Review of D-$\overline{D}$ Mixing

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Abstract

This report is part of the author's completion of the A-exam to become a Ph.D. candidate. It reviews the basic formalism, Standard Model calculation and current experimental status of D-$\overline{D}$-mixing.

1 Introduction

The extraordinary success of the Standard Model in explaining all of the current particle physics data has been both a curse and a blessing. It requires us to search for the effects of new physics at either extremely high energies, or to look for tiny effects on low-energy observables. The Tevatron (today) and the LHC (tomorrow?) are two accelerators that operate on the energy frontier. Flavor factories like BaBar, Belle and CLEO on the other hand explore the luminosity frontier — they gather colossal amounts of data on processes which occur only at loop level in the Standard Model, hoping to reveal whatever small quantum corrections the new physics could provide.

In this report, I will review the theory and current state of experimental searches for D-$\overline{D}$-mixing. The D system is special in that it is the only meson in the up-sector that is thought to undergo oscillation, which suppresses the effect due to the relatively lighter intermediate down-sector. The mixing effect is so minute that it has only been discovered a few years ago, and high-precision data is still lacking. However, measurement of this tiny effect has the potential to discover or constrain theories beyond the Standard Model, since many of the BSM scenarios currently under investigation could have effects that would either increase the mixing or the CP-violation far above the minute levels predicted by the SM.

This report is outlined as follows: In Section 2 I will review the basic quantum-mechanical formalism of meson-mixing, with special focus on applications to D-mixing. Section 3 explores why the SM prediction of mixing is both tiny and difficult to calculate. In Section 4 I explain how to measure D-mixing experimentally, and outline some recent analysis by the BaBar and CLEO collaborations. I conclude in Section 5, and Appendix A contains some Linear Algebra results used in Section 2.

2 Meson-Mixing Formalism

I present the formalism for describing the oscillation and decay of a charged or neutral pseudoscalar meson M, which would be D, K, B, or B_s. The material in this section is a compilation of the reviews in [1] and [2].

2.1 Definition of Decay Amplitudes

We define decay amplitudes of the *weak eigenstate* M (which could be charged or neutral) and its CP conjugate $\overline{M}$ to a multi-particle final state f and its CP conjugate $\overline{f}$ as

$$\begin{aligned} A_f &= \langle f|H_w|M\rangle & \overline{A}_f &= \langle f|H_w|\overline{M}\rangle \\ A_{\overline{f}} &= \langle \overline{f}|H_w|M\rangle & \overline{A}_{\overline{f}} &= \langle \overline{f}|H_w|\overline{M}\rangle \end{aligned} \tag{2.1}$$

where H_w is the Hamiltonian governing weak interactions. For our purposes, this will be made up of weak operators in the effective SM Lagrangian responsible for flavor-changing neutral currents, along with an uncalculable (but measurable) QCD piece responsible for hadronization and dispersive effects.

To analyze the action of CP on the various states, it is useful to define spurious CP-phases:

$$CP \begin{pmatrix} \overline{M} \\ M \end{pmatrix} = \begin{pmatrix} 0 & e^{-i\xi_M} \\ e^{+i\xi_M} & 0 \end{pmatrix} \begin{pmatrix} \overline{M} \\ M \end{pmatrix} \tag{2.2}$$

(similarly for $|f\rangle, |\overline{f}\rangle$), so that $CP^2 = 1$. If CP is conserved by the dynamics, then $[CP, H_w] = 0$ and $A_f, \overline{A}_{\overline{f}}$ are the same up to an unphysical phase, which we are free to set to zero.

2.2 Neutral-Meson Mixing

Say we produce a state that is initially given as some superposition of M^0 and $\overline{M}^0$:

$$|\psi(0)\rangle = a(0)|M^0\rangle + b(0)|\overline{M}^0\rangle. \tag{2.3}$$

This state will evolve in time, which includes oscillation between M and $\overline{M}$ as well as possible decays into multiparticle final states $f_1, f_2, \ldots$:

$$|\psi(t)\rangle = a(t)|M^0\rangle + b(t)|\overline{M}^0\rangle + c_1(t)|f_1\rangle + c_2(t)|f_2\rangle + \ldots \tag{2.4}$$

If we are primarily interested in oscillation, and are working with time scales $\gg \Lambda_{\mathrm{QCD}}^{-1}$, then we can treat time evolution in a simplified formalism with an effective 2×2 Hamiltonian matrix $\mathbf{H} = \{h_{ij}\}$ with the Schrödinger Equation:

$$i\frac{\partial}{\partial t} \begin{pmatrix} \overline{M}^0 \\ M^0 \end{pmatrix} = \mathbf{H} \cdot \begin{pmatrix} \overline{M}^0 \\ M^0 \end{pmatrix}. \tag{2.5}$$

This effective hamiltonian will *not be hermitian*, since we are dealing with an incomplete Hilbert space of only the $M^0, \overline{M}^0$ states, which are "leaking" (i.e. decaying) into the final states $f_1, \ldots$. It is therefor expected that we would have a non-hermitian component of H which is responsible for the decay. We explicitly separate out the anti-hermitian component by writing H in terms of hermitian matrices M, Γ as follows:

$$\mathbf{H} = \mathbf{M} - \frac{i}{2}\Gamma. \tag{2.6}$$

The value of this decomposition is best demonstrated by example.[1] In $D^0 - \overline{D}^0$ mixing, the mass difference between the two mass eigenstates is tiny, and it is valid to write

$$\left(\mathbf{M} - \frac{i}{2}\Gamma\right)_{ij} = \frac{1}{2m_D}\langle D_i|H_{\mathrm{eff}}|D_j\rangle \tag{2.7}$$

This will be an extremely useful equation later on, when we want to actually calculate $\mathbf{H}$ from SM amplitudes. Let us further expand the LHS:

$$m_D^{(0)}\delta_{ij} + \frac{\langle D_i|H_w|D_j\rangle}{2m_D} + \frac{1}{2m_D}\sum_f \frac{\langle D_i|H_w|f\rangle\langle f|H_w|D_j\rangle}{m_D^{(0)} - E_f + i\epsilon} \tag{2.8}$$

[1]We will be assuming that CPT is a good symmetry of the Lagrangian from now on (part of Lorentz-invariance), which dictates $h_{11} = h_{22}$.

The first term is time evolution without oscillation. The second term describes flavor oscillation due to purely weak processes, i.e. box diagrams. The third term comes from real intermediate states, e.g. oscillation via hadronic states. We can see that if the intermediate state is on-shell, the propagator becomes imaginary and contributes to $\mathbf{\Gamma}$. *Hence* $\mathbf{M}$ *and* $\mathbf{\Gamma}$ *are associated with transitions involving off- and on-shell intermediate states respectively. Furthermore, on- and off-diagonal elements of each correspond to flavor-preserving and -changing processes.*

2.3 Switch from Weak Basis to Mass Basis

The matrix $\mathbf{H}$ can be diagonalized by the matrix T defined in Appendix A. CPT sets $z = 0$ and simplifies the expressions considerably. Therefor we can define the light/heavy mass eigenstates as follows:

$$\begin{aligned} |M_L\rangle &= p|M^0\rangle + q|\overline{M}^0\rangle \\ |M_H\rangle &= p|M^0\rangle - q|\overline{M}^0\rangle, \end{aligned} \tag{2.9}$$

where $|q|^2 + |p|^2 = 1$ and the eigenvalues are given by

$$\omega_{L,H} = h_{11} \mp \sqrt{h_{12}h_{21}}. \tag{2.10}$$

The real and imaginary parts of $\omega_{L,H}$ give the masses and decay widths of the eigenstates, respectively. The mass and width splittings are

$$\begin{aligned} \Delta m &\equiv m_H - m_L = \mathrm{Re}\,(\omega_H - \omega_L) \\ \Delta\Gamma &\equiv \Gamma_H - \Gamma_L = -2\mathrm{Im}\,(\omega_H - \omega_L). \end{aligned} \tag{2.11}$$

Often they are discussed in terms of the x and y parameters:

$$x \equiv \frac{\Delta m}{\Gamma} \qquad\qquad y \equiv \frac{\Delta\Gamma}{2\Gamma} \tag{2.12}$$

In the D-$\overline{D}$ system CP-violation is a very small effect, since it effectively only involves the first two generations, and we can write

$$\frac{\langle D|H_{\mathrm{eff}}|\overline{D}\rangle}{m_D} = \Delta m - \frac{i}{2}\Delta\Gamma = \Gamma(x + iy) \tag{2.13}$$

This will be a formula we use a lot later.

