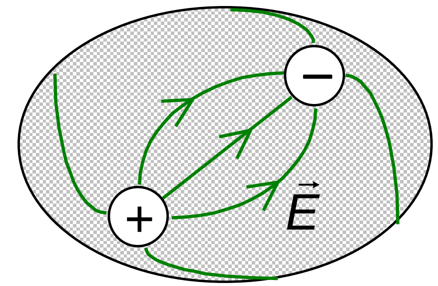
No!



2. **Any isolated charge on a conductor in electrostatic equilibrium can only be on its surfaces.**

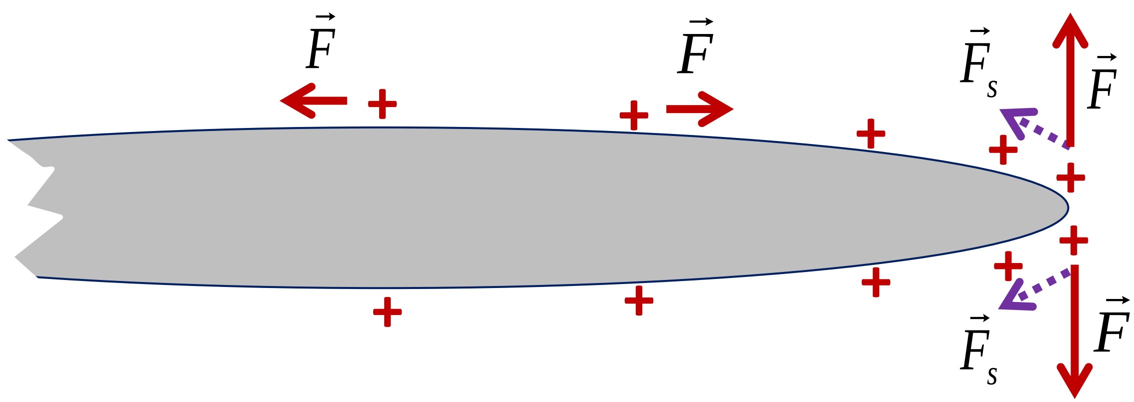
If isolated (separated) charges were present in the conductor, then electric field lines would start or end on each charge, and $\vec{E}$ would $\neq 0$ in there.

No!



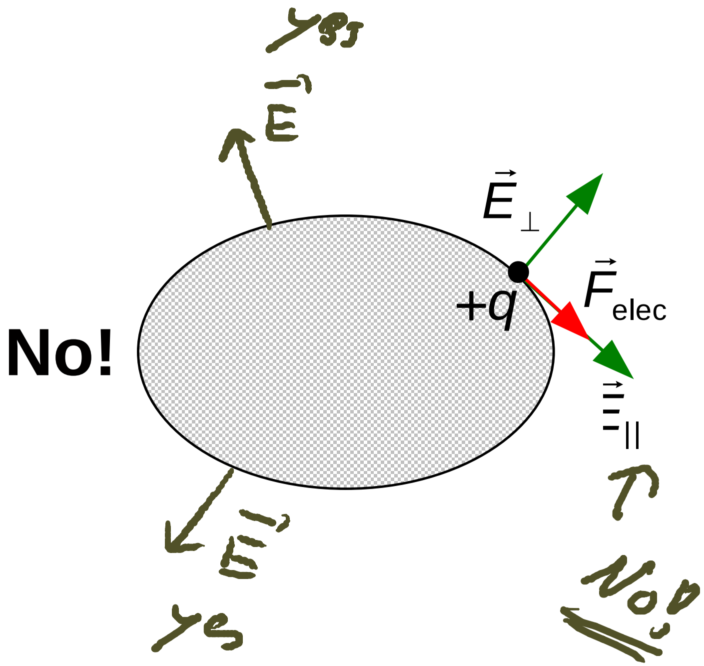
3. **The excess charge on a conductor in electrostatic equilibrium is more concentrated in regions of greater curvature (no external electric field).**

Force component parallel to surface pushes surface charges apart



4. $\vec{E}$ just outside the surface of a conductor in electrostatic equilibrium must be perpendicular (⊥) to the surface.

