BSM Journal Club 3/15/2010 David Curtin

Complete Mass Determination at Hadron Colliders

References:

hep-ph/0304226 : old-skool M_{T2} review.

0711.4526 Cho et.al : analytical expressions for M_{T2} event-by-event w/o ISR, M_{T2} edges

0810.5576 Matchev et.al : good review of non-M_{T2} methods, definition of M_{T2} subsystem vars, detailed derivation for various endpoints & kinks of $M_{T2}^{(n,p,c)}$ w/ & w/o ISR (massless SM FS & sym. decay chains only)

0910.3679 Matchev et.al : good review of M_{T2} properties, definition of $M_{T2\perp}$, $M_{T2\parallel}$, method for using $M_{T2\perp}$ for full mass determination of 1-step decay chain (with ISR).

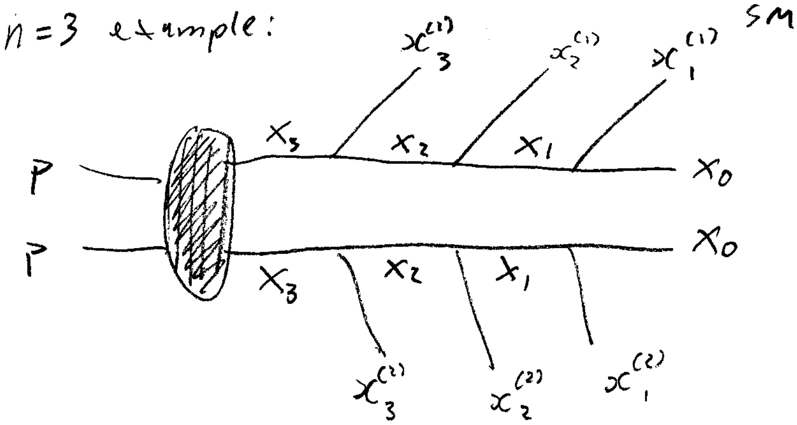
Outline

1. Review of older Methods
2. M_{T2} review
3. Complete Mass determination with M_{T2}
   - 3.1 2-step decay chain : $M_{T2}^{(2,1,0)}$
   - 3.2 1-step decay chain : $M_{T2\perp}$

1. Review of older Methods

(mostly Matchev 2008)

$n=3$ example:



Problem: X_0 are invisible.

We want to measure the masses of X_3, X_2, X_1

For long decay chains ($n \geq 3$) we can use Endpoint or Polynomial methods.

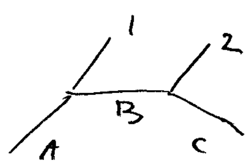
I) Endpoint Method

- we can form invariant mass distributions $M^2_{x_{i_1} x_{i_2} \ldots x_{i_k}} = \left(\sum p_{i}\right)^2$

(The longer the chain, the more distributions we can form $\Rightarrow$ more information!)

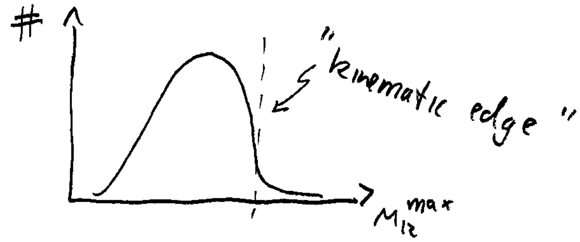
- These distributions have endpoints which depend on the BSM masses

↳ eg.



For $m_1 = m_2 \approx 0$,

$$M_{12}^{max} = \frac{(m_A^2 - m_B^2)(m_B^2 - m_C^2)}{m_B^2}$$



- When $n \geq 3$, we can measure enough independent endpoints to determine the BSM masses entirely!

