Cornell University

Department of Physics

Phys 651 October 3, 2006

Relativistic Quantum Field Theory, Fall 2006

Homework Assignment # 6

(Due Wednesday, October 11 (or Monday, October 16), after the lecture.)

Lectures and Reading Assignments:

Readings are from *"An Introduction to QFT"* by Peskin and Schroeder.

- Lec 12, 10/04 (Wed): Interacting Fields and Perturbation Theory **Sec. 4.1.**
- **No lecture 10/09 (Mon): Fall Break!**
- Lec 13, 10/11 (Wed): S-matrix, Invariant Matrix Element and Cross Section. **Sec. 4.5**

Problems: Numbered problems are from Peskin and Schroeder, Ch. 3.

1. **Discrete Symmetries of the Dirac Theory:**

   (a) Prove the identities used in the derivation of the Dirac field transformation under the charge conjugation symmetry:

   $$u^s(\mathbf{p}) \,=\, i\gamma^2\,(v^s(\mathbf{p}))^*\,, \qquad v^s(\mathbf{p}) \,=\, i\gamma^2\,(u^s(\mathbf{p}))^*\,. \tag{1}$$

   (b) Show that the Dirac lagrangian is invariant under the time reversal transformation, $\Psi(t,\mathbf{x}) \to -\gamma^1\gamma^3\Psi(-t,\mathbf{x})$. Note: for consistency, one has to assume that the usual numbers ("c-numbers") get complex-conjugated under time reversal. For details, see P& S, pp. 67–69.

2. Problem 3.7.

3. **The "Higgs" Field:**

   (a) Consider a theory of a single real Klein-Gordon field described by a Lagrangian

   $$\mathcal{L}_0 \,=\, \frac{1}{2}\,(\partial_\mu\phi)^2 \,+\, \frac{1}{2}\mu^2\phi^2. \tag{2}$$

   (This can be thought of as a standard Klein-Gordon Lagrangian with an imaginary mass, $m = i\mu$.) Argue that this is not a sensible field theory.

   (b) Now, modify the Lagrangian (2) by adding a ϕ^4 interaction term:

   $$\mathcal{L}_1 \,=\, \mathcal{L}_0 \,-\, \frac{\lambda}{4}\,\phi^4. \tag{3}$$

   Argue that this theory is well defined when $\lambda > 0$. *Note:* Eq. (3) is similar to the Lagrangian of the Higgs field in the standard model of electroweak interactions, except that the Higgs field is actually complex.

(c) Derive equations of motion from the Lagrangian (3). Show that these equations have a constant (that is, space- and time-independent) solution: $\phi(x) = v, v \neq 0$. Find v in terms of μ and λ. *Note:* v is called the "vacuum expectation value" of the field ϕ.

(d) Define a new field $h(x) = \phi(x) - v$. Rewrite the Lagrangian in Eq. (3) in terms of $h(x)$. What is the mass of the field h?

(e) Consider adding a Dirac field $\Psi(x)$ with no mass term and a Yukawa coupling to ϕ:

$$\mathcal{L}_2 \,=\, \mathcal{L}_1 \,+\, i\bar{\Psi}\gamma^\mu\partial_\mu\Psi \,+\, g\phi\bar{\Psi}\Psi. \tag{4}$$

Rewrite this Lagrangian in terms of the field h. Derive the equation of motion for the field Ψ. What is the mass of the Ψ particle?

4. **Allowed Interactions:**

If ϕ is a real scalar field and Ψ is a Dirac field, which of the following terms can appear in the Lagrangian density of a sensible field theory? Which ones can appear in principle, but become irrelevant at sufficiently low energies?

$$\begin{aligned} &c_1\phi^7(x); \qquad c_2\partial_\mu\phi(x)\bar{\Psi}(x)\gamma^\mu\Psi(x); \quad c_3\Phi(x)\Psi^\dagger(x)\Psi(x); \quad c_4\Phi(x)\bar{\Psi}(x)\gamma^5\Psi(x); \\ &c_5\left(\bar{\Psi}(x)\Psi(x)\right)^2; \qquad c_6\phi^4(x)\partial_\mu\phi(x); \quad c_7\Phi(x)\Phi(x+\delta)\bar{\Psi}(x)\Psi(x+\delta). \end{aligned} \tag{5}$$