If $\vec{E}$ had a component parallel (||) to the surface ($\vec{E}_{||} \neq 0$), then a mobile charge on the surface would be acted on by a force $\vec{F}_{\text{elec}} = q\vec{E}_{||} \neq 0$, and would therefore have a nonzero acceleration and would not be in equilibrium.



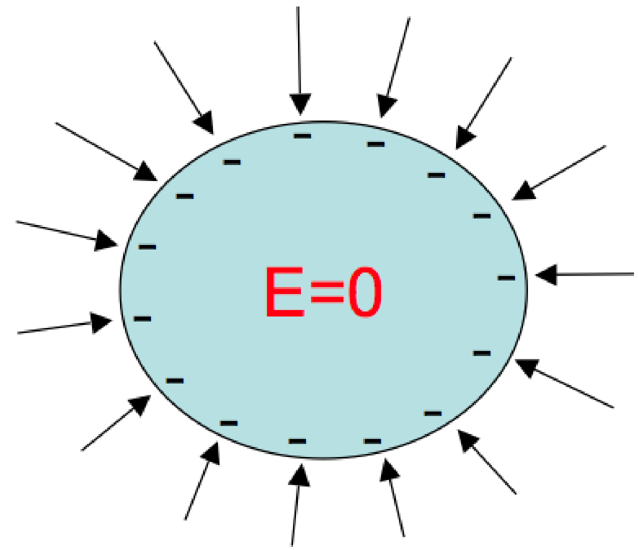
Summary: Conductors in Electrostatic Equilibrium