2.4 Time-evolution of Mass Eigenstates

A bit of algebra yields expressions for the time-evolution of states that were initially pure $M^0, \overline{M}^0$:

$$|M^0_{\mathrm{phys}}(t)\rangle = g_+(t)|M^0\rangle - \frac{q}{p}g_-(t)|\overline{M}^0\rangle \tag{2.14}$$

$$|\overline{M}^0_{\mathrm{phys}}(t)\rangle = g_+(t)|\overline{M}^0\rangle - \frac{p}{q}g_-(t)|M^0\rangle \tag{2.15}$$

where

$$g_\pm(t) = \frac{1}{2}\left(e^{-im_H t - \frac{1}{2}\Gamma_H t} \pm e^{-im_L t - \frac{1}{2}\Gamma_L t}\right). \tag{2.16}$$

We will often deal with decays where $D^0 \to \overline{f}$ occurs much more often than $D^0 \to f$. In that case, we define the *Right-Sign (RS) Amplitudes* $A_{\overline{f}}, \overline{A}_f$ and *Wrong-Sign (WS) Amplitudes* $A_f, \overline{A}_{\overline{f}}$. Then we can express the *time-dependent wrong-sign decay rate*, normalized to the right-sign amplitude, as

$$\begin{aligned} r(t) &= \frac{|\langle f|D^0_{\rm phys}(t)\rangle|^2}{|\overline{A}_f|^2} = \left|\frac{q}{p}\right|^2 \left|g_+(t)\lambda_f^{-1} + g_-(t)\right|^2 \\ \overline{r}(t) &= \frac{|\langle \overline{f}|\overline{D}^0_{\rm phys}(t)\rangle|^2}{A_{\overline{f}}|^2} = \left|\frac{p}{q}\right|^2 \left|g_+(t)\lambda_{\overline{f}} + g_-(t)\right|^2 \end{aligned} \tag{2.17}$$

where

$$\lambda_f \equiv \frac{q}{p}\frac{\overline{A}_f}{A_f} \qquad \text{and} \qquad \Gamma = \frac{\Gamma_H + \Gamma_L}{2} \tag{2.18}$$

2.5 Classification of Phases

Consider amplitudes A_f for $M \to f$ its CP-conjugate $\overline{A}_{\overline{f}}$ for $\overline{M} \to \overline{f}$. There are two types of phases that can appear in those amplitudes:

- **weak phases** ϕ which appear in the Lagrangian directly. In the SM, these are only present in the W-boson couplings. *They are opposite for A_f, $\overline{A}_{\overline{f}}$.*
- **strong phases** δ due to intermediate on-shell states in the decay process. They are due to CP-conserving interactions (mostly QCD) and are *the same for A_f, $\overline{A}_{\overline{f}}$.*

Hence we might obtain an expression like

$$A_f = |a_1|e^{i(\delta_1+\phi_1)} + |a_2|e^{i(\delta_2+\phi_2)} \qquad \overline{A}_{\overline{f}} = |a_1|e^{i(\delta_1-\phi_1)} + |a_2|e^{i(\delta_2-\phi_2)}. \tag{2.19}$$

Note that only weak phases cause CP-violation! Furthermore, only *differences* between phases are physical and convention-independent.

Unless qualified otherwise, the use of the expression *"weak phase"* refers to

$$\phi = \arg\,(q/p). \tag{2.20}$$

If either $|q/p| \neq 1$ or $\phi \neq 0$, this would signal CP violation. An alternative definition of the weak phase is achieved by writing

$$M_{12} = |M_{12}|e^{i\phi_M} \qquad \Gamma_{12} = |\Gamma_{12}|e^{i\phi_\Gamma} \tag{2.21}$$

which yields (see Appendix A)

$$\omega_H - \omega_L = 2\left[|M_{12}|^2 - \frac{1}{4}|\Gamma_{12}|^2 - i|M_{12}\Gamma_{12}|\cos(\phi_M - \phi_\Gamma)\right]^{1/2} \tag{2.22}$$

$$\left|\frac{q}{p}\right| = \left[\frac{|M_{12}|^2 + \frac{1}{4}|\Gamma_{12}|^2 + |M_{12}\Gamma_{12}|\sin(\phi_M - \phi_\Gamma)}{|M_{12}|^2 + \frac{1}{4}|\Gamma_{12}|^2 - |M_{12}\Gamma_{12}|\sin(\phi_M - \phi_\Gamma)}\right]^{1/4}. \tag{2.23}$$

and allows us to evaluate ϕ in terms of ϕ_M and ϕ_Γ.

Classification of CP-violating effects

We can classify two different kinds of CP-violation (CPV) based on our discussion of weak phases:

1. **Indirect CP-violation**: can be accounted for by only having a single weak phase, no strong phases.

2. **Direct CP-violation**: needs both weak and strong phases.

Direct CPV is generally harder to deal with, since the strong phases cannot be calculated reliably in QCD. Instead, one must measure them separately.

A (complementary) classification scheme of CPV is based on where the CP-violation occurs in expressions for decay rates, e.g. eq. (2.17):

1. **CP-violation in Decay** occurs when the meson and its CP-conjugate decay at different rates into the same (up to CP) final state. It is characterized by

$$\left|\frac{\overline{A}_{\overline{f}}}{A_f}\right| \neq 1. \tag{2.24}$$

   and is the only possible source of CPV in charged meson decays, where mixing is absent:

$$\mathcal{A}_{f^\pm} \equiv \frac{\Gamma(M^- \to f^-) - \Gamma(M^+ \to f^+)}{\Gamma(M^- \to f^- + \Gamma(M^+ \to f^+)} = \frac{|\overline{A}_{\overline{f}}/A_f|^2 - 1}{|\overline{A}_{\overline{f}}/A_f|^2 + 1} \tag{2.25}$$

   This is *direct* CP-violation, and the presence of strong phases makes it difficult to extract the weak CPV phases.

2. **CP-violating in Mixing**is characterized by

$$\left|\frac{q}{p}\right| \neq 1. \tag{2.26}$$

   and is the only source of CPV in semi-leptonic final states like $M^0 \to l^+X$, $\overline{M}^0 \to l^-X$, where the wrong-sign decay amplitudes are zero. (This is the case in the SM and most of its sensible extensions, to an extremely high degree of accuracy.) In that case the only source of WS decays is oscillation $M^0 \to \overline{M}^0 \to l^- X$, and their asymmetry determines $|q/p|$:

$$A_{\rm SL} = \frac{\Gamma(\overline{M}^0_{\rm phys}(t) \to l^+ X) - \Gamma(M^0_{\rm phys}(t) \to l^- X)}{\Gamma(\overline{M}^0_{\rm phys}(t) \to l^+ X) + \Gamma(M^0_{\rm phys}(t) \to l^- X)} = \frac{1 - |q/p|^4}{1 + |q/p|^4} \tag{2.27}$$

This is indirect CPV, but extracting the weak phases nevertheless requires knowledge of certain hadronic quantities and can be difficult.

