# Physics 116 Formula Sheet: Final Exam

## Vectors and Polar Coordinates

$$|\mathbf{a}| = \sqrt{a_x^2 + a_y^2 + a_z^2}$$

$$\mathbf{a}\cdot\mathbf{b} = a_x b_x + a_y b_y + a_z b_z \qquad \mathbf{a}\cdot\mathbf{b} = |\mathbf{a}|\,|\mathbf{b}|\,\cos\theta_{ab}$$

$$\mathbf{a}\times\mathbf{b} = (a_y b_z - a_z b_y)\hat{\mathbf{i}} + (a_z b_x - a_x b_z)\hat{\mathbf{j}} + (a_x b_y - a_y b_x)\hat{\mathbf{k}}$$

$$|\mathbf{a}\times\mathbf{b}| = |\mathbf{a}|\,|\mathbf{b}|\,\sin\theta_{ab}$$

$$x = r\cos\theta, \qquad y = r\sin\theta$$

## Kinematics

$$\mathbf{v}(t) = \mathbf{v}(t_0) + \int_{t_0}^{t}\mathbf{a}(t')dt' \qquad \mathbf{r}(t) = \mathbf{r}(t_0) + \int_{t_0}^{t}\mathbf{v}(t')dt'$$

$$\mathbf{v} = \dot{r}\hat{\mathbf{r}} + r\dot{\theta}\hat{\boldsymbol{\theta}} \qquad \mathbf{a} = (\ddot{r} - r\dot{\theta}^2)\hat{\mathbf{r}} + (r\ddot{\theta} + 2\dot{r}\dot{\theta})\,\hat{\boldsymbol{\theta}}$$

## Force Laws

$$m\mathbf{a} = \textstyle\sum_i \mathbf{F}_i; \quad \mathbf{F}_{12} = -\mathbf{F}_{21}; \quad \mathbf{F}_g = -\frac{G_N M_1 M_2}{r_{12}^2}\,\hat{\mathbf{r}}_{12}; \quad \mathbf{W} = M\mathbf{g}$$

$$F_{\text{fric}}^{\text{stat}} \le \mu N; \qquad F_{\text{fric}}^{\text{kin}} = \mu N; \qquad F_{\text{spr}} = -k(x - x_{\text{eq}})$$

## Momentum and Center of Mass

$$\text{particle: } \mathbf{p} = m\mathbf{v}; \quad \frac{d\mathbf{p}}{dt} = \mathbf{F}$$

$$\text{system: } \mathbf{P} = \textstyle\sum_i m_i\mathbf{v}_i; \quad \frac{d\mathbf{P}}{dt} = \sum \mathbf{F}_{\text{ext}}$$

$$\text{center of mass: } \mathbf{R}_{\text{cm}} = \frac{1}{M}\textstyle\sum_i m_i\mathbf{r}_i, \text{ where } M = \sum_i m_i$$

$$M\ddot{\mathbf{R}}_{\text{cm}} = \textstyle\sum \mathbf{F}_{\text{ext}}$$

$$\text{impulse: } \mathbf{I} = \int_0^t \mathbf{F}dt; \quad \mathbf{I} = \mathbf{P}(t) - \mathbf{P}(0)$$

## Energy

work-energy theorem: $W_{ba} = K_b - K_a, \quad K = \frac{1}{2}M\mathbf{v}^2$

$W_{ba} = \int_{x_a}^{x_b} F(x)dx$ (1D); $\quad W_{ba} = \int_{\mathbf{r}_a}^{\mathbf{r}_b} \mathbf{F}(\mathbf{r}) \cdot d\mathbf{r}$ (3D)

potential energy: $U(\mathbf{r}) = -\int_0^{\mathbf{r}} \mathbf{F}(\mathbf{r}) \cdot d\mathbf{r}$

mechanical energy conservation: $E \equiv U + K = const$

gravity: $U(z) = mgz + C$ (on Earth); $\quad U(r) = -\frac{mMG}{r} + C$ (in general)

spring: $U(x) = \frac{1}{2}k(x - x_{\text{eq}})^2 + C$; $\quad$ force (in 1D): $F = -\frac{dU}{dx}$

