BSM JC 11/22/10

Seiberg Witten Theory

$\mathcal{N}=2$ SU(2) SYM

<u>References</u>: • hep-th/9701069 (review)
- hep-th/9407087 (original paper by SW)
- Monopole Review by Preskill 1984
- Terning Ch 13

1 Brief Monopole Review

1.1 Preliminaries

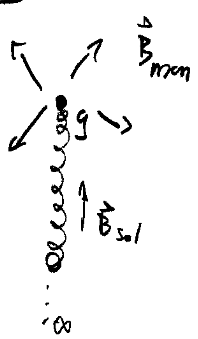
$U(1)$ gauge theory $\quad A_\mu(\phi, \vec{A})\quad , \quad F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu , \quad \tilde{F}_{\mu\nu} = \epsilon_{\mu\nu\rho\sigma} F^{\rho\sigma}$

Define electric & magnetic currents $\quad j^\mu = (\rho, \vec{j}) \quad k^\mu = (\sigma, \vec{k})$

$\Rightarrow$ Maxwell Eqns $\quad \partial_\nu F^{\mu\nu} = -j^\mu \quad , \quad \partial_\nu \tilde{F}^{\mu\nu} = -k^\mu$

are invariant under
$$\left.\begin{aligned}(F, j) &\to (\tilde{F}, k)\\ (\tilde{F}, k) &\to -(F, j)\end{aligned}\right\}$$
actually $SO(2)$ symmetry: $\boxed{\text{E-M DUALITY}}$

1.2 Monopoles $\Rightarrow$ Charge Quantization



- Imagine monopole @ origin. To satisfy $\vec{\nabla} \cdot \vec{B} = 0$ need to introduce Dirac string (∞ solenoid) so that the singularities in A^μ cancel

$$\vec{B} = \frac{g}{4\pi r^2} + g\,\Theta(-z)\,\delta(x)\,\delta(y)\,\hat{z}$$

- position of singularity in $A^\mu \leftrightarrow$ orientation of Dirac String is GAUGE DEP.
  - ↳ observable consequence: Aharonov-Bohm Phase
    - ↳ REQUIRE TO VANISH: $\boxed{eg = 2\pi n}$

More generally for two dyons (q_i, g_i): $\quad \boxed{q_1 g_2 - q_2 g_1 = 2\pi n_{12}}$

Note: the above is what we call an ELEMENTARY MONOPOLE

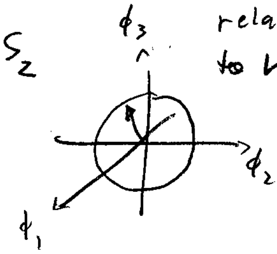
↳ "put in by hand" as opposed to a solitonic gauge field configuration (TOPOLOGICAL MONOPOLE, requires SSB)

↳ can have any mass (at least w/o SUSY)

1.3 Topological Monopoles: Georgi - Glashow Model

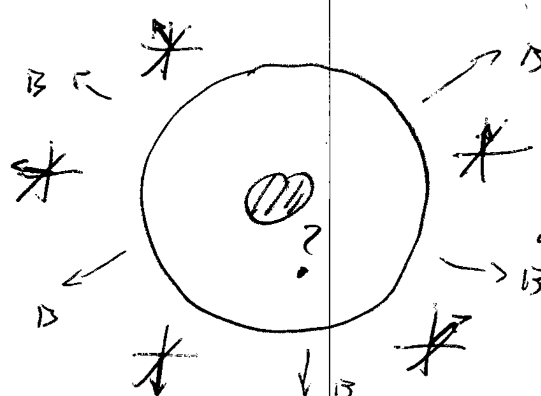
- topological monopoles can arise dynamically in theories with SSB.
  - ↳ solitonic field solutions, stable due to preserved topological charge

