Phys. 116 Final
9:00-11:30 am, Dec. 16, 2003

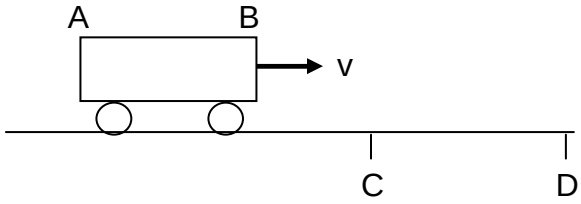
1. (5 points) In one inertial reference frame, event A and event B occur at the same position, with A preceding B in time. Will A precede B in all other inertial reference frames? Explain.

2. (5 points. Dedicated to people with manual-transmission cars) Consider two disks, rotating about a common axis, with moments of inertia I_1 and I_2. Initially they have rotational velocities ω_1 and ω_2. Suppose that they are not subject to any external torques. A driver then brings the two disks into contact, and they stick together. Find the final angular velocity of the combined system.

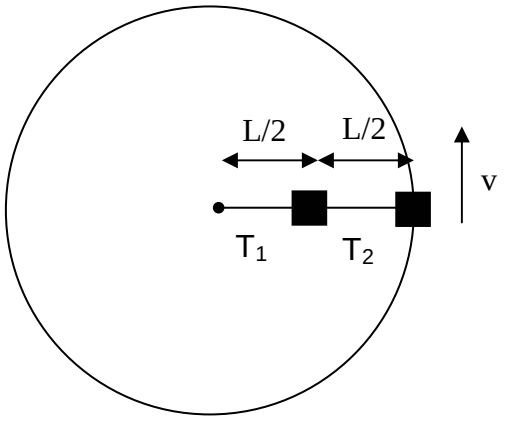
3. (5 points) A mass on a spring is released from rest with amplitude x_0. Because of viscous frictional forces, the mass does not return to its original position after one oscillation period. Rather, its maximum displacement is reduced by a distance Δx, where $\Delta x << x_0$. Determine the quality factor of this oscillator using the definition

$$Q = \frac{\text{energy stored}}{\text{energy dissipated per radian}}.$$

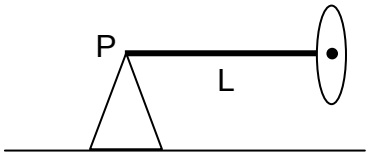
4. (5 points) A cart moves on a track with a constant relativistic velocity v. A and B are observers at the two ends of the cart, and C and D are stationed along the track. We define event AC as the occurrence of A passing C, and event BD as B passing D. Suppose that events AC and BD are simultaneous in the cart's reference frame. Are they simultaneous in the track's reference frame? If not, which event is earlier in the track's frame?



5. (5 points) Two identical weights with mass *m* are connected to massless ropes as shown. One end of rope 1 is fixed, so that the two weights move together in horizontal circles. (You may ignore gravity.) The outer weight moves with a speed v. Determine the tensions, T_1 and T_2, in the two rope segments.



6. (10 points) A gyroscope, a uniform disc with mass M and moment of inertia $I=MR^2/2$ at the end of a massless rod of length L, is mounted on the floor of an elevator going up with a constant acceleration a = g/5. The gyroscope is supported at the fulcrum point P. The gyroscope remains horizontal and precesses with an angular velocity Ω. Neglecting friction, find the angular velocity ω with which the gyroscope must be spinning.

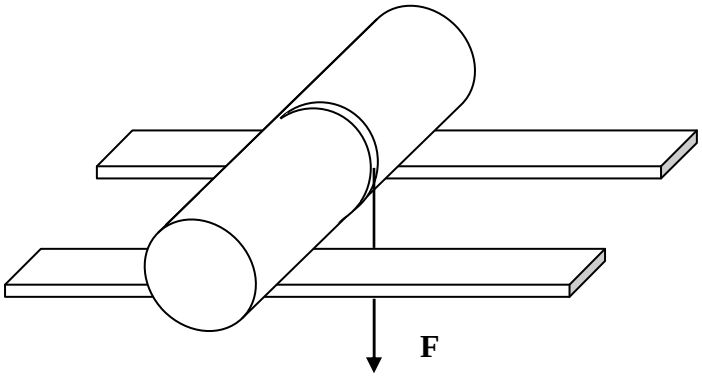


7. (10 points) It is possible for a high-energy photon to collide with one electron to produce the original electron plus an additional electron-positron pair. This reaction can be written:

