Cornell University

Department of Physics

Phys 651 September 12, 2006

Relativistic Quantum Field Theory, Fall 2006

Homework Assignment # 3

(Due Wednesday, September 20, before the lecture.)

Lectures and Reading Assignments:

Readings are from *"An Introduction to QFT"* by Peskin and Schroeder.

- Lec 6, 9/13 (Wed): Representations of the Lorentz Group (**Sec. 3.1**. A more detailed and very nice discussion is in *"Field Theory: a Modern Primer"* by Ramond, **Ch. 1.**)
- Lec 7, 9/18 (Mon): Dirac Equation and its Solutions. **Sec. 3.2, 3.3.**
- Lec 8, 9/20 (Wed): Quantization of the Dirac Field. **Sec. 3.5.**

Problems:

Numbered problems are from Peskin and Schroeder, Ch. 3.

1. Problem 3.1 (a) and (b).
2. **Fermion bilinears**

   NOTE: see Ramond's book for help if you're stuck on this problem.

   (a) Show that the quantity $\psi_L^T \sigma^2 \psi_L$ behaves like a scalar with respect to the Lorentz transformations. (**HINT:** start by proving the following identity of Pauli matrices: $\sigma^2 \sigma^i \sigma^2 = -\sigma^{i*}$.)

   (b) Show that the quantity $\psi_L^\dagger \sigma^\mu \psi_L$, where $\sigma^\mu = (1, \sigma^i)$, is a four-vector, by considering its behaviour under *infinitesimal* Lorentz transformations.

3. **Symmetries of the Dirac Lagrangian**

   Consider the Dirac Lagrangian,

$$\mathcal{L} = \bar{\Psi}(x)\,(i\gamma^\mu \partial_\mu - m)\,\Psi(x), \tag{1}$$

   where $\Psi = (\psi_L, \psi_R)^T$.

   (a) Show that this Lagrangian is invariant under the transformation $\psi_L \to e^{i\alpha}\psi_L, \psi_R \to e^{i\alpha}\psi_R$, where α is a constant number (*not* a function of x). (Note: in technical literature, this property goes by the fancy name of "vector $U(1)$ global symmetry".)

   (b) Construct the conserved current corresponding to this symmetry.

   (c) Now, consider the transformation $\psi_L \to e^{i\alpha}\psi_L, \psi_R \to e^{-i\alpha}\psi_R$, where α is again a constant. (This transformation is called "axial $U(1)$".) Is it a symmetry of the Dirac Lagrangian? Consider two cases: (a) $m \neq 0$, and (b) $m = 0$.