3. **CP-violation in Interference of Decays with/without Mixing** only occurs in final states f common to both M^0 and $\overline{M}^0$, and is characterized by

$$\mathrm{Im}\,\lambda_f \neq 0. \tag{2.28}$$

E.g. for final states of definite CP eigenvalue $\eta_f = \pm 1$, one can measure the asymmetry

$$A_{\rm f_{CP}}(t) = \frac{\Gamma(\overline{M}^0 \to f_{\rm CP}) - \Gamma(M^0 \to f_{\rm CP})}{\Gamma(\overline{M}^0 \to f_{\rm CP}) + \Gamma(M^0 \to f_{\rm CP})}. \tag{2.29}$$

For B-mesons, we can make the approximation $\Delta\Lambda = 0$, $|q/p| = 1$. This yields

$$A_{\rm f_{CP}}(t) = S_f \sin(\Delta m t) - C_f \cos(\Delta m t) \tag{2.30}$$

$$S_f \equiv \frac{2\mathrm{Im}\,\lambda_f}{1 + |\lambda_f|^2}, \quad C_f \equiv \frac{1 - |\lambda_f|^2}{1 + |\lambda_f|^2} \tag{2.31}$$

which contains no hadronic quantities!

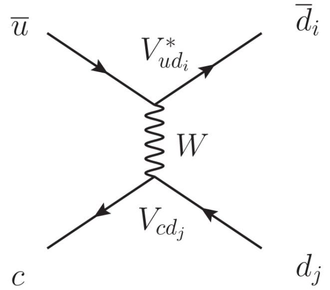
In the D-system, the SM predicts negligible CPV since to a very good approximation it only involves the first two quarks.

3 Calculation of $D^0 - \overline{D}^0$-mixing in the SM

In this section I will explain why we can not calculate D-$\overline{D}$-mixing to any real precision in the SM, and provide an order-of-magnitude estimate of its expected size. The main references are [3–5].

3.1 The SM GIM Mechanism and the SU(3)-Limit

In the Standard Model, the only source of flavor-violation is the Yukawa couplings between the fermions and the higgs. The Yukawas can be diagonalized with a biunitary transformation, which rotates the off-diagonal terms into the W-couplings, whereas the Z-couplings



$$H_w^{\Delta c=1} \propto \sum_{i,j} V^*_{ud_i} V_{cd_j} (\overline{u}\Gamma^\mu d_i)(\overline{d}_j \Gamma_\mu c)$$

$$\text{where} \qquad \Gamma^\mu = \gamma^\mu P_L$$

Figure 1: The W-mediated effective 4-fermion vertex changing $\overline{u}c \to \overline{d}_i d_j$

stay flavor-universal. *This SM GIM Mechanism guarantees that there are no tree-level flavor-changing neutral couplings.* Hence, the $\Delta F = 2$ processes of neutral meson-mixing will have to be mediated by loop diagrams, and are accordingly suppressed.

In the quark mass-basis, all the flavor violation in the SM is packed into the unitary CKM-matrix, which contains three angles and a single complex phase (after absorbing 5 unphysical phases into the quark wave functions).

$$-\mathcal{L}_{W^\pm} = \frac{g}{\sqrt{2}} \overline{u}_{L_i} \gamma^\mu (V_{\text{CKM}})_{ij} d_{Lj} W^+_\mu + h.c. \tag{3.1}$$

Its off-diagonal terms allows the W-boson to mediate flavor-changing charged current interactions, see fig. 1.

Since experimental measurements seem to constrain $\theta_{\text{QCD}}$to be effectively zero (strong CP-problem), the single complex phase of the CKM matrix is also the only source of CP violation in the SM. We *know* that cannot be the full story, since an additional source of CPV is required to explain the matter-antimatter asymmetry of the universe. It is hoped that precise measurements of meson-mixing processes could point towards new sources of CP-violation.

Like all kinds of neutral meson oscillation, $D^0 - \overline{D}^0$-mixing is mediated by two W-exchanges, see fig. 2. The CKM matrix is extremely hierarchical:

$$V_{\text{CKM}} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \sim \begin{pmatrix} 1 & \lambda & \lambda^3 \\ \lambda & 1 & \lambda^2 \\ \lambda^3 & \lambda^2 & 1 \end{pmatrix}, \tag{3.2}$$

where $\lambda \sim 0.2$. This means that the diagram with intermediate bottom quarks is extremely suppressed, despite the b's larger mass — see eq. (3.5). Therefor, we can treat D-$\overline{D}$-mixing as a 2-generation process:

$$V_{\text{CKM}} \to \begin{pmatrix} \cos\theta_c & \sin\theta_c \\ -\sin\theta_c & \cos\theta_c \end{pmatrix} \tag{3.3}$$

This means that CP-violation is expected to be minimal, since we can always rotate away the complex phase of a 2×2-mixing matrix.

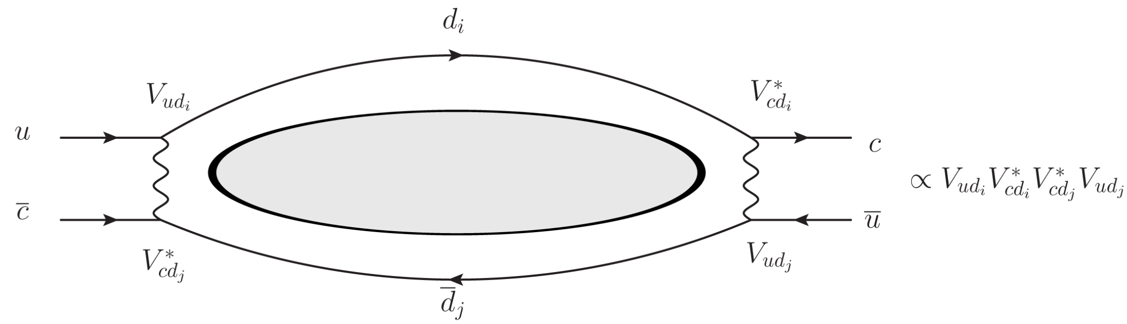


Figure 2: General form of a $D \to \overline{D}$ oscillation process (not shown: contributing diagram with W's in "s-channels"). The grey blob could be *anything*. If it's nothing: box diagram. If it's lots of glue, we have hadronic intermediate states.

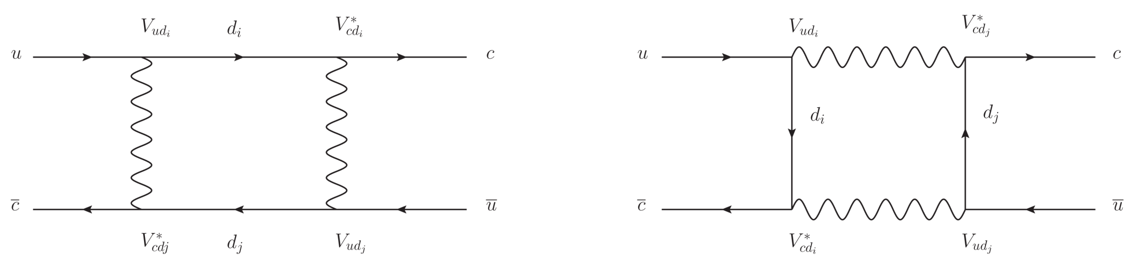


Figure 3: The SM box diagram mediating D-$\overline{D}$ mixing.