- GG Model: $SO(3) \longrightarrow SO(2) \sim U(1)$ by higgs $\phi^a \sim \square$
  - ↳ vacuum is defined by $\vec{\phi}_{vac} \cdot \vec{\phi}_{vac} = a^2$, $\underbrace{D_\mu \phi_{vac} = 0}$ relates gauge configurations to higgs configuration
    - ↳ $\phi_{vac} \in M$ = vacuum manifold $\simeq S_2$



$$\boxed{\begin{array}{l}\text{In general, for } G \to H \text{ SSB,} \\ \phi_{vac} \in M \simeq G/H\end{array}}$$

- topological monopole solution:
  - ↳ define S_2 @ $\infty = \Sigma$. On Σ, fields take vacuum configuration $D\phi = 0$, $\phi^2 = a$
  - ↳ $\phi \in M$ can vary across Σ. Some configurations have nonzero **WINDING NUMBER** and are stable with nonzero magnetic charge



higgs configuration $\in M$ varies along

$$g_\Sigma = \int_\Sigma B^i ds^i = -\frac{1}{2\pi e a^3} \int_\Sigma \epsilon_{ijk} \vec{\phi}_{vac} \cdot (\partial^j \vec{\phi}_{vac} \times \partial^k \vec{\phi}_{vac}) ds^i = -\frac{4\pi}{e} \nu$$

winding # of higgs field config @ ∞

- ↳ ϕ_0 at ∞ defines a map: $\phi_0 : \mathcal{M} \to S_2$
  this map is a member of a certain HOMOTOPY CLASS (labelled by winding #)
  (two field configs are homotopic if they can be ctsly deformed into each other)

$$\boxed{\Pi_2(G/H) = \text{group of homotopy classes of these maps}}$$

- ↳ Specific Soln: t'Hooft-Polyakov Monopole, Julia-Zee Dyon
  - set g, q by $\int B, E$ over Σ
  - get M by $\int$ energy density
  - OBEY CHARGE QUANT.

1.4 The Bogomol'nyi Bound & BPS States

- applies to topological monopoles / dyons (more important in $N=2$ SUSY)
- can show that $M \geq M_B = a\sqrt{q^2+g^2}$ in GG model (for example)
  - ↳ t'Hooft Polyakov monopole $M \geq a\frac{4\pi}{e} = \frac{v}{\alpha} M_W$ ← HEAVY
  - ↳ $M = M_B$: "BPS States". $\boxed{\text{VERY important in } N=2 \text{ SUSY}}$

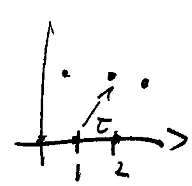
1.5 Miscellaneous Remarks

- Can show $\pi_2(G/H) \simeq \pi_1(H)_G$ = paths in H that, upon embedding in $G \supset H$, are deformable to the trivial path
  - ↳ Not all theories allow for topological dyons!

    Rule of thumb: if H does not have more U(1) factors than G, then $\pi_1(H)_G = 0$ → no top. monopoles
    - ↳ $\boxed{SU(2)+U(1) \to U(1) \text{ does NOT have top. mon.}}$ (only elementary, w/ Dirac strings)
- Charge quantization for color triplets is generalized to all our $\frac{1}{3}$-charges!
- Montonen Olive Conjecture:

electric		magnetic
H	⟷	$\check{H}$
e	⟷	$g \sim 1/e$
perturbative dof ($W^\pm$) / solitons	⟷	solitonic topological solns / pert. dof
Noether / topological currents	⟷	topological currents / Noether

- Useful for later: arrange allowed states in $(q+ig)$-plane



$\tau = \frac{\theta}{2\pi} + \frac{4\pi i}{e^2}$

$q + ig = e(n + m\tau)$

$M \geq M_B = \sqrt{2}\,|ae(n+m\tau)|$

2. Necessary $N=2$ SUSY Facts

- SUSY rep's w/o central charges:

$\underline{N=1}$ χSF (ψ, ϕ) : 4 dof $m\chi SF$: 4 dof

VSF (A_μ, λ) : 4 dof $mVSF$: 8 dof

$\underline{N=2}$: hyper $= \chi SF \oplus \overline{\chi SF}$: 8 dof

vector $= \chi SF \oplus VSF$: 8 dof. $\Big\}$ massless

massive : 16 dof

- $N=2$ algebra w/ central charge:

$$\{Q, \bar{Q}\} = 2\sigma P$$
$$\{Q, Q\} = 2\sqrt{2}\, \epsilon\, \epsilon\, Z$$
$$\{\bar{Q}, \bar{Q}\} = 2\sqrt{2}\, \epsilon\, \epsilon\, Z$$

single real central charge (JUST A NUMBER)

↳ NOTE: the eigenvalues of Z can be different for each particle !!!!

