Cornell University

Department of Physics

Phys 116 April 12, 2006

Mechanics and Special Relativity, Spring 2006

Homework Assignment # 9

(Due Wednesday, April 19, before the lecture.)

Lectures and Reading Assignments:

Readings are from *"An Introduction to Mechanics"* by Kleppner and Kolenkow.

- Lec 33, 4/14 (Fri): The Forced Harmonic Oscillator. **Sec. 10.3 (pp. 421–432).**
- Lec 34, 4/17 (Mon): The Michelson-Morley Experiment and the Postulates of Special Relativity. **Sec. 11.1–11.3 (pp. 442–452).**
- Lec 35, 4/19 (Wed): The Galilean and Lorentz Transformations. **Sec. 11.4, 11.5 (pp. 453–458).**

Problems:

Numbered problems are from *"An Introduction to Mechanics"* by Kleppner and Kolenkow, Chapter 7 (pp. 334–337) and Chapter 10 (pp. 438–440).

1. Problem 7.4
2. A truck of mass $m = 1000$ kg *without shock absorbers* (so that there is no damping) rests on suspension springs of a combined effective spring constant $k_{\rm eff}$ such that the full weight of the truck compresses the springs by 2.5 cm. Before $t = 0$ the truck is riding smoothly along (the springs are in their equilibrium position), but at $t = 0$ the truck hits a bump which gives it an initial upward velocity of 10 cm/s from the equilibrium position. What is the amplitude of the resulting oscillations?
3. Problem 10.1
4. An object of mass m is attached to a spring with spring constant k and is subject to a linear friction force, $f = -b\dot{x}$. (The motion is one-dimensional.) Check that the function

$$x(t) \;=\; A\,e^{-\beta t}\,\cos(\omega t + \phi_0) \tag{1}$$

   is a solution of the equation of motion of this system for *any* A and ϕ_0. Find the expressions for β and ω in terms of m, b and k.
5. Problem 10.5
6. To avoid the possibility of driving bridges into large oscillations by the periodic impact of footsteps, the US Army infantry now never marches instep across bridges. In this problem, we will evaluate whether this concern is realistic by computing how closely timed the footsteps must be to match a bridge's natural frequency. To answer this question, we shall use a simple model of the bridge as a driven, damped oscillator.

   Within this model, we take the march steps to correspond to a periodic driving force $F = F_o\cos(\omega t)$, where $\omega = \frac{2\pi}{T_{\rm st}}$ with $T_{\rm st}$ being the time between the footsteps of a unit marching in formation. To make the model realistic, we assume the following information:

- We take the bridge to have a mass m and spring constant k such that the bridge has a natural angular frequency $\omega_0 = 2\pi/T_o$, where $T_o \equiv 0.5$ sec is something close to the period of a march step (time between single steps).
- We assume that the natural damping of the bridge's motion is such that, if initially put in motion but then left undriven, the bridge will noticeably vibrate up and down about 20 times. Specifically, take the damping parameter b to be such that the amplitude of *undriven* oscillations decays to $1/e$ of its original value in a time $t = 10T_o$

(a) Use the above information to extract the numerical parameters (ω_o, b) needed to make a computer or calculator plot of $A/(\frac{F_o}{m})$ versus the marching frequency ω, where A is the amplitude of the bridge's oscillation. Turn in a properly labeled sketch or a copy of the resulting plot.

(b) Read off from your plot the range within which $T_{\rm st}$ must fall in order for the amplitude to be within 1/2 of its maximum value. (An *approximate* range read off from your plot is fine.)

(c) Comment in a brief sentence or two on whether you feel the concern about driving such a bridge into resonance is realistic.