The light quarks u, d, s are much lighter than the heavy quarks c, b, t. For many applications it is a valid approximation to treat the three quarks as *massless*, meaning the strong force has an additional $SU(3)$ global symmetry that exchanges u, d, s. In this approximation, D-$\overline{D}$-mixing vanishes.

This is not hard to see. We can group all the intermediate states into sets of 4 which, by $SU(3)$-symmetry, only differ by the CKM-matrix element appearing in the vertices in fig. 2. For $(i, j) = (1, 1), (1, 2), (2, 1), (2, 2)$ the diagrams in each set evaluate (up to a constant) to $\cos^2\theta_c \sin^2\theta_c, -\cos^2\theta_c \sin^2\theta_c, -\cos^2\theta_c \sin^2\theta_c, \cos^2\theta_c \sin^2\theta_c$, i.e. they add up to zero.

To illustrate by example, let the grey blob in fig. 2 be "nothing". We get the SM box diagram fig. 3, and adding up all 4 possibilities of internal quark lines gives exactly zero in SU(3)-limit. Similarly, if we let the grey blob be a virtual up-quark-loop, along with a bunch of glue, we can represent a 2-particle intermediate hadronic state in fig. 4, which also gives zero net contribution. As long as the stuff inside the grey blob in fig. 2 only contains u, d, s-quarks, it will all cancel, and contributions by states involving heavier quarks are very small.

However, for the D-$\overline{D}$-system, this global $SU(3)$ symmetry is badly broken. This can be easily determined by experimental measurement of the $D \to \pi^+\pi^-, \pi^-K^+, \pi^+K^-, K^-K^+$ decay rates, which in the $SU(3)$-limit should occur with relative rates $\cos^2\theta_c \sin^2\theta_c : \sin^4\theta_c :$

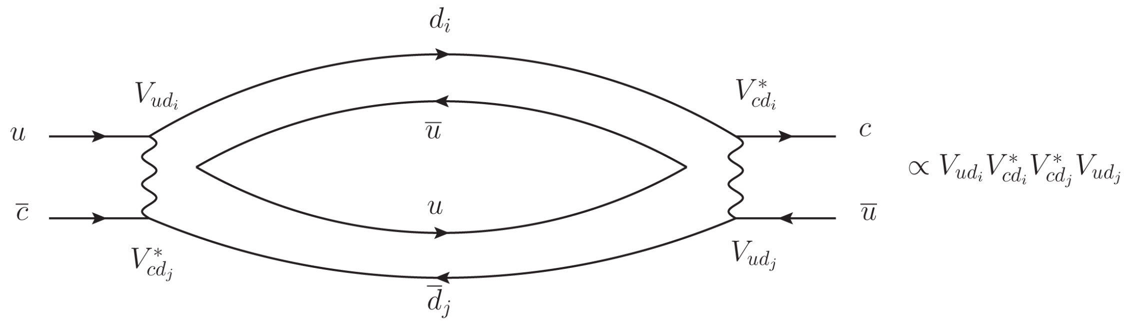


Figure 4: A 2-particle intermediate hadronic state in D-$\overline{D}$-mixing (glue not shown). $(i, j) = (1,1),(1,2),(2,1),(2,2)$ corresponds to charged pseudoscalar intermediate state $\pi^+\pi^-, \pi^-K^+, \pi^+K^-, K^-K^+$ respectively, and in the SU(3)-limit they all cancel to zero.

$\cos^4\theta_c : \cos^2\theta_c \sin^2\theta_c$. The real data shows $O(1)$ deviations from that relation, and therefor the cancelation is ineffective.

3.2 Estimating the size of short-distance and dispersive contributions to mixing

We will now estimate the contribution to $\Delta m = \Gamma x$ in D-$\overline{D}$ mixing, by both short-range (box diagram) and long-range (hadronic intermediate states) effects. This discussion will closely follow [3]. As we will see, the short-range effects are completely drowned out by non-perturbative dispersive contributions, making precise SM predictions for D-$\overline{D}$ mixing difficult.

Short-Range Contributions: Box Diagram

Calculating the box diagram for D-$\overline{D}$ mixing is quite involved, because the external momenta cannot be neglected like for the Kaon-System. This results in complicated integrals over Feynman parameters. The resulting full expressions are rather cumbersome, but the calculation has been performed in [4] with the perfectly sufficient $m_c \gg m_u$ approximation. The result is

$$\mathcal{L}^{\Delta c=2} = \frac{G_F^2}{8\pi^2}\xi_s\xi_d\frac{(m_s^2 - m_d^2)^2}{m_c^2}(\mathcal{O} + 2\mathcal{O}') \tag{3.4}$$

$$\xi_i = V^*_{cd_i}V_{ud_i}, \qquad \mathcal{O} = \overline{u}\gamma^\mu(1+\gamma_5)c\;\overline{u}\gamma_\mu(1+\gamma_5)c, \qquad \mathcal{O}' = \overline{u}(1-\gamma_5)c\;\overline{u}(1-\gamma_5)c$$

The operator $\mathcal{O}$ is expected and shows up in the expression for the Kaon box diagram, but $\mathcal{O}'$ is a new contribution due to the non-negligible charm mass. Pulling a M_w^2 out of one of the G_F's, we see that the dimensionless factor $(m_s^2 - m_d^2)^2/M_w^2 m_c^2$ is smaller by a factor of $\sim 5\times 10^4$ than the corresponding m_c^2/M_w^2 factor in the Kaon box diagram. This suppression has two sources: Firstly, the dominant intermediate state in Kaon mixing is the charm quark, which is much heavier than the strange quark dominating D-mixing. Secondly, an additional suppression $(m_s^2 - m_d^2)/m_c^2 \sim 0.04$ arises because momentum from the heavy external charm-quark leg must be squeezed through the light s, d propagators.

As an aside, we note that the bottom quark contribution is dominated by the light quarks (just like the top quark contribution is tiny for Kaon mixing). Its mass enhancement is canceled out by the strong GIM suppression:

$$\frac{m_b^2}{m_c^2} \frac{V_{cb}^* V_{cs}^* V_{ub} V_{us} + V_{cb}^* V_{cd}^* V_{ub} V_{ud}}{V_{cd}^* V_{cs}^* V_{ud} V_{us}} \approx 0.1 \tag{3.5}$$

Of course, a $\sim 10\%$ contribution could be significant if we were doing precision studies, but it does not matter here.[1]

We can now estimate the mixing contribution of the box diagram. The expression for Δm is

$$\Delta m_D^{\text{box}} = \frac{\langle D|H|\overline{D}\rangle}{m_D} = \frac{1}{m_D} \frac{G_F^2}{8\pi^2} \xi_s \xi_d \frac{(m_s^2 - m_d^2)^2}{m_c^2} (\langle D|\mathcal{O}|\overline{D}\rangle + 2\langle D|\mathcal{O}'|\overline{D}\rangle). \tag{3.6}$$

The hadronic matrix elements can be parameterized thusly:[2]

$$\langle D|\mathcal{O}|\overline{D}\rangle = \frac{8}{3} m_D^2 f_D^2 B_D \tag{3.7}$$

$$\langle D|\mathcal{O}'|\overline{D}\rangle = -\frac{5}{3} \left(\frac{m_D}{m_c}\right)^2 m_D^2 f_D^2 B_D' \tag{3.8}$$

The unknown parameters B_D, B_D' are $O(1)$-numbers, and in the vacuum approximation (which is sufficient for an order-of-magnitude estimate) we can set them to 1. We obtain

$$\Delta m_D^{\text{box}} \sim 10^{-18}\,\text{GeV} \times \left(\frac{f_D}{200\,\text{MeV}}\right)^2 \tag{3.9}$$

Long-Range Contributions: Hadronic Intermediate States

Consider two-particle intermediate hadronic states, like in fig. 5. Working in chiral perturbation theory, we can regard this diagram as a correction to the vacuum polarization of the D-propagator

$$i\mathcal{M} = A(g) \log \left(\frac{-p^2}{\Lambda_{\text{QCD}}^2}\right) + \ldots \tag{3.10}$$

[1] Note that we cannot apply this formula to estimate the top contribution to Kaon oscillation — since the top is more massive than the W, we cannot expand for small fermion mass, and get a somewhat different expression for its contribution.