2. Polynomial Method

- Try to solve system completely.

  knowns : 4-momenta of SM decay products (8n)

  unknowns : LSP momenta (8), BSM masses (n+1)

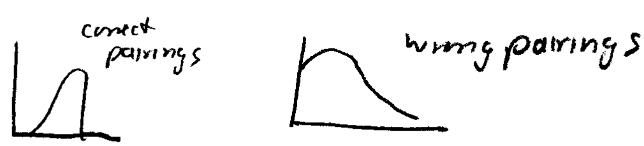
- can also play with simultaneously analyzing several events (masses are "common" unknowns)

- can also be used to extract all masses for $n \geq 3$

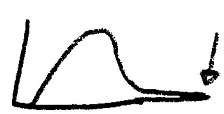
Some common issues

- Combinatorics errors : need to be able to correctly "place" each daughter particle in the decay chain

  ↳ not too bad for endpoint method :



  ↳ Dalitz plots can help! (don't yet know how though :))

- In general, measuring edges/endpoints is problematic
  - ↳ when you can't fit to a shape, endpoint info is entirely carried by small # of events → STATISTICS
  - ↳ what if you have an endpoint like 

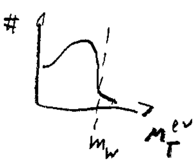
- error propagation! (complicated expressions).

MT2 Review

hep-ph/0304226, Cho '07

For two particles, we can define transverse mass

eg W mass measurement from $W \to e\nu$



$$m_T^2 = \left(p_1^T + p_2^T\right)^2 \le s_{12} = (p_1 + p_2)^2$$

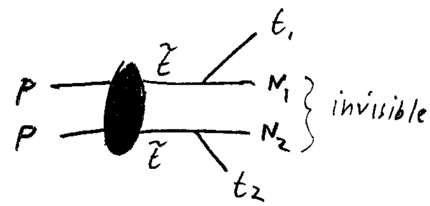
where $p_T = (E_T, \vec{p}_T)$ and $E_T = \sqrt{m^2 + \vec{p}_T^{\,2}}$

similarly for more particles:

$$M_T(p_1, \dots, p_k) = \left(\sum_{i=1}^{k} p_T\right)^2 \le s_{12\dots k} = \left(\sum p\right)^2$$

Original M_{T2} Variable

- Consider pair production of heavy particle which decays into two identical SM + invis.



- Can we construct some useful observable which carries information on masses $m_{\tilde{t}}$, m_N?

(1) If we knew $p_{N_1}^T$, $p_{N_2}^T$, then $\max\{M_T^{(1)}, M_T^{(2)}\} \le m_{\tilde{t}}$ would give the best estimate (highest lower bound) of $m_{\tilde{t}}$

(2) However, we only know total $\not{p}_T$ $\Rightarrow$ MINIMIZE the above wrt all possible splittings of $\vec{\not{p}}_T = \vec{p}_{N_1}^T + \vec{p}_{N_2}^T$ to get the MOST CONSERVATIVE $m_{\tilde{t}}$ LOWER BOUND (ie necessarily not incorrect).

$$\Rightarrow \min_{\vec{\not{p}}_T = \vec{p}_{N_1}^T + \vec{p}_{N_2}^T} \left\{ \max\{M_T^{(1)}, M_T^{(2)}\} \right\} \le m_{\tilde{t}}$$

(can show that sometimes have equality)

(3) But we don't even know invisible mass! $\Rightarrow$ Must use a testmass χ:

$$\boxed{M_{T2}^2(\vec{p}_{t_1}^T, \vec{p}_{t_2}^T, \chi) = \max_{\vec{q}_1^T + \vec{q}_2^T = \vec{\not{p}}^T} \left[\max\left\{m_T^2(\vec{p}_{t_1}^T, \vec{q}_1^T, \chi),\ m_T^2(1 \to 2)\right\}\right]}$$

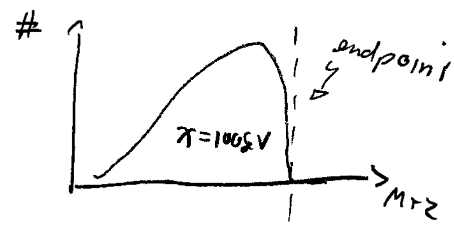
- $M_{T2}(\chi)$ distribution always has an endpoint. When $\chi = m_N$, that endpoint is $m_{\tilde{\ell}}$:

$$\boxed{\begin{aligned} M_{T2\,max}(\chi) &= \max_{\text{(all events)}} \{ M_{T2} \} \\ &= m_{\tilde{\ell}} \quad \text{when } \chi = m_N \end{aligned}}$$