↳ Massive states must obey BPS bound $\boxed{M \geq 2|Z|}$

↳ If $M = 2|Z|$, the algebra looks like massless case → SHORT MULTIPLETS

↳ $M > 2|Z|$: LONG MULTIPLETS

$\boxed{\text{Short multiplets} = \text{BPS States}}$ → Their relationship between M and Z does NOT get modified by quantum corrections

↳ VERY IMPORTANT

↳ massive hypers have $|Z| = M/2$

↳ SUSY GG Model: $Z = a(q + ig) = a\,e(n + m\tau)$ is DYNAMICALLY generated

(calculated by solving field configs → calculating supercurrents in terms of fields → $\int$ to get $Q, \bar{Q}$ → get anticommutator → Z)

- $\mathcal{N}=1$: local reps & Lagrangian:

$$\mathcal{L} = \frac{1}{8\pi} \operatorname{Im}\Big(\underbrace{\tau \operatorname{Tr} \int d^2\theta\, W^\alpha W_\alpha}_{\text{gauge kinetic}} \Big) + \Big(\underbrace{\int d^2\theta\, W + hc}_{\text{superpot}} \Big) + \underbrace{\int d^4\theta\, \Phi e^{-2V} \Phi}_{\substack{\text{canonical} \\ \text{Kähler term}}}$$

$$V = \sum_i \frac{\partial W}{\partial A_i} - \frac{1}{2} g^2 \left(A^\dagger T^a A\right)^2$$

- $\mathcal{N}=2$ SYM Lagrangian (in $\mathcal{N}=1$ SF language)

non-canonical Kähler for χSF in Vector is enforced by $\mathcal{N}=2$ SUSY

↳ microscopic theory: $\mathcal{L}_{SYM} = \frac{1}{8\pi 2} \operatorname{Im} \operatorname{Tr} \left\{ \tau \left(\int d^2\theta\, W^\alpha W^\dagger_\alpha + 2 \int d^4\theta\, \Phi^\dagger e^{-2V} \Phi \right) \right\}$

with $V = -\frac{1}{2g^2} \operatorname{Tr}\left([A^\dagger, A]^2 \right)$ where A is the scalar in the χSF in the $\mathcal{N}=2$ Vector (so $\mathcal{N}=2$ SYM actually has a higgs that can SSB!)

↳ IN GENERAL, THEORY IS SPECIFIED BY $\boxed{\text{PREPOTENTIAL } \mathcal{F}}$

$$\mathcal{L}_{SYM} = \frac{1}{8\pi} \operatorname{Im}\left(\int d^2\theta\, \mathcal{F}_{ab}(\Phi) W^{a\alpha} W^b_\alpha + 2 \int d^2\theta\, d^2\bar\theta \left(\Phi^\dagger e^{2gV} \right)^a \mathcal{F}_a(\Phi) \right)$$

↳ $\mathcal{F}_a(\Phi) = \frac{\partial \mathcal{F}}{\partial \Phi_a}$ etc

$K = \operatorname{Im}\left(\Phi^{\dagger a} \mathcal{F}_a(\Phi) \right)$

$g_{ab} = \operatorname{Im}(\partial_a \partial_b \mathcal{F})$ is the special Kähler metric

↳ $F(\Phi) = \frac{1}{2} \tau \Phi^2$ for microscopic theory.

$\boxed{\text{For IR theory, could be ANYTHING}}$ ← FOUND BY SW → exactly solve low-E SYM theory!