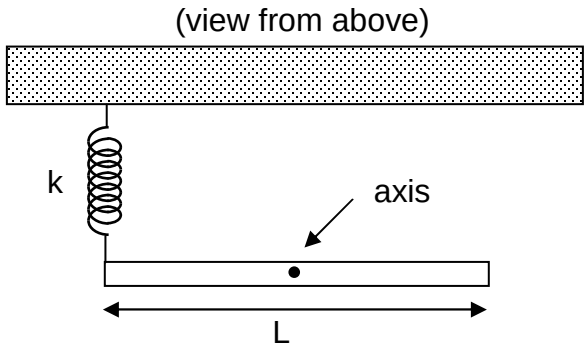
$$\gamma + e^- \rightarrow e^- + \left(e^- + e^+\right)$$

The rest masses of both the electron and the positron are m_e. Consider the reaction in the center-of-mass frame. (When dealing with photons, this might better be called the center-of-momentum frame.) Determine the minimum energy required for the photon in this frame in order for the reaction to proceed.

8. (15 points) A uniform solid cylinder of mass M and radius R rests on two horizontal planks. The coefficient of friction between the cylinder and the planks is μ. A thread is wound on the cylinder. The hanging end of the thread is pulled vertically downward with a constant force F, causing the cylinder to begin to roll along the planks. What is the maximum force which may be applied before the cylinder will begin to slip on the planks, instead of simply rolling without slipping?



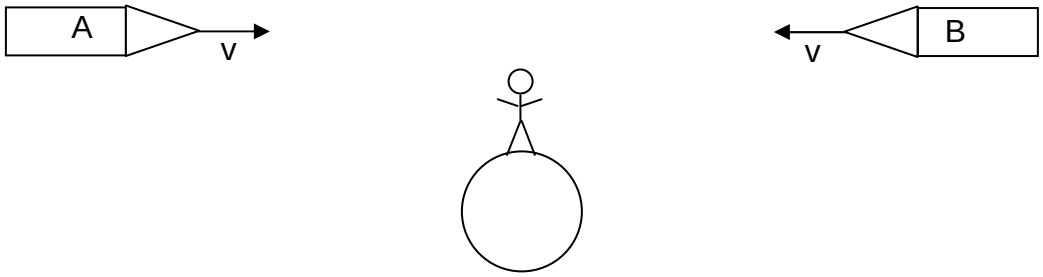
9. (15 points) A uniform rod of length L and mass M is free to rotate in a horizontal plane about a vertical axis through its center. A spring with force constant k is connected horizontally between the end of the rod and a fixed wall as shown, so that in equilibrium the rod is parallel to the wall. Let θ be the angle of the rod when it is displaced slightly from equilibrium.



(a) Write an equation of motion for θ, assuming a small deviation from equilibrium. You may use that the moment of inertia of the rod about the pivot point is $ML^2/12$.

(b) What is the period of oscillation for small deviations from equilibrium?

10 (15 points) Imagine that there exists a species of space-faring elephants, who are 1 meter tall at birth, and who grow at a rate of 1 m per year throughout childhood, measured according to time in their own frame. An observer at rest in some inertial reference frame observes two spaceships (A and B), both moving at a speed of v = 4c/5, traveling toward him in opposite directions. At time t = -5/4 year, when each ship is 1 light-year from his position, an elephant is born in each one. (One light year is defined as the distance that light travels in one year.) At time t = 0, the two ships pass by each other at his position, at which point the elephants have grown to the same height.



(a) (5 points) What is this height? (Assume that the observer measures the height of the elephants in a direction perpendicular to the velocities of the ships, so that there is no length contraction for the heights.)

(b) (10 points) A different observer on ship A would say that the clocks on ship B move more slowly than her own clock, which means that she would observe the elephant on ship B to grow at a slower rate than her own elephant on ship A. Relative to an observer on ship A, will the elephants be the same height at the moment when the two ships pass? For a complete answer, you should determine the coordinates of both births according to an observer on ship A as well as the heights of both elephants as measured by observer A at the moment when the ships pass.

11. (10 points) You are biking along level ground with a speed v. The mass of you plus the bike is M. The distance between the bottom of your two tires is L. The center of mass of you plus the bike is centered between the wheels and is a distance *d* above the ground. Assume that the bike wheels have negligible mass, so that you can ignore the angular momentum associated with their rotation about their axles. You can also ignore air resistance.

Suddenly you apply your *front* brake, but not the rear one, so that your front wheel is locked and you begin to skid. What is the maximum magnitude of deceleration that could be achieved for the system of you plus the bike before your rear wheel just loses contact with the ground, and you risk somersaulting over your handlebars?