Simple & Robust usage (works even when your sample includes ISR)

1) Pick a testmass χ. Compute M_{T2} for each event, plot distribution:

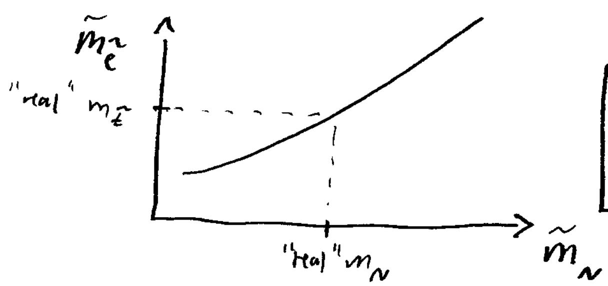
this is computationally intensive!!



2) Measure endpoint: gives $M_{T2\,max}(\chi)$ for that testmass.

3) Repeat for many different testmasses, building up list of $M_{T2\,max}(\chi)$

4) Plot $m_{\tilde{\ell}}$ "as a fn of m_N":

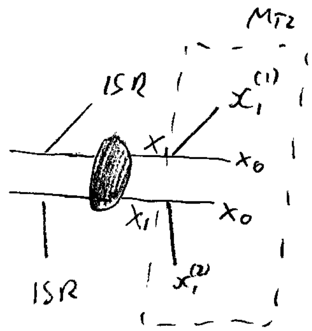


$\boxed{\Rightarrow \text{roughly speaking gives value of } m_{\tilde{\ell}}^2 - m_N^2}$

5) Measure m_N somehow $\Rightarrow$ get $m_{\tilde{\ell}}$

Remark about KINKS

- We did not consider ISR.

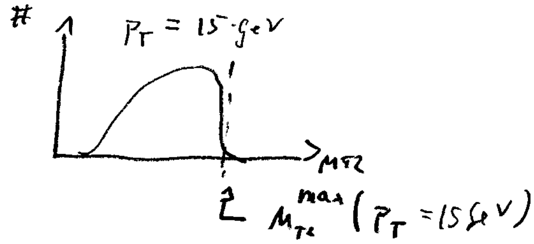


ISR can give a transverse boost to our M_{T2}-system!

To consider $\vec{p}_T^{\,ISR}$ dependence:

1) Say we can distinguish ISR jets from interesting stuff.
2) Put events into different $|\vec{p}_T^{\,ISR}|$ bins.
3) For each bin, numerically calculate M_{T2} distribution:

eg

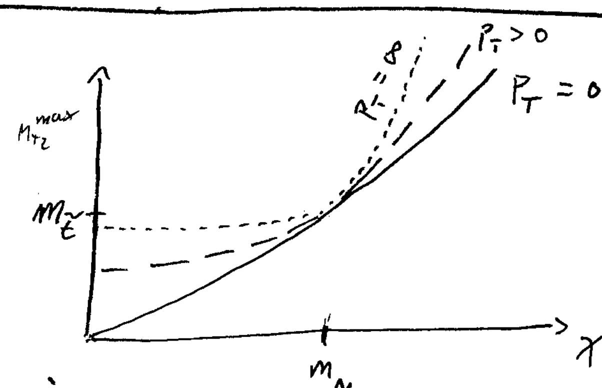


$\Rightarrow$ $\boxed{\text{get } M_{T2}^{max} \text{ as a function of } P_T^{ISR}}$

Can show that

$$\boxed{M_{T2}^{max}(P_T, \chi) \geq M_{T2}^{max}(0, \chi) \text{ with equality when } \chi = m_N}$$

- This leads to the famous MT2-KINK:



$\Rightarrow$ If we can measure this kink position, we can find $m_{\tilde{E}}$, m_N independently!