↳ In the microscopic theory, for topological dyons $Z = a(n_e + \tau_{cl} n_m)$

for $\tau_{cl} = \frac{\theta}{2\pi} + \frac{4\pi i}{g^2}$

for a more general effective action, $\boxed{Z = a n_e + a_D n_m \quad \text{where } a_D = \frac{\partial \mathcal{F}}{\partial a}}$

3. The SW Analysis of $\mathcal{N}=2$ $SU(2)$ SYM

3.1 Overview

- $SU(2)$ $\mathcal{N}=2$ SYM: known microscopic Lagrangian. Moduli space parameterized by $u = \mathrm{Tr}\, A^2 = \frac{1}{2} a^2$ where a = eigenvalue of ϕ
  ↳ this relation holds classically!

- to know low-energy theory we must find $u(a)$ and $\mathcal{F}(u(a))$ ↗ just need to know instanton coefficients!
  ↳ since only derivatives of $\mathcal{F}$ are physical, can also use $h(a) = \frac{\partial \mathcal{F}}{\partial a}$: $\boxed{u(a), h(a)}$

- EM Duality: $A, F_{\mu\nu}, h(A) \longleftrightarrow \underset{h(A)}{\overset{\shortparallel}{A_D}}, F_{D\mu\nu}, h_D(A_D)$, $\; h_D = -A$

$$\tau = \frac{\partial h}{\partial a} \longrightarrow \tau' \; (SL(2,\mathbb{Z}) \text{ transformed})$$

$\Rightarrow$ instead of $a(u), h(a(u))$ can describe theory with $\boxed{a(u), a_D(u)}$

- Monodromies are transformations of quantities as we walk around a singularity in Moduli Space.
  ↳ FIND TWO SINGULARITIES NEAR ORIGIN OF MS $\rightarrow$ MASSLESS DYONS!
  ↳ Softly break $\mathcal{N}=2 \rightarrow \mathcal{N}=1$ SUSY: vacuum degeneracy is lifted and $\langle u \rangle$ is pushed towards region where dyon or monopole becomes massless.
  $\boxed{\text{MONOPOLES GET A VEV} \rightarrow \text{GAUGINO CONDENSATION IN } \mathcal{N}=1 \text{ SYM!!}}$

- Solving for explicit $a(u), a_D(u)$: Identify MS of SYM vacua with MS of genus-(something) Riemann Surfaces. a, a_D are related to the PERIODS of these surfaces and can be calculated using "(?) Seiberg-Witten Curves. $\Rightarrow$ COMPLETELY SOLVE LOW-E THEORY! } NOT COVERED HERE!

3.2 Parametrization of Moduli space: coordinate u

- $\mathcal{N}=2$ SYM has classical potential $V=\frac{1}{2g^2}Tr([A^\dagger, A]^2)$ for adjoint scalar

  $\Rightarrow [A_{vac}^\dagger, A_{vac}]=0 \Rightarrow A = A^a T^a \in$ Cartan Subalgebra H of group G

- different choices of A^a give different physical theories, MODULO WEYL REFLECTIONS
  "renaming of Generators"

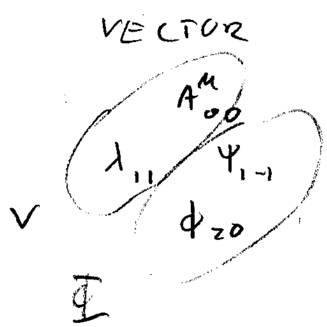
$\Rightarrow$ MS is parametrized by Weyl inv. functions of $A \in H \subset G$

- $SU(2)$: $u = Tr(A^2) = \frac{1}{2}a^2$ $\quad a =$ e-value of A

  ↳ $u=\frac{1}{2}a^2$ holds classically and can only be trusted far away from the origin. We do not know $u(a)$ in general.

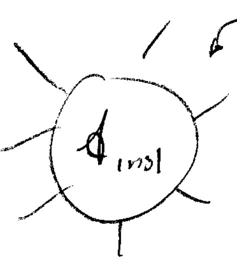
3.3 $U(1)_R$ - breaking by the chiral anomaly

- This will be important in constraining the form of $\mathcal{F}(a)$
- $\mathcal{N}=2$ SUSY has $U(2)_R = SU(2)_R \times U(1)_R$ sym of generators.



