Cornell University

Department of Physics

Phys 651 September 20, 2006

Relativistic Quantum Field Theory, Fall 2006

Homework Assignment # 4

(Due Wednesday, September 27, after the lecture.)

Lectures and Reading Assignments:

Readings are from *"An Introduction to QFT"* by Peskin and Schroeder.

- Lec 9, 9/25 (Mon): Quantization of the Dirac Field. **Sec. 3.5.**
- Lec 10, 9/27 (Wed): Angular Momentum, Spin and Helicity in the Dirac Theory. **Sec. 3.3, 3.5.**

Problems:

Numbered problems are from Peskin and Schroeder, Ch. 3.

1. **Solutions of the Dirac Equation:**

   (a) Prove, by explicit substitution, that the spinors

$$\begin{aligned} u(\mathbf{p}) &= (\sqrt{p\cdot\sigma}\,\xi, \sqrt{p\cdot\bar\sigma}\,\xi)^T, \\ v(\mathbf{p}) &= (\sqrt{p\cdot\sigma}\,\eta, -\sqrt{p\cdot\bar\sigma}\,\eta)^T, \end{aligned} \tag{1}$$

   where ξ and η are arbitrary two-component spinors, satisfy their respective Dirac equations:

$$\begin{aligned} (\not p - m)\,u(\mathbf{p}) &= 0, \\ (\not p + m)\,v(\mathbf{p}) &= 0. \end{aligned} \tag{2}$$

   (b) With the definitions of Eq. (1) above, and spinors ξ and η normalized by $\xi^\dagger\xi = \eta^\dagger\eta = 1$, show that $\bar u(\mathbf{p})u(\mathbf{p}) = -\bar v(\mathbf{p})v(\mathbf{p}) = 2m$, $u^\dagger(\mathbf{p})u(\mathbf{p}) = v^\dagger(\mathbf{p})v(\mathbf{p}) = 2E_{\mathbf{p}}$, and $\bar u(\mathbf{p})v(\mathbf{p}) = 0$. Work out $u^\dagger(\mathbf{p})v(\mathbf{p})$ and $u^\dagger(\mathbf{p})v(-\mathbf{p})$.

2. Problem 3.2 (see P&S Sec. 3.4 for the definition of $\sigma^{\mu\nu}$).

3. **Supersymmetry!**

   (a) Problem 3.5 (a) and (b).

   (b) For extra credit, solve 3.5 (c).