PROBLEM: Would have to construct M_{T2} distributions seperately for each P_T^{ISR}-bin $\Rightarrow$ very poor statistics (few events/bin)
The edges are VERY slight ($\sim O(1\%)$ gradient change), so we'd need extremely high precision measurements
$\Rightarrow$ Practically impossible

- M_{T2}-subsystem vars have other kinks too: general feature when we have up stream transverse momentum. Some might be more pronounced & measurable.
- Even though kinks are not useful, the mathematical property ⊛ which causes them is! (see later)

If you have enough $p_T^{ISR} = 0$ events, the analysis becomes simpler:

- The minimization required to compute M_{T2} for a given event is very complicated, but Cho found an analytical expression (that magnificient bastard!) for $p_T^{ISR} = 0$.

$$\boxed{M_{T2}^2 = m_N^2 + A_T + \sqrt{\left(1 + \frac{2m_N^2}{A_T - m_t^2}\right)\left(A_T^2 - m_t^4\right)} \qquad \text{when} \quad A_T = E_{t_1}^T E_{t_2}^T + \vec{p}_{t_1}^{\,T} \cdot \vec{p}_{t_2}^{\,T}}$$

- Can also predict the <u>endpoint</u> of the distribution:

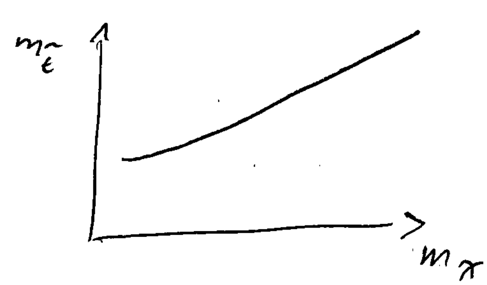
$$\boxed{M_{T2}^{max} = \frac{m_{\tilde t}^2 - m_{\tilde\chi^0}^2 + m_t^2}{2\, m_{\tilde t}^2} + \sqrt{m_N^2 + \frac{\left[(m_{\tilde t} + m_t)^2 - m_{\tilde\chi^0}^2\right]\left[(m_{\tilde t} - m_t)^2 - m_{\tilde\chi^0}^2\right]}{4\, m_{\tilde t}^2}}}$$

$\hookrightarrow$ generalization to $p_T^{ISR} \neq 0$ exists

- Hence do the following:

  1) Calculate M_{T2} for each event with <u>zero</u> testmass & plot distribution

  2) Extract endpoint from distribution. Using above analytical expression, this defines $m_{\tilde t}$ as a fn of $m_{\tilde\chi}$

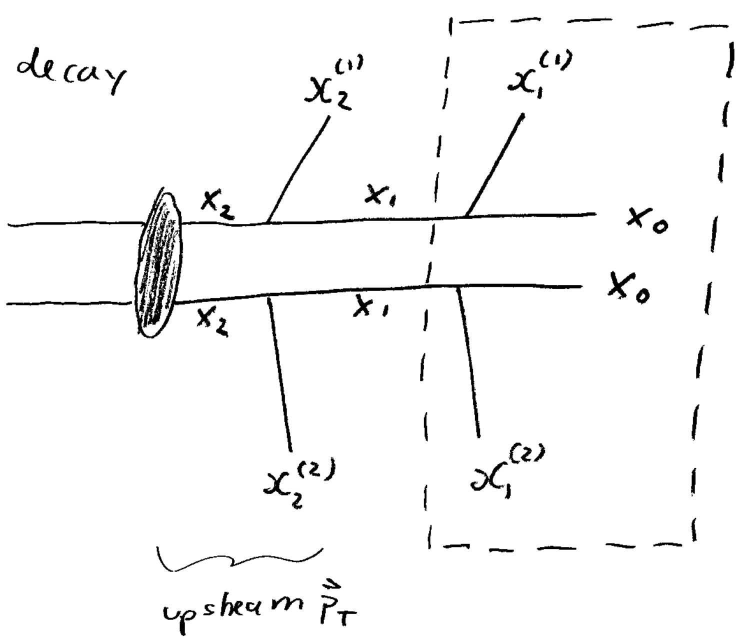


- This method of doing analyses not very useful in practise since you loose all your $p_T^{ISR} \neq 0$ events, but we will use these eqns <u>later</u> when we define $M_{T2\perp}$!!