- $SU(2)_R$: $\leftrightarrow$
- shown: $U(1)_R$, $U(1)_J \subset SU(2)_R$ charges.

- $U(1)_R$ is broken by instantons (chiral anomaly) $\quad$ Terning Eqn 7.49 p 114



1 zero mode fermion insertion for each $\square, \bar{\square}$
$2N_c$ '' Adj

$\mathcal{N}=2$ SYM: λ, ψ have $R=1 \Rightarrow A_{inst} \to e^{i\alpha 4N_c} A_{inst}$

$\Rightarrow \boxed{U(1)_R \to \mathbb{Z}_{4N_c}}$ and $\boxed{\partial_\mu J_5^\mu = -\frac{N_c}{8\pi^2} F_{\mu\nu}\tilde{F}^{\mu\nu}}$

3.4 Write down low-energy Effective Action

- $SU(2) \to U(1)$ by $\langle A \rangle$

$$\mathcal{L}_{eff} = \frac{1}{4\pi} \operatorname{Im}\left[\int d^4\theta \, \frac{\partial \mathcal{F}(A)}{\partial A} \bar{A} + \int d^2\theta \, \frac{1}{2} \frac{\partial^2 \mathcal{F}}{\partial A^2} W^\alpha W_\alpha \right]$$

metric on field space: $ds^2 = \operatorname{Im} \frac{\partial^2 \mathcal{F}}{\partial a^2} \, da \, d\bar{a} = (\operatorname{Im} \tau) \, da \, d\bar{a}$

- $$\boxed{\mathcal{F}(A) = \frac{i}{2\pi} A^2 \ln \frac{A^2}{\Lambda^2} + \sum_{k=1}^{\infty} \mathcal{F}_k \left(\frac{\Lambda}{A}\right)^{4k} A^2 \quad (\mathcal{F}_k \text{ indep of fields})}$$

1st term: Perturbative contributions

↳ $\partial_\mu J_5^\mu = -\frac{N_c}{8\pi^2} F_{\mu\nu} \tilde{F}^{\mu\nu} \Rightarrow$ under $U(1)_R$, $\delta \mathcal{L} = -\frac{\alpha N_c}{8\pi^2} F\tilde{F}$

↳ shifts θ-angle: $\theta \to \theta$ w log.

↳ $\delta \mathcal{L} = \delta \mathcal{L}_{eff} \Rightarrow$ only comes from $O(F^2)$ terms, get ODE for $\mathcal{F}$

$$\mathcal{F}'''(A) = \frac{N_c}{\pi} \frac{i}{A} \to \boxed{\mathcal{F}_{1\text{-loop}} = \frac{i}{2\pi} A^2 \ln \frac{A^2}{\Lambda^2}}$$

(No higher loop corrections due to $N=2$ holomorphy)

2nd term: NP contributions

↳ instanton contribution $\propto e^{k(-8\pi^2/g^2)} = \left(\frac{\Lambda}{a}\right)^{4k}$

↳ can restore $U(1)_R$ by $R(\Lambda) = 2 = R(A) \Rightarrow R(\mathcal{F}) = 4$

$$\Rightarrow \sim A^2 \qquad \Rightarrow \mathcal{F}_{inst} = \sum_{k=1}^{\infty} \mathcal{F}_k \left(\frac{\Lambda}{A}\right)^{4k} A^2$$

- **FINDING $\mathcal{F}_k$ is the goal of SW Analysis**

3.5 Electric – Magnetic Duality

- WHY do we need duality?

  ↳ far out in field space $\mathcal{F} = \mathcal{F}_{1\text{-loop}} \Rightarrow \tau(a) = \frac{\partial^2 \mathcal{F}}{\partial a^2} = \frac{i}{\pi}\left(\ln \frac{a^2}{\Lambda^2} + 3\right)$

  ↳ $\operatorname{Im}\tau$ is harmonic fn (twice diff'ble) ⇒ NO GLOBAL MAX/MIN

  ↳ $\operatorname{Im}\tau < 0$ gives $-$ve KE ⇒ $\boxed{\text{NEED DUAL DESCRIPTION OF THEORY WHEN } \operatorname{Im}\tau < 0}$

- How do we DERIVE the duality for $N=2$ SYM?

