Cornell University

Department of Physics

Phys 651 August 28, 2006

Relativistic Quantum Field Theory, Fall 2006

Homework Assignment # 1

(Due Wednesday, September 6, before the lecture.)

Lectures and Reading Assignments:

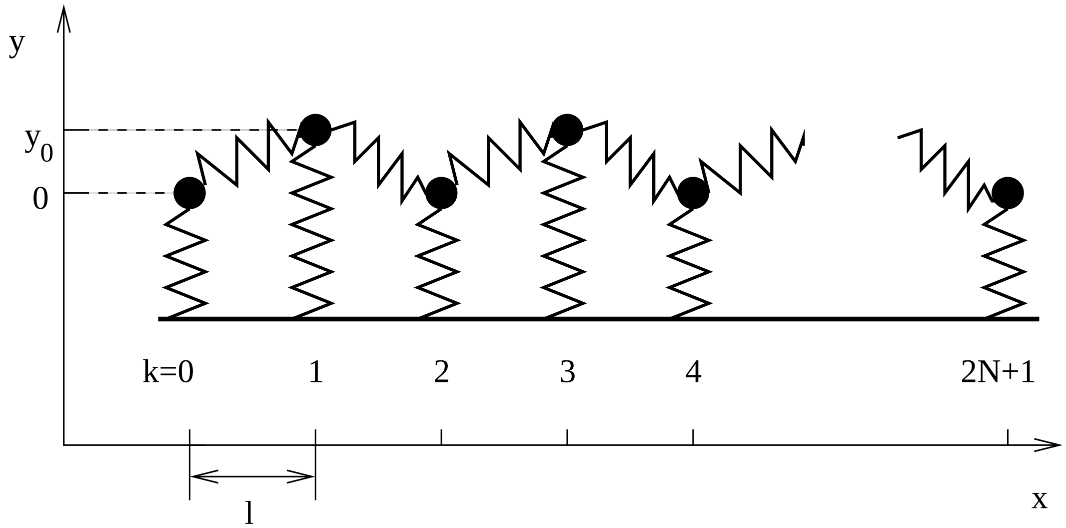
Readings are from *"An Introduction to QFT"* by Peskin and Schroeder.

- Lec 1, 8/28 (Mon): Course organization. Motivation. Why QFT? **Sec. 2.1.**
- Lec 2, 8/30 (Wed): Why QFT? (cont'd.) Klein-Gordon equation. Quantization of the scalar field. **Sec. 2.1, 2.3.**
- Lec 3, 9/04 (Mon): Quantization of the scalar field, cont'd. **Sec. 2.3, 2.4.**
- Lec 4, 9/06 (Wed): Lagrangians and all that. **Sec. 2.2.**

Problems:

1. **Getting used to new units:** Here are a few excersises to get some feel for the system of units we will use in this class, $c = \hbar = 1$.
   (a) What is the length of an Olympic swimming pool (50 m) expressed in eV^{-1}?
   (b) Find the frequency (in sec) and the wavelength (in cm) of a photon with an energy of 1 GeV.
   (c) What is the value (including the units) of the Newton's gravitational constant G_N in our new system? The *"Planck mass"* is defined by $M_{\mathrm{Pl}} = (G_N)^\alpha$, and its units should be obvious from the name. Find α and M_{Pl} (in GeV). Find the wavelength (in cm) of a photon whose energy is equal to M_{Pl} – the so-called *"Planck distance"*.
   (d) At what temperature (in K) does a gas of electrons become relativistic? How does it compare with the temperature at the center of the Sun? (**Hint:** if you don't remember how hot the Sun is, use Google to find out!)

2. **Mattress Theory:** An ordinary mattress provides a useful mechanical analogue for the Klein-Gordon field theory. In this problem, we will consider a one-dimensional "mattress" consisting of $2N+1$ masses (mass M each) labelled by $k = 0 \ldots 2N$, connected by springs (spring constant K each) to the foundation and to the neighboring masses on both sides (see the figure on the next page). The distance between two neighboring masses along the x axis is fixed (equal to l). The masses are free to move in the y direction. When the masses with even k are at $y = 0$ and the masses with odd k are at $y = y_0$, the system is in equilibrium.
   (a) Write down the equation of motion for the y coordinate of the kth mass. Consider both even and odd k, but not the endpoints ($k = 0$ and $k = 2N$). Use the variable $\tilde{y}_k = y_k - y_k^{\mathrm{eq}}$, where y_k^{eq} is the equilibrium position of the kth mass. Restrict your attention to the case of small displacements from equilibrium, $\tilde{y}_k \ll l, y_0$.



(b) Show that by appropriate relabelings your equations can be made to look identical to the Klein-Gordon equation on a discretized one-dimensional space (see lecture 2).

(c) Assuming that the masses with $k = 0$ and $k = 2N$ are held rigidly at $y = 0$ (e.g. by welding them to the steel frame of the mattress), derive the spectrum of the waves on the mattress. (**Note:** Quantizing these waves results in an as yet undiscovered elementary particle – the mattresson!)