3. Complete Mass Determination with M_{T2}

3.1 2-step Decay Chain: $M_{T2}^{(2,1,0)}$

Consider the decay



form $M_{T2}^{(2,1,0)}$ subsystem variable: exactly like normal M_{T2}, except always have lots of upstream transverse momentum

(Notation:
$M_{T2}^{n,p,c}$
n = length of chain
p = parent
c = child)

Due to upstream $\vec{p}_T$, we cannot use analytical formula to calculate $M_{T2}^{(2,1,0)}$ for each event $\Rightarrow$ must numerically perform minimization

$$\boxed{M_{T2}^{(n,p,c)} = \min_{\substack{\text{total} \\ \text{child } \vec{p}_T}=\substack{\text{total} \\ \text{upstream} \\ \vec{p}_T}} \left\{ \max\left[M_T^{(1)}, M_T^{(2)} \right] \right\}}$$

However, we <u>can</u> analytically predict the <u>endpoint</u> of $M_{T2}^{(2,1,0)}$ for a given test mass χ.

(For simplicity, assume SM decay products are massless, ie no tops. Can presumably generalize fairly easily)

$\rightarrow$ For $\vec{p}_T^{\,ISR} = 0$ and $m^i_{SM} = 0$ (can generalize)

$$M^{(2,1,0)}_{T2\,max}(\chi) = \begin{cases} F_L^{(2,1,0)}(\chi) & \text{for } \chi < m_{\chi_0} \\ F_R^{(2,1,0)}(\chi) & \chi > m_{\chi_0} \end{cases}$$

Note:

$$\mu_{npc} = \frac{M_n}{2}\left(1 - \frac{M_c^2}{M_p^2}\right)$$

where
$$F_L^{(2,1,0)}(\chi) = \left\{ \left[\mu_{2,2,0} - \mu_{2,2,1} + \sqrt{\mu^2_{2,2,0} + \chi^2} \right]^2 - \mu^2_{2,2,1} \right\}^{1/2}$$

$$F_R^{(2,1,0)}(\chi) = \left\{ \left[\mu_{2,1,0} + \sqrt{(\mu_{2,2,1} - \mu_{2,1,0})^2 + \chi^2} \right]^2 - \mu^2_{2,2,1} \right\}^{1/2}$$

This looks troublesome: not knowing what m_{χ_0} is, how do we know which branch to use for a given test mass χ?

$\rightarrow$ <u>IN FACT, IT IS AWESOME!</u> We can extract <u>3</u> endpoints from this one $M_{T2}^{2,1,0}$ variable:

1) Set $\chi = 0$: certainly selects lower branch: $M^{(2,1,0)}_{T2\,max}(0) = 2\sqrt{\mu_{220}(\mu_{220} - \mu_{221})}$

2) Set $\chi = E_b$ (beam energy). Certainly selects <u>upper</u> branch:

$M^{(2,1,0)}_{T2\,max}(E_b) = F_R^{(2,1,0)}(E_b)$ $\leftarrow$ χ-dependent part is composed of <u>two</u> mass combinations: μ_{210} and $(\mu_{221} - \mu_{210})$

$\Rightarrow$ a <u>second</u> measurement of this edge with different χ will actually reveal <u>new</u> information!

$\Rightarrow$ 3) set $\chi = E_b' > E_b$ and get $M_{T2}^{210}(E_b') = F_R^{210}(E_b')$

$\Rightarrow$ Get three edges, each an independent combination of M_1, M_2, M_3

$\Rightarrow$ $\boxed{\text{using a } p_T^{ISR}=0 \ M_{T2}^{(2,1,0)} \text{ measurement, we can uniquely solve for all three masses } M_1, M_2, M_3}$

This is due to the two-branch structure of $M_{T2}^{2,1,0}$. Other M_{T2}-subsystem variables also have similar features $\rightarrow$ VERY POWERFUL!

Remarks

1) p_T^{ISR}-dependence complicates things. The above eqns generalize to $p_T^{ISR} \neq 0$, but analysing each p_T-bin seperately reduces statistics and might make this particular analysis impossible in practise.

(Damn you p_T^{ISR}!! *shakes fist*)

2) However, the "simple & robust" analysis we did with simple M_{T2} applies to subsystem variables too: we could simply throw all the events together and build M_{T2}-subsystem distributions for different test masses. Then each subsystem variable would only give us one measurement, but we can define several subsystem variables for each decay chain

$\Rightarrow$ might still be able to extract a lot of mass information

3) Might have to worry about combinatorics errors & being able to correctly assign each particle to its place in the "upper" or "lower" decay chain (depends on subsystem variable).

