Cornell University

Department of Physics

Phys 651 October 31, 2006

Relativistic Quantum Field Theory, Fall 2006

Homework Assignment # 9

(Due Monday, November 6, after the lecture.)

Lectures and Reading Assignments:

Readings are from *"An Introduction to QFT"* by Peskin and Schroeder.

- Lec 17, 11/01 (Wed): Feynman Rules for Yukawa Theory and Quantum Electrodynamics. **Sec. 4.7, 4.8**
- Lec 18, 11/06 (Mon): The process $e^+e^- \to \mu^+\mu^-$. Spin Sums and Trace Technology. **Sec. 5.1–5.3**

Problems:

1. **Dirac Propagator:**

   Propagator for a Dirac fermion S_F is defined by

   $$S_F(x-y) \,=\, \left\langle 0\right| T\left(\Psi(x)\bar{\Psi}(y)\right) \left|0\right\rangle \tag{1}$$

   where $T\left(\Psi(x)\bar{\Psi}(y)\right)$ is equal to $\Psi(x)\bar{\Psi}(y)$ for $x^0 > y^0$ and to $-\bar{\Psi}(y)\Psi(x)$ (note the sign!) for $y^0 > x^0$. Show that

   $$S_F(x-y) \,=\, \int \frac{d^4p}{(2\pi)^4} \frac{i(\not{p}+m)}{p^2-m^2+i\epsilon}\, e^{-ip\cdot(x-y)} \tag{2}$$

   where $i\epsilon$ in the denominator means that Feynman prescription for going around the poles in the p^0 plane should be used.

2. **Learning to Trust Feynman Diagrams I:**

   Consider the Yukawa theory defined by

   $$\mathcal{L} \,=\, \frac{1}{2}\left((\partial\phi)^2 - m^2\phi^2\right) \,+\, \bar{\Psi}(i\not{\partial} - m)\Psi \,-\, g\phi\bar{\Psi}\Psi. \tag{3}$$

   Compute the invariant matrix element $\mathcal{M}$ for the process

   $$\text{fermion}\,(\mathbf{p}_A, s) \,+\, \text{ scalar}\,(\mathbf{p}_B) \to \text{fermion}\,(\mathbf{p}_1, s') \,+\, \text{ scalar}\,(\mathbf{p}_2), \tag{4}$$

   at the leading order (g^2). Do the calculation in two ways:

   - By brute force (start with the matrix element of $\text{Texp}(-i\int dtH_I)$, expand to order g^2, expand the fields in terms of creation/annihilation operators, (anti)commute them until you reach normal ordering, discarding the "disconnected" terms with no momentum exchange).

- By using the Feynman rules on p. 118 of Peskin and Schroeder.

Verify that they give the same result.

NOTE: This problem and the next involve a lot of writing - you **do not** need to put all the details on paper, as long as what you write is sufficient to show that you understand where all the different terms go.

3. **Learning to Trust Feynman Diagrams II:**

   Repeat the above exercise, within the same Yukawa theory, for the process

$$\text{fermion}\,(\mathbf{p}_A, s_A) + \text{ antifermion}\,(\mathbf{p}_B, s_B) \rightarrow \text{fermion}\,(\mathbf{p}_1, s_1) + \text{ antifermion}\,(\mathbf{p}_2, s_2). \tag{5}$$