[2] There are several different versions of this parametrization in the literature, and when the dust settles there is overall agreement up to a factor of 2. This is practically irrelevant due to the negligible size of the box contribution, so we don't worry about it…

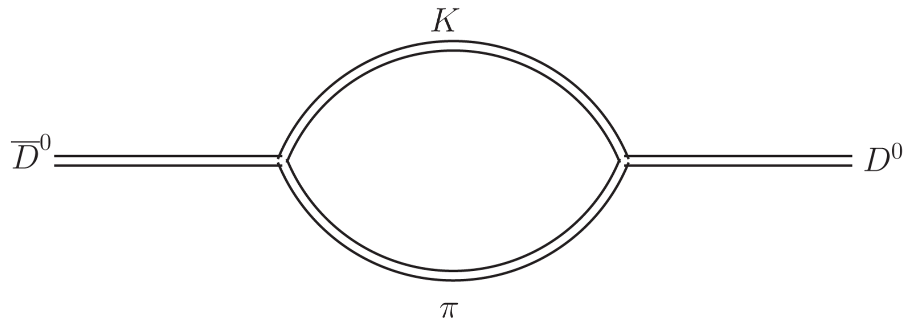


Figure 5: D-$\overline{D}$-oscillation by exchange of a Pion and a Kaon.

The divergent log is regulated by strong cutoff and $A(g)$ is quadratic in the coupling constant g. The ellipses denote various constant factors whose numerical value depends on the specific vertex, but the logarithm is universal since its imaginary part

$$\log(-p^2) = \log p^2 + i\pi \tag{3.11}$$

must yield the decay rate into that intermediate channel. We can see this by applying the optical theorem:

$$\mathrm{Im}[\text{—○—}] = \tfrac{1}{2}\int d\Pi |\text{—(}|^2 = m\Gamma$$

(where Γ is the decay rate into the intermediate channel). Applying this to our $i\mathcal{M}$ and using eq. (3.10) yields $m\Gamma = \pi A(g)$, and hence

$$\mathrm{Re}\,(i\mathcal{M}) = \frac{m\Gamma}{\pi}\log\frac{p^2}{\Lambda_{\mathrm{QCD}}} \tag{3.12}$$

which yields

$$\Delta m = \mathrm{Re}\,\frac{\langle\overline{D}|H|D\rangle}{m_D} = \frac{\Gamma}{\pi}\log\frac{m_D^2}{\Lambda_{\mathrm{QCD}}^2}, \tag{3.13}$$

where in the first step we assumed CPV to be a small effect and in the second we have set $p^2 = m_D^2$ (certainly fine approximations for an order-of-magnitude estimate in the D-$\overline{D}$-system).

Now we can estimate the Δm contribution of, say, two-particle charged pseudoscalar meson intermediate states. Note that the *relative* sign of the contributions is given by the sign of the CKM factor, just like in the $SU(3)$ approximation:

$$\Delta m_D^{\mathrm{disp}} \sim \frac{1}{\pi}\log\frac{m_D^2}{\Lambda_{\mathrm{QCD}}^2}\left[\Gamma(D^0\to K^+K^-) + \Gamma(D^0\to\pi^+\pi^-) - \Gamma(D^0\to K^+\pi^-) - \Gamma(D^0\to K^-\pi^+)\right] \tag{3.14}$$

So we open PDG and look up those decay rates, and see that the $K^-\pi^+$ rate dominates ($Br = 3.8\%$, $\Gamma_{\text{tot}} \approx 1.6 \times 10^{-12}\,\text{GeV}$). This yields

$$\Delta m_D^{\text{disp}} \sim 5 \times 10^{-14}\,\text{GeV} \gg \Delta m_D^{\text{box}} \sim 10^{-18}\,\text{GeV}, \tag{3.15}$$

so we see that the non-perturbative long-range effects are several orders of magnitude larger than the calculable short-distance contributions, making the process impossible to calculate with high precision.

3.3 Toy Problem: Calculating the box diagram in the $m_c \ll m_s$ Limit

Why are the hadronic contributions so much more important for D-mixing than for K- or B-mixing? There are several reasons for this:

1. The box diagram for D-mixing is severely suppressed due to the heavy charm-quark (compared to the intermediate quarks), yielding a double-suppression by the W-mass as well as the c-mass.

2. Due to the heavy charm, more hadronic intermediate states open up. This is not a huge effect, since the hadronic contributions are not necessarily a lot bigger in the D-system than in the K- and B-system, but reducing the number of available hadronic states would reduce the non-perturbative effect.

As a result, D-mixing is very difficult to calculate and, due to the very low values of the mixing variables $x, y \ll 1$, very challenging to measure. Compare this to other mixing systems:

- K-$\overline{K}$: $x, y \sim 1$. The constituent d, s-quarks are a lot lighter than the c-quark, which dominantly contributes to the mixing, so that particular source of suppression is not present. Nevertheless, the hadronic contribution is comparable to the short-range contribution. [5].

- B-$\overline{B}$: $x \sim 1, y \ll 1$. The constituent d, b quarks are much lighter than the extremely heavy top quark, which supplies the greatest contribution to the mixing despite the small 13-elements of the CKM matrix.

What if we made the c-quark much lighter than the s-quark? In that case, the K-intermediate states would not contribute much to the mixing, but much more importantly the contribution of the box-diagram would increase dramatically. We calculate the box diagram for this (unrealistic) limit.