4) Error propagation: if we measure some complicated fn of the masses, the determination of the masses themselves might suffer from large uncertainties.

Well, that all sounds great, but at this point M_{T2} still faces two main problems in using it for complete mass determination at hadron colliders:

1) We can do a lot of cute tricks when we have a large $p_T^{ISR}=0$ sample. In practise, that would reduce our statistics a lot at a hadron collider.

$\Longrightarrow$ We want to use these powerful methods on the whole event sample, including with ISR.

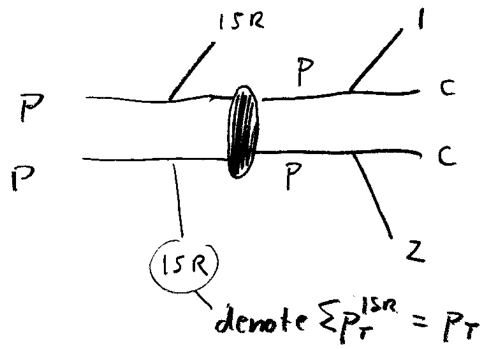
2) Extracting endpoint of any distribution, including M_{T2}'s, relies on a small # of events at the tail.

↳ no fully well-defined method for endpoint extraction exists when you have BG too, people use "empirical" fit functions

↳ If we knew the shape of the distribution, we could fit a fn to it and extract the endpoint very reliably $\Longleftrightarrow$ the whole distribution (all events) would carry information, not just the tail (few events)

3.1 1-Step Decay Chain: $M_{T2\perp}$ (0910.3679 Matchev et al.)

- This will solve both problems outlined on previous page!!
- Consider the shortest possible decay chain:



(Again for simplicity assume massless SM decay products. The results can be generalized.)

- In theory, M_{T2} can reveal full mass information for P & C if we could measure the kink, but this is impossible in practice.

  ↳ We need some way to make use of the kink property that is able to utilize the whole data sample, not just a single p_T^{ISR}-bin. (ie solve problem 1 on p 11)

- Consider the following corollary of the kink property:

Defining $\tilde{M}_P(\tilde{M}_c, p_T) \equiv M_{T2}^{max}(\tilde{M}_c, \underset{\text{ISR}}{p_T})$

we notice that $\tilde{M}_P$ is p_T-independent only if $\tilde{M}_c = M_c$, in which case $\tilde{M}_P = M_P$.

$$\Rightarrow \boxed{\begin{array}{l}\tilde{M}_P(\tilde{M}_c, p_T) - \tilde{M}_P(\tilde{M}_c, 0) \geq 0 \\ \text{with equality when } \tilde{M}_c = M_c\end{array}} \quad \circledast$$

- Using this, assuming we can reliably determine $\tilde{M}_P(\tilde{M}_c, 0)$ (more on this later), we can define a function of $\hat{M}_c$ which has a MINIMUM at $\hat{M}_c = M_c$:

$$N(\tilde{M}_c) = \sum_{\substack{\text{all} \\ \text{events}}} H\left(M_{T2}(\tilde{M}_c) - \tilde{M}_P(\tilde{M}_c, 0)\right)$$

<u>Explanation:</u> Consider some event with $M_{T2}(\hat{M}_c, P_T)$.
It certainly has $M_{T2} \leq M_{T2}^{max}(\hat{M}_c, P_T)$
but since $M_{T2}^{max}(\hat{M}_c, 0) < M_{T2}^{max}(\hat{M}_c, P_T)$ for $\hat{M}_c \neq M_c$,
it <u>can</u> contribute $+1$ to $N(\hat{M}_c)$.
If $\tilde{M}_c = M_c$, then $M_{T2}^{max}(M_c, P_T) = M_{T2}^{max}(M_c, 0) \geq M_{T2}$
and $N = 0$ → MINIMUM

↳ In practice not zero due to detector effects etc, but <u>location</u> of minimum at $M_c = \tilde{M}_c$ should be robust.