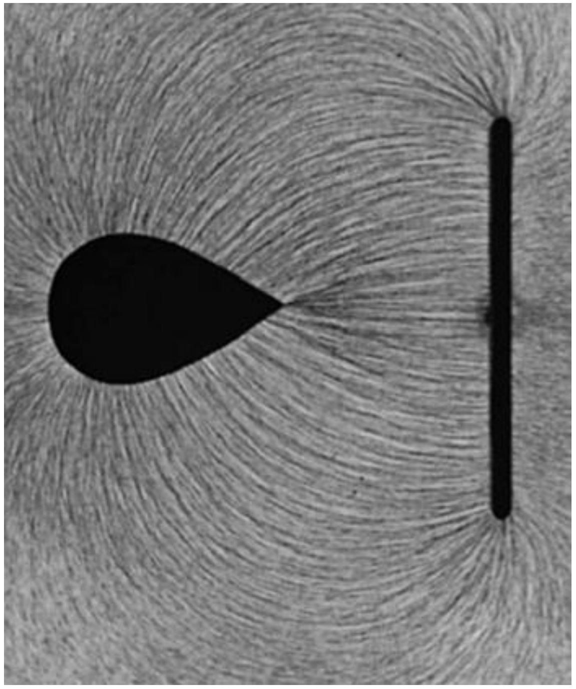
1. $\vec{E} = 0$ inside

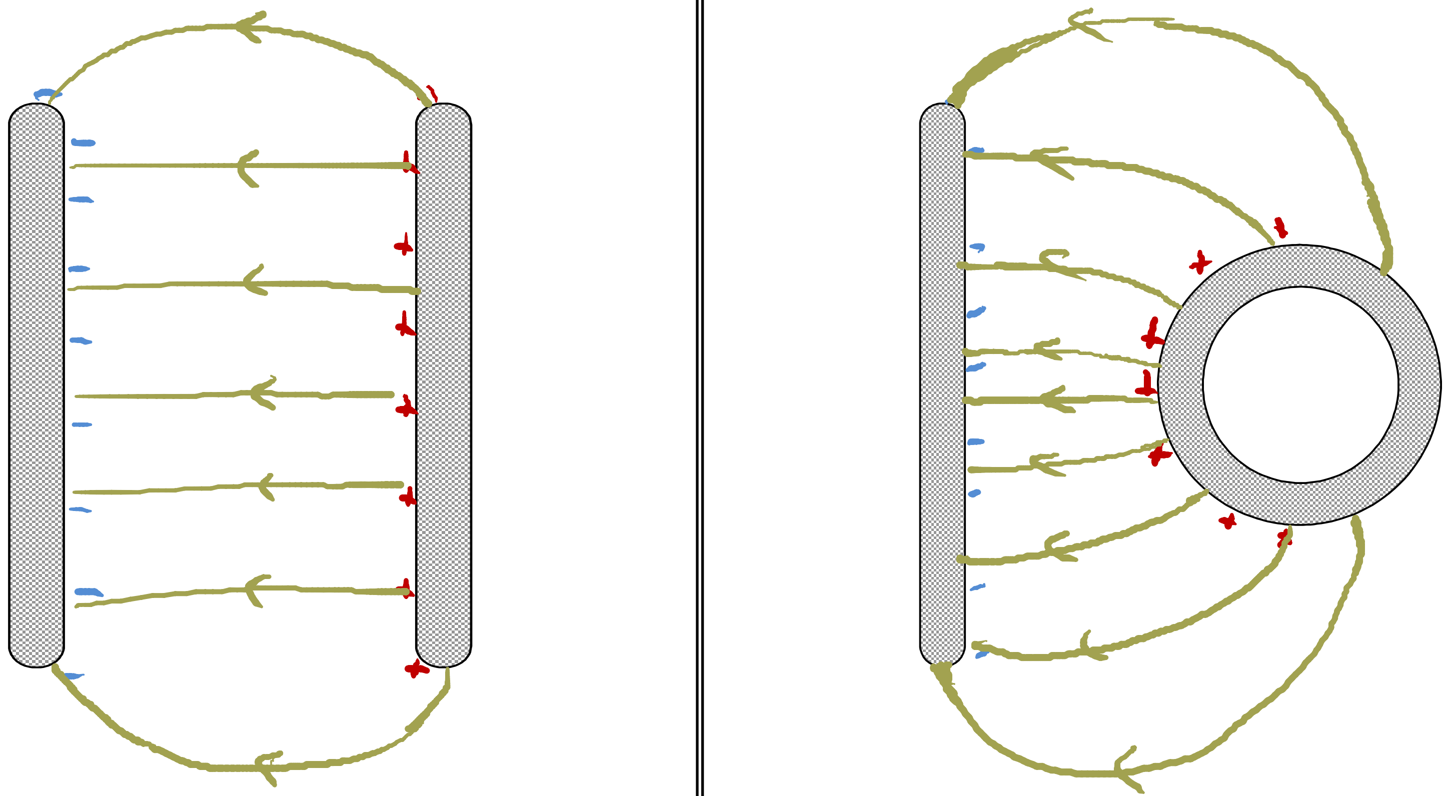
2. Excess charge can only be on its surfaces

3. Excess charge is more concentrated in regions of greater curvature

4. $\vec{E}$ at surface must be perpendicular to surface





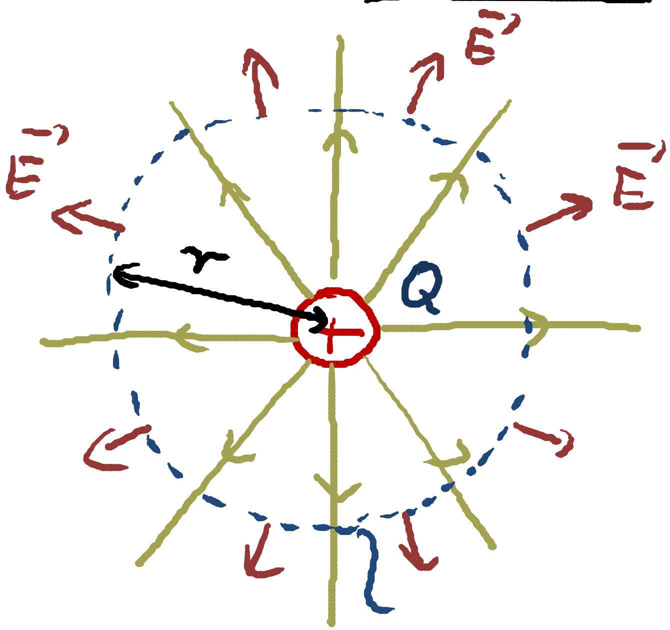


In each of the above cases, the conductors have charges that are equal in magnitude but opposite in sign. In each case, the positively charged conductor is the one on the right.