## Rotational Motion

particle: $\mathbf{L} = \mathbf{r} \times \mathbf{p}, \quad \boldsymbol{\tau} = \mathbf{r} \times \mathbf{F}, \quad \frac{d\mathbf{L}}{dt} = \boldsymbol{\tau}$

moment of inertia: $I = \sum_i m_i \rho_i^2 \to \int \rho^2 dm$

rigid body, pure rotation: $L_z = I\omega, \quad \tau_z = I\frac{d\omega}{dt}, \quad K = \frac{1}{2}I\omega^2$

rigid body, rotation+translation: $L_z = (\mathbf{R} \times \mathbf{P})_z + I_0\omega,$

$M\ddot{\mathbf{r}}_{\text{c.m.}} = \sum \mathbf{F}_{\text{ext}}, \quad \tau_0 = I_0\frac{d\omega}{dt}, \quad K = \frac{1}{2}MV^2 + \frac{1}{2}I_0\omega^2$

no-slip condition: $V = \omega R$

static equilibrium: $\sum \mathbf{F} = 0, \quad \sum \boldsymbol{\tau} = 0$

## moments of inertia:

hoop (axis thru c.of m., $\perp$ to the plane of the hoop) $I = MR^2$

solid disk (axis through c.of m., $\perp$ to the plane of the disk) $I = MR^2/2$

solid cylinder (axis along the axis of the cylinder) $I = MR^2/2$

uniform stick (axis through c. of m., $\perp$ to the stick) $I = ML^2/12$

solid sphere (axis through center): $I = 2MR^2/5$

paralel axis theorem: $I = I_0 + Ml^2$

## Oscillators

**undamped:** $x(t) = A\cos(\omega_0 t + \phi_0), \quad \text{where } \omega_0 = \sqrt{k/m}$

**damped:** $f = -b\dot{x}, \quad x(t) = Ae^{-(\gamma/2)t}\cos(\omega_1 t + \phi_0),$

where $\gamma = b/m, \quad \omega_1 = \sqrt{\omega_0^2 - \gamma^2/4}$

**forced:** $F = F_0 \cos\omega t, \quad x(t) = A\cos(\omega t + \phi),$

where $A = \frac{F_0}{m}\frac{1}{\sqrt{(\omega_0^2-\omega^2)^2+\omega^2\gamma^2}}, \quad \phi = \arctan\left(\frac{\gamma\omega}{\omega^2-\omega_0^2}\right)$

## Relativity

Lorentz transformations: $x' = \gamma(x - vt), \quad t' = \gamma(t - \frac{v}{c^2}x),$

where $\gamma = \frac{1}{\sqrt{1-v^2/c^2}}$

time dilation: $T = \gamma\tau$; Lorentz contraction: $L = l_0/\gamma$

velocity addition: $u_x' = \frac{u_x - v}{1 - vu_x/c^2}$

Doppler effect: $\nu_D = \nu_0\sqrt{\frac{1+v/c}{1-v/c}}, \quad \lambda = c/\nu$

energy & momentum: $E = \gamma m_0 c^2, \quad \mathbf{p} = \gamma m_0 \mathbf{v}, \quad E^2 = (\mathbf{p}c)^2 + (m_0c^2)^2$

photon: $E = |\mathbf{p}|c, \quad E = h\nu$

## Fundamental Constants:

speed of light $c = 3 \times 10^8$ m/sec;

Newton's gravitational constant $G_N = 6.67 \times 10^{-11}$ N·m$^2$/kg$^2$

Planck's constant: $h = 6.63 \times 10^{-34}$ J/Hz

## Math

Taylor expansion: $F(x) = \sum_{n=0}^{\infty} \frac{1}{n!}\frac{d^nF(x_0)}{dx^n}(x - x_0)^n$

$= F(x_0) + F'(x_0)(x - x_0) + \frac{1}{2}F''(x_0)(x - x_0)^2 + \ldots$