⇒ We could then just plot $N(\tilde{M}_c)$, look for the minimum and find M_c! Then we can find M_P using usual M_{T2} methods.

So now the real problem is determining the function $\tilde{M}_P(M_c, 0)$ with the greatest possible precision, otherwise we cannot reliable find the minimum of N.

Problem 1!!

Find $\tilde{M}_P(M_C, 0)$

- The p_T^{ISR}-dependence of M_{T2} makes this challenging: we could just use $p_T = 0$ events and plot M_{T2} distributions, but then we are again have the problem of reduced statistics.

$\Rightarrow$ We need to project out the p_T^{ISR}-dependence!

Then we can use the whole data sample.

- Define new variable $M_{T2\perp}(\tilde{M}_c)$: exactly like M_{T2}, except replace all $\vec{p}_i^{\,T} \to \vec{p}_{i\perp}^{\,T}$ (component of $\vec{p}_i^{\,T}$ $\perp$ to $\vec{p}_{ISR}^{\,T}$ the event)

$\hookrightarrow$ (also on transverse energies)

$\hookrightarrow$ the minimization is now $\sum p_{T\perp} = 0$, ie INDEPENDENT OF ISR p_T !!

(this is where ISR p_T would show up)

$\Rightarrow$ $\boxed{\text{can use existing analytical formulas to calculate } M_{T2\perp} \text{ for each event}}$

- Can also predict its endpoint:

$$M_{T2\perp}^{max}(\tilde{M}_c) = \mu + \sqrt{\mu^2 + \tilde{M}_c^2}$$

where $\mu = \frac{M_P}{2}\left(1 - \frac{M_c^2}{M_P^2}\right)$

$\hookrightarrow$ note independent of p_T^{ISR}! $\Rightarrow$ $\boxed{\text{Solves problem 1}}$

$\hookrightarrow$ If we can measure μ somehow, then we have $M_{T2\perp}^{max}(\tilde{M}_c) = M_{T2}^{max}(\tilde{M}_c, 0) = \tilde{M}_P(\tilde{M}_c, 0)$, ie we are done

$\Rightarrow$ Find $\tilde{M}_p(\tilde{M}_c, 0)$ = Find μ

- MAIN POINT: For $M_{T2\perp}$, we can predict not only its <u>endpoint</u> analytically, but also the <u>shape</u> of the distribution!

$$\frac{dN}{dM_{T2\perp}} = \frac{M_{T2\perp}^4 - \tilde{M}_c^4}{\mu^2 M_{T2\perp}^3} \ln\left(\frac{2\mu M_{T2}}{M_{T2\perp}^2 - \tilde{M}_c^2}\right)$$

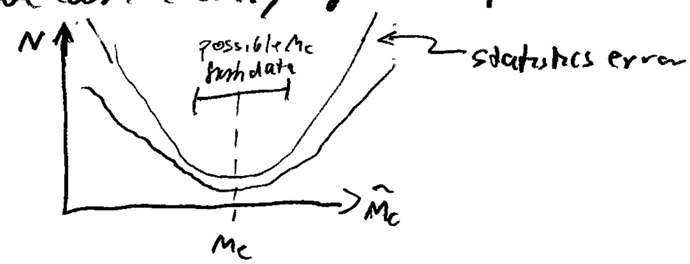
$\Rightarrow$ We can construct $M_{T2\perp}$ distribution for an <u>arbitrary</u> $\tilde{M}_c$, fit the above fn and extract the parameter μ

$\hookrightarrow$ THE WHOLE <u>SHAPE</u> CONTAINS μ-INFO, NOT JUST THE ENDPOINT, and we can use <u>all</u> events

$\Rightarrow$ avoid both edge-measurement problem due to p_T-dep. AND small-statistics problem

Solves Problem 2

- Hence we can easily get $\tilde{M}_\phi(\tilde{M}_c, 0)$ and hence plot $N(\tilde{M}_c)$



with enough statistics, completely determine M_c and hence M_P

<u>Caveats</u>

- Cuts affect $M_{T2\perp}$-distribution-shape. Compensate, or get endpt the old way.
- might be difficult to get unique minimum of N due to statistical error.

BUT THINGS LOOK VERY GOOD FOR THE LHC!! ☺