Gauss' Law for Electric Fields

- Relates the electric field on a closed surface ("Gaussian Surface") to the net charge inside the closed surface.

Look at point charge:



3D sphere
= gaussian surface

field of point charge:

$$|E| = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2} = \frac{1}{\underbrace{(\text{surface area of sphere})}_{}} \cdot \frac{Q}{\varepsilon_0}$$

$$\Rightarrow Q = \varepsilon_0 \underbrace{EA}_{\Phi} \qquad A = 4\pi r^2$$

Φ: electric flux

$\Rightarrow$ Gauss Law:

$$\boxed{Q^{net}_{\text{enclosed by gaussian surface}} = \varepsilon_0 \, \Phi_{\text{net through closed gaussian surface}}}$$

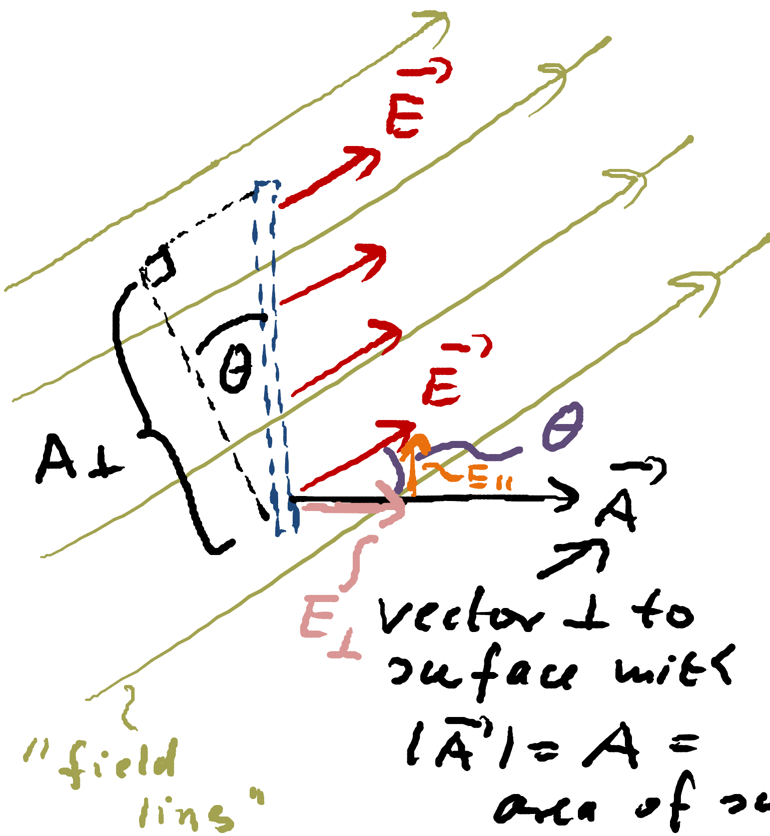
Note:

① If we know the <u>net</u> flux Φ through an enclosed surface $\Rightarrow$ know <u>net</u> charge inside of surface

② $Q_{net,\,enclosed} \Longleftrightarrow \Phi_{net,\,gaussian\ surface} = 0$

③ <u>Electric flux Φ through given surface</u>

flux $\Phi_A \propto$ # of electric "flux lines" crossing through surface of area A