The box diagram with all momenta indicated is shown in fig. 6. In the limit of $m_c, m_u \ll m_s$, we can ignore the external momenta and set them to zero. The corresponding matrix

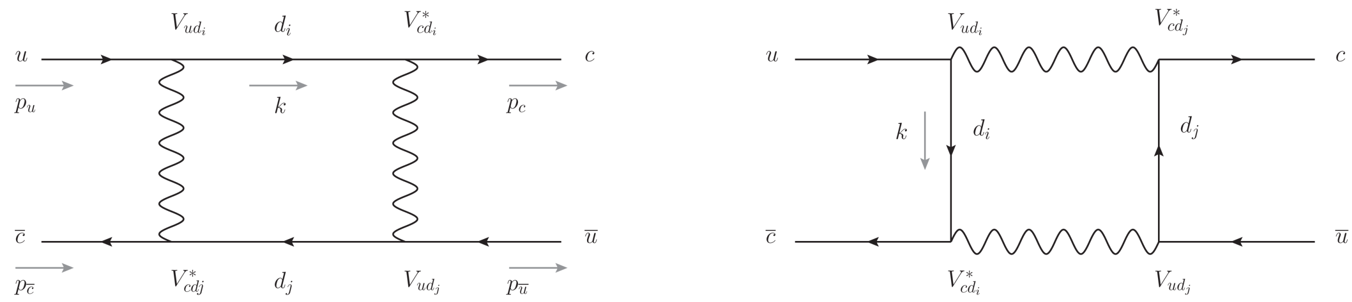


Figure 6: The box diagram amplitude we will calculate. Note that for each diagram, there is another corresponding diagram where the W's are crossed.

element is then given by[3]

$$i\mathcal{M} = \sum_{d_i,d_j} \frac{g^4}{4} V^*_{cd_i} V_{ud_i} V^*_{cd_j} V_{ud_j} I^{ij} \times \quad (3.16)$$

$$\Big\{ -(\overline{u}_c \gamma^\mu \gamma^\alpha \gamma^\nu P_L u_u)(\overline{v}_{\overline{c}} \gamma_\nu \gamma_\alpha \gamma_\mu P_L v_{\overline{u}}) + (\overline{u}_c \gamma^\mu \gamma^\alpha \gamma^\nu P_L u_u)(\overline{v}_{\overline{c}} \gamma_\mu \gamma_\alpha \gamma_\nu P_L v_{\overline{u}})$$

$$+ (\overline{v}_{\overline{c}} \gamma^\mu \gamma^\alpha \gamma^\nu P_L u_u)(\overline{u}_c \gamma_\nu \gamma_\alpha \gamma_\mu P_L v_{\overline{u}}) - (\overline{v}_{\overline{c}} \gamma^\mu \gamma^\alpha \gamma^\nu P_L u_u)(\overline{u}_c \gamma_\mu \gamma_\alpha \gamma_\nu P_L v_{\overline{u}}) \Big\}$$

$$\text{where} \quad I^{ij} = \frac{1}{32\pi^2} \int dk \frac{k^5}{(k^2 - m_{d_i}^2)(k^2 - m_{d_j}^2)(k^2 - m_W^2)^2}$$

and $P_L = \frac{1}{2}(1 - \gamma_5)$.[4]

We will be working with 2 quark generations only, so our CKM matrix is given by eq. (3.3) and consequently

$$V^*_{cd_i} V_{ud_i} V^*_{cd_j} V_{ud_j} = \sin^2\theta_c \cos^2\theta_c \begin{pmatrix} +1 & -1 \\ -1 & +1 \end{pmatrix} \quad (3.17)$$

Working in the $m_c, m_u \ll m_s \ll m_W$ limit, we get

$$I^{uu} + I^{ss} - 2I^{us} = \frac{1}{32\pi^2} \left(\frac{m_d^4 - m_s^4 + 4 m_d m_s \log\left(\frac{m_s}{m_d}\right)}{2(m_d^2 - m_s^2) m_W^4} \right) \quad (3.18)$$

[3]In my calculations, the signs of the 1st and 2nd spinor term are $(+,-)$ instead of $(-,+)$ as shown. However, this would cause the amplitude to cancel exactly! This is obviously not right, and I know the two halves of the amplitude are equal and add…I consulted with Maxim Perelstein, and at the time of writing he also didn't know where the extra minus sign comes from…So at this point I am happy to admit defeat and just put in that minus sign because I "know it's there somehow".

[4]We are working in the Peskin&Schröder gamma basis, which might be different from what previous results in this report used.

It is easy to check that this expression $\to 0$ as $m_s \to m_d$, i.e. it *vanishes in the $SU(3)$ limit.* In the case of physical interest, $m_s \gg m_d$ and

$$I^{uu} + I^{ss} - 2I^{us} = \frac{1}{32\pi^2}\left(\frac{m_s^2}{2m_W^4} + O\left(\frac{m_d}{m_s}\right)^2\right). \tag{3.19}$$

Finally, we can simplify the spinor product using Fierz Identities:

$$\begin{aligned}(\overline{u}_c\gamma^\mu\gamma^\alpha\gamma^\nu P_L u_u)(\overline{v}_{\overline{c}}\gamma_\nu\gamma_\alpha\gamma_\mu P_L v_{\overline{u}}) &= 4(\overline{u}_c\gamma^\mu P_L u_u)(\overline{v}_{\overline{c}}\gamma_\mu P_L v_{\overline{u}})\\ (\overline{u}_c\gamma^\mu\gamma^\alpha\gamma^\nu P_L u_u)(\overline{v}_{\overline{c}}\gamma_\mu\gamma_\alpha\gamma_\nu P_L v_{\overline{u}}) &= 16(\overline{u}_c\gamma^\mu P_L u_u)(\overline{v}_{\overline{c}}\gamma_\mu P_L v_{\overline{u}})\\ (\overline{v}_{\overline{c}}\gamma^\mu P_L u_u)(\overline{u}_c\gamma_\mu P_L v_{\overline{u}}) &= -(\overline{u}_c\gamma^\mu P_L u_u)(\overline{v}_{\overline{c}}\gamma_\mu P_L v_{\overline{u}})\end{aligned}$$

Putting everything together, we get the following simple result for the box-diagram amplitude:

$$i\mathcal{M} = \frac{3g^4}{32\pi^2}\sin^2\theta_c\cos^2\theta_c\frac{m_s^2}{m_W^4}(\overline{u}_c\gamma^\mu P_L u_u)(\overline{v}_{\overline{c}}\gamma_\mu P_L v_{\overline{u}}) \tag{3.20}$$

4 Experimental Measurements

We finally turn our attention to measuring D-$\overline{D}$-mixing. The mixing parameters are determined by measuring the time-dependence of decay rates, see eq. (2.17), which is especially challenging due to the tiny signal for D-$\overline{D}$-oscillation predicted by the SM. However, it is this very suppression that makes D-mixing an attractive discovery channel for new phenomena (in particular new sources of CP-violation), since any oscillation significantly larger than the SM-estimate or CPV $\gtrsim 10^{-3}$ would signal physics beyond the Standard Model. Most of the discussion in this section is derived from [2, 6].

Basic Definitions: CF, SCS and DCS Decays

It is useful to classify decays based the amount of "flavor violation" required for it to proceed:

- **Cabibbo-Favored (CF)** decays involve only diagonal elements of V_{CKM}, and they are not flavor-suppressed. One example is $A(D^0 \to K^-\pi^+) \propto V_{cs}V_{ud}^*$.
- **Singly-Cabibbo-Suppressed (SCS)** decays involve one off-diagonal CKM-matrix element, for example $A(D \to K^+K^-) \propto V_{cs}V_{us}^*$. They are therefor suppressed by 1 to 3 powers of $\lambda \sim 0.23$.
- **Doubly-Cabibbo-Suppressed (DCS)** decays involve two off-diagonal CKM-matrix elements, e.g. $A(\overline{D}^0 \to K^-\pi^+) \propto V_{us}V_{cd}^*$. They are suppressed by 2 to 5 powers of λ.

Note: I found no formal definition of these terms, so the above is my own extrapolation based on what I have seen in the literature. It seems a bit strange to me that no distinction is made between 12-, 23- and 13-elements.

4.1 Measuring Mixing using Wrong-Sign Semi-Leptonic Final States

Consider the final state $f = K^+ l^- \nu$. The Right-Sign amplitude $A(D^0 \to K^- l^+ \nu)$ is CF, whereas the Wrong-Sign amplitude $A(D^0 \to K^+ l^- \nu)$ is practically nil in the SM. We can therefor set $A_f = \overline{A}_{\overline{f}} = 0$ in our analysis. This means that *the only way D^0 can decay into the WS state is via mixing $D^0 \to \overline{D}^0 \to K^+ l^-$*. It therefor seems natural to measure the WS decay rate to determine the mixing parameters.