$\vec{A}$: vector $\perp$ to surface with $|\vec{A}| = A$ = area of surface

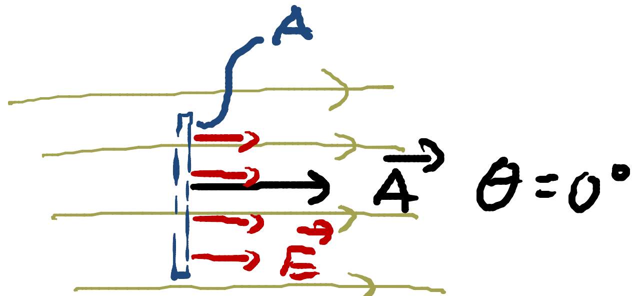
$$\Phi_A = \vec{E}\cdot\vec{A} = EA\cos\theta$$

$$= EA_\perp = E_\perp A$$

θ: angle between $\vec{E}$ and $\vec{A}$

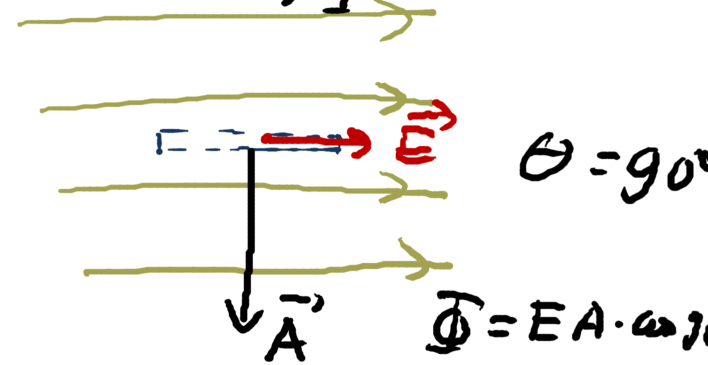
$A_\perp$: effective area $\perp$ to $\vec{E}$

$E_\perp$: component of $\vec{E}$ $\perp$ to surface, i.e. in direction of $\vec{A}$



no field lines cross surface

$\Rightarrow \Phi = 0$



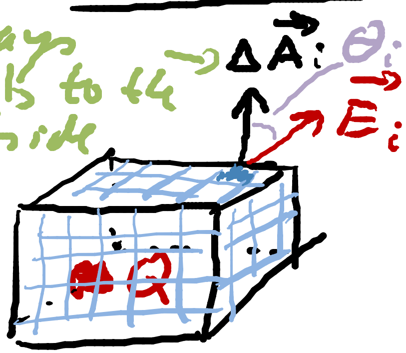
$$\Rightarrow \Phi_A = EA\cos 0° = EA$$

$$\Phi = EA \cdot \cos 90° = 0$$

$\Rightarrow$ only $E_\perp$ contributes to Φ_A, $E_\parallel$ does not

$\Rightarrow$ for closed, Gaussian surface:

always points to the outside



need to sum over all surfaces

$$\Phi_{\text{net, gaussian closed surface}} = \sum_{i=1}^{N} \underbrace{\vec{E}_i \cdot \Delta\vec{A}}_{} = \sum_i E_i \Delta A_i \cos\theta_i$$

small section of surface with $\vec{E}$ = const over given small area

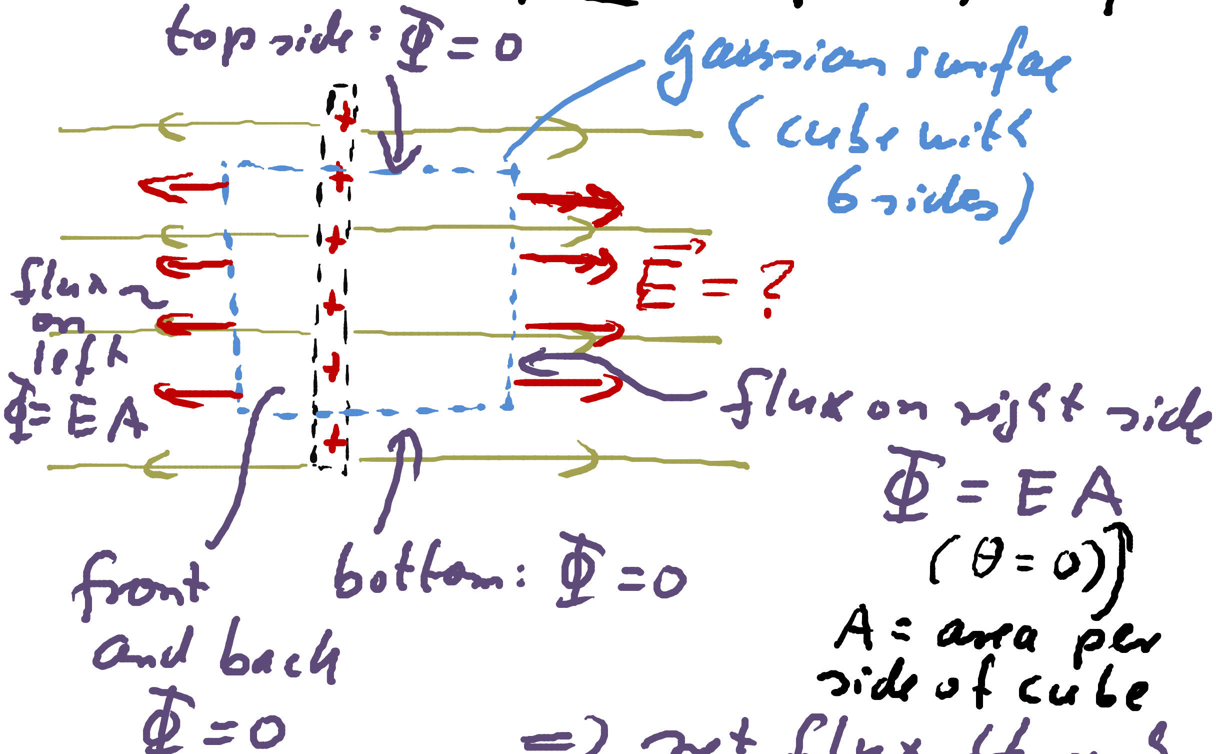
$$\Phi_{net} = \oint \vec{E} \cdot d\vec{A}$$

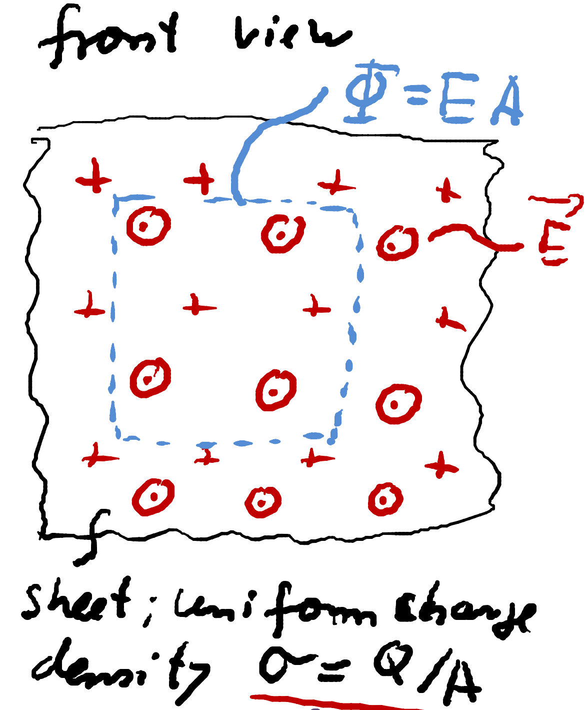
integral taken over entire closed surface

$\Phi_{net} > 0 \Leftrightarrow Q_{net} > 0$

$\Phi_{net} < 0 \Leftrightarrow Q_{net} < 0$

Example: infinite, uniform sheet of charge





⇒ net flux through gaussian surface

$$\Phi_{net} = 0 + 0 + 0 + 0 + EA + EA$$

⇒ from Gauss Law:

$$Q_{inside} = \varepsilon_0 \Phi_{net} = \varepsilon_0 2AE \Rightarrow \boxed{E_{\infty\ \text{sheet}} = \frac{1}{2\varepsilon_0}\frac{Q}{A} = \frac{1}{2\varepsilon_0}\sigma}$$