Our expression eq. (2.17) simplifies considerably for $A_f = \overline{A}_{\overline{f}} = 0$. In particular, all strong phases disappear from the time-dependent WS decay rate since there are no interference terms:

$$r(t) = \left|\frac{q}{p}\right|^2 |g_-(t)|^2 \approx \frac{e^{-\Gamma t}}{4} \left|\frac{q}{p}\right|^2 (x^2 + y^2)(\Gamma t)^2 \tag{4.1}$$

This means we should be able to extract the mixing parameters with great precision from a time-dependent measurement of $r(t)$ — the measurement is *theoretically clean.* At many flavor factories like BaBar, a D^0 is produced from the decay $D^{*+} \to \pi_s^+ D^0$, where the π_s^+ is a "slow" pion. Detection of the "WS-pair" $\pi_s^+ l^-$ is an unambiguous mixing signature.

As promising as that sounds, this channel has serious drawbacks.

1. It is impossible to determine x and y directly, only their sum in quadrature.

2. The presence of the undetected neutrino in the final state complicates the measurement.

3. The most serious disadvantage is the very fact that this measurement is so clean. Since there are no other terms in the rate, it goes as $\propto (\text{mix})^2$. But the mixing amplitude in the D-system is so tiny that actually performing this measurement with good statistics becomes very difficult. As a result, it is not the favored discovery channel for D-mixing.

We need some way of enhancing the tiny mixing signature...

4.2 Measuring Mixing using Wrong-Sign $K\pi$ Final State

Consider the RS process $D^0 \to K^- \pi^+$. This is CF and has a large branching ratio, which might lead one to suspect this would be a good way to measure the mixing. Not so. The decay occurs too rapidly to discern the very slow oscillation, which has to be followed by a slow DCS decay (with about $\tan\theta_c \approx 0.3\%$ the rate of the CF decay):

$$\begin{aligned} \text{Amplitude} \quad = \quad & D^0 \xrightarrow{\text{CF}} K^- \pi^+ && \text{(large)} \\ & + \\ & D^0 \xrightarrow{\text{mix}} \overline{D}^0 \xrightarrow{\text{DCS}} K^- \pi^+ && \text{(tiny)(tiny)} \end{aligned}$$

Speaking more quantitatively, we can expand the time-dependent decay rate (similar to eq. (2.17), but this time for the RS process) for small $x, y \ll 1$ to obtain a simple polynomial time-dependence:

$$\Gamma(D^0 \to K^-\pi^+) = |A_{K\pi}|^2 e^{-\Gamma t}\left(1 + \left|\frac{q}{p}\right| r_{K\pi}\left[x\sin(\delta_{K\pi}-\phi) + y\cos(\delta_{K\pi}-\phi)\right](\Gamma t) + \ldots\right) \tag{4.2}$$

$$\begin{aligned} A_{K\pi} &= A(D^0 \to K^-\pi^+) \\ -r_{K\pi}e^{-i\delta_{K\pi}} &= \frac{A(\overline{D}^0 \to K^-\pi^+)}{A(D^0 \to K^-\pi^+)} = \frac{A(D^0 \to K^+\pi^-)}{A(D^0 \to K^-\pi^+)} = \frac{\text{WS}}{\text{RS}} \end{aligned} \tag{4.3}$$

(There is effectively no "CPV in decay" for this process.) Note the presence of the strong phase $\delta_{K\pi}$, which arises due to the interference of two amplitudes and has to be measured separately (see below). We might think that the interference term, linear in t, is exactly the enhancement of the mixing signal that we are looking for, since it multiplies the small mixing contribution by a large CF-amplitude. However, even so the linear term already is extremely small, suppressed by both the small mixing ($x, y \ll 1$) *and* the small DCS decay amplitude ($r_{K\pi} \ll 1$).

$$\Gamma \propto (\text{CF})^2 + 2(\text{CF})(\text{DCS})(\text{mix}) + (\text{DCS})^2(\text{mix})^2 \tag{4.4}$$

The time-dependent term of the rate is so negligible compared to the constant that this process can only be used to determine $A_{K\pi}$. So while we did get an enhancement of the mixing signal, it was drowned out by an overwhelming time-constant background.

On the other hand, all contributions to the WS mode $D^0 \to K^+\pi^-$ are roughly of the same order, so no one term is drowned out:

$$\begin{aligned} \text{Amplitude} \;=\; & D^0 \xrightarrow{\text{DCS}} K^+\pi^- && \text{(tiny)} \\ & + \\ & D^0 \xrightarrow{\text{mix}} \overline{D}^0 \xrightarrow{\text{CF}} K^+\pi^- && \text{(tiny)} \end{aligned}$$

Crucially, the DCS amplitude is still *larger* than the mixing one, meaning that the interference term (DCS)(mix) is *larger* than (mix)^2 and hence easier to detect! That is what makes this mode a preferred discovery channel for D-mixing, and it is to be compared to the theoretically appealing but practically unfavorable wrong-sign semi-leptonic final state, where we only had the tiny (mix)^2 contribution.

The expression for the WS time-dependent decay rate eq. (2.17) is

$$\Gamma(D^0 \to K^+\pi^0) = |A_{K\pi}|^2 r_{K\pi}^2 e^{-\Gamma t}\left[1 + \left|\frac{q}{p}\right| \frac{y'_{K\pi}\cos\phi - x'_{K\pi}\sin\phi}{r_{K\pi}}(\Gamma t) + \left|\frac{q}{p}\right|^2 \frac{{x'_{K\pi}}^2 + {y'_{K\pi}}^2}{4r_{K\pi}^2}(\Gamma t)^2\right]$$

$$x'_{K\pi} = x\cos\delta_{K\pi} + y\sin\delta K\pi \qquad\qquad y'_{K\pi} = y\cos\delta_{K\pi} - x\sin\delta K\pi \tag{4.5}$$

Careful time-dependent measurement of this process allows determination of $\left|\frac{q}{p}\right|, x'_{K\pi}, y'_{K\pi}$ and, if it were large enough, ϕ. Keep in mind that true determination of x, y relies on separate measurement of the strong phase $\delta_{K\pi}$.

Discovery of D-$\overline{D}$-mixing at BaBar

D-mixing was discovered just recently, first by BaBar in March 2007 [7,8], followed later in the year by the BELLE and CDF collaborations [9–11]. The original BaBar analysis [7] used time-dependent measurements of the WS decay $D^0 \to K^+\pi^-$, where the D^0 was identified by its production via $D^{*+} \to \pi_s^+ D^0$. They fitted the time-dependence to eq. (4.2), assuming CP conservation,

$$\frac{\Gamma(D^0 \to K^+\pi^0)}{e^{-\Gamma t}} \propto 1 + \left|\frac{q}{p}\right| \frac{y'_{K\pi}}{r_{K\pi}}(\Gamma t) + \left|\frac{q}{p}\right|^2 \frac{{x'_{K\pi}}^2 + {y'_{K\pi}}^2}{4r_{K\pi}^2}(\Gamma t)^2, \tag{4.6}$$

allowing the extraction of $y\prime$ from the linear term and x' from the quadratic term. CP-violation is expected to be extremely small, but they checked for its effect by fitting the above function to $D^0 \to K^+\pi^-$ and $\overline{D}^0 \to K^-\pi^+$ decays separately, and found no evidence for $\phi \neq 0$. Their results for the mixing parameters are

$$\begin{aligned} r_{K\pi}^2 &= (3.03 \pm 0.16 \pm 0.10) \times 10^{-3}, \\ {x'}^2 &= (-0.22 \pm 0.30 \pm 0.21) \times 10^{-3}, \\ {y'}^2 &= (9.7 \pm 4.4 \pm 3.1) \times 10^{-3}, \end{aligned}$$

where the errors are statistical and systematic, respectively. Note that they allowed for unphysical negative values of ${x'}^2$ in their fit. The systematic error in their y' measurement was significantly reduced in a subsequent analysis assuming CP-conversion [8],

$$y' = (12.4 \pm 3.9 \pm 1.3) \times 10^{-3},$$

constituting discovery of mixing at the 3σ level. Taking together all the current BaBar, BELLE and CDF measurements excludes the no-mixing scenario at 9.8σ [6].

Measuring the Strong Phase at CLEO-c

CESR and CLEO-c have a unique feature enabling them to measure the $\delta_{K\pi}$ strong phase. They collide e^+e^- near the $\Psi(3770)$-resonance at $\sqrt{s} = 3.77\,\text{GeV}$. The produced $\Psi(3770)$ is a $CP = -1$ eigenstates, and decays almost exclusively into $D^0\overline{D}^0$ in a CP-odd state. The two D's are quantum-mechanically entangled, and if one decays into a CP-eigenstate, we immediately know the CP-eigenstate of the surviving D, at which point it's "clock starts ticking" and we can get very precise measurements of decay rates for different CP-eigenstates of D. (Note that this method implicitly assumes CP-conservation, which according to BaBar etc. measurements is a justified approximation.) This is the D-system equivalent of the $\Upsilon(4S) \to B\overline{B}$ process at B-factories.

Let us illustrate this with an example. Define the two CP-eigenstates with CP-eigenvalues $\eta = \pm 1$ to be

$$D_\pm^0 = \frac{1}{\sqrt{2}}\left(D^0 \pm \overline{D}^0\right) \tag{4.7}$$

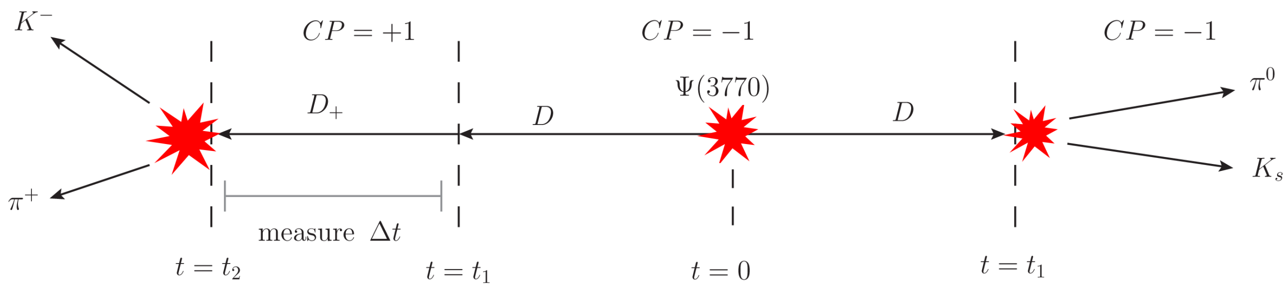


Figure 7: A $\Psi(3770)$ decays at time $t = 0$ into a $D - \overline{D}$-pair which is quantum-mechanically entangled because the total CP-eigenvalue has to be -1. If at time $t = t_1$ one of the daughters decays into a CP-odd state like $\pi^0 K_s$, that immediately collapses the other $D's$ wave function to the CP-*even* D^0_+. At that point we start our clock and measure the decay-time Δt between the collapse of the D^0_+ wave function until its decay.

Now consider the process shown in figure fig. 7. As soon as one of the D's decays into, say, a CP-odd state like $\pi^0 K_s$, the surviving D is immediately *tagged* as D^0_+, at which point we start our clock and measure its decay, in this example to the highly interesting WS state $K^-\pi^+$. That way, we can measure $A(D^0_+ \to K^-\pi^+)$ and $A(D^0_- \to K^-\pi^+)$ separately, from which we can construct the separate decay rates for D^0 and $\overline{D}^0$, since

$$A(D^0_\pm \to K^-\pi^+) \quad = \quad \frac{1}{\sqrt{2}} \left[A(D^0 \to K^-\pi^+) \pm A(\overline{D}^0 \to K^-\pi^+) \right] \tag{4.8}$$

By comparing the different branching ratios for CP-eigenstates we can extract the strong phase $\delta_{K\pi}$:

$$\begin{aligned} 1 \pm 2 r_{K\pi} \cos\delta_{K\pi} &= 2 \frac{B(D^0_\pm \to K^-\pi^+)}{B(D^0 \to K^-\pi^+)} \\ \Longrightarrow \cos\delta_{K\pi} &= \frac{1}{2 r_{K\pi}} \frac{B(D^0_+ \to K^-\pi^+) - B(D^0_- \to K^-\pi^+)}{B(D^0_+ \to K^-\pi^+) + B(D^0_- \to K^-\pi^+)} \end{aligned} \tag{4.9}$$

The measurement for this and other final states was performed at CLEO-c [12], with the result

$$\cos\delta_{K\pi} = 1.03^{+0.31}_{-0.17} \pm 0.06. \tag{4.10}$$

Current State of Mixing Parameter Measurements

Putting everything together, the 2008 PDG [13] quotes the current best values for the D^0-$\overline{D}^0$ mixing parameters as

$$
\begin{aligned}
x &= \left(9.72^{+2.71}_{-2.91}\right) \times 10^{-3} \\
y &= \left(7.8^{+1.8}_{-1.9}\right) \times 10^{-3} \\
\left|\frac{q}{p}\right| &= 0.86 \pm 0.31 \\
\cos\delta_{K^+\pi^-} &= 1.03^{+0.32}_{-0.18}
\end{aligned}
$$

5 Conclusion

We reviewed the theory and current experimental status of D-$\overline{D}$-mixing, and discussed its special status as the only portal to potential CP-violation and FCNCs in the up-sector. Current experimental results are still of low precision, but we can say that mixing has been unequivocally discovered. It remains to be seen whether future results reveal significant deviations from the Standard Model, or whether calculational advances (e.g. using lattice QCD) allow us to improve the crude SM prediction and glean more information from the data.

Acknowledgements

I would like to thank Anders Ryd for helpful discussion.

A Linear Algebra Facts

Let

$$H = \begin{pmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{pmatrix} \tag{A.1}$$

be an arbitrary complex 2×2 matrix. Also define

$$T = \begin{pmatrix} q\sqrt{1+z} & p\sqrt{1-z} \\ -q\sqrt{1-z} & p\sqrt{1+z} \end{pmatrix}. \tag{A.2}$$

Then

$$THT^{-1} = \begin{pmatrix} \omega_L & \\ & \omega_H \end{pmatrix} \tag{A.3}$$

if

$$\frac{q}{p} = -\sqrt{\frac{h_{21}}{h_{12}}} \tag{A.4}$$

$$z = \frac{h_{22} - h_{11}}{\omega_H - \omega_L} \tag{A.5}$$

$$\omega_{L,H} = \frac{1}{2}\left[h_{11} + h_{22} \mp \sqrt{(h_{11} - h_{22})^2 + 4h_{12}h_{21}}\right] \tag{A.6}$$

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