$\boxed{\text{Bosonic gauge field action}}$

↳ enforce Bianchi identity $\partial F = 0$ by introducing dual vector field V_D^μ as Lagrange multiplier coupled to $\epsilon^{\lambda\mu\nu\rho}\partial_\mu F_{\nu\rho}$ ($= 8\pi\delta^3(x)$ for MONOPOLES) and integrate out original gauge fields $\longrightarrow$ $\boxed{\text{Duality for gauge fields}}$

↓ $N=2$ SUSY relations

$\boxed{\text{Duality for scalars } A, A_D}$

- Duality for gauge fields

$F_{\mu\nu}$ coupling to charges $\longrightarrow$ $F_{D\,\mu\nu}$ coupling to monopoles

$\tau \longrightarrow \tau_D = -1/\tau$

Full duality is larger, since $\tau \to \tau + 1$ also leaves action invariant

⇒ Full Duality transformation is $\tau \to \frac{a\tau + b}{c\tau + d}$ $SL(2,\mathbb{Z})$ $a,b,c,d \in \mathbb{Z}$, $ad - bc = 1$

- Duality for Scalars

  - Introduce $h(A) = \frac{\partial \mathcal{F}}{\partial A}$, $\tau(A) = \frac{\partial h}{\partial A}$
  - dual description $A_D, \mathcal{F}_D, h_D(A_D)$:

$$\tau_D = -\frac{1}{\tau} = -\frac{\partial A}{\partial h} = \frac{\partial h_D}{\partial A_D} \Rightarrow \boxed{A_D = h(A)} \quad h_D = -A$$

$$\begin{pmatrix} A_D \\ A \end{pmatrix} \longrightarrow \begin{pmatrix} a & b \\ c & d \end{pmatrix}\begin{pmatrix} A_D \\ A \end{pmatrix}$$

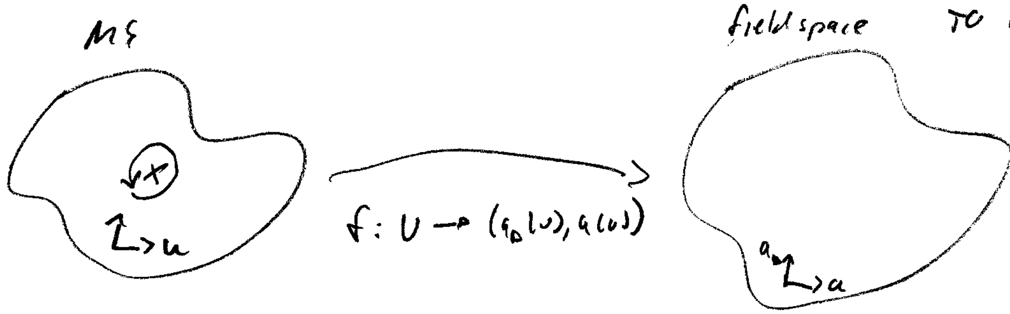
↑ full description of theory!

3.6 Monodromies on the MS of SU(2) SYM

- The field space is parameterized by (a_D, a) (or "theory space")

  The vacuum manifold is parameterized by coord u (the moduli space)

  and defines a subspace of the field space $\underbrace{(a_D(u),\ a(u))}_{\text{THIS IS WHAT WE WANT TO FIND LOW-E THEORY}}$

- 

  x = singularity

  ⟲ = path around sing

  MONODROMY OF SINGULARITY

  As you walk around singularity in MS, $v = \begin{pmatrix} a_D(u) \\ a(u) \end{pmatrix} \to v' = Mv$

  where $M \in$ monodromy group $\subset SL(2, \mathbb{Z})$

- WE CAN USE MONODROMIES TO EXPLORE SINGULARITIES OF MS → LEARN ABOUT WHERE WHICH PARTICLES BECOME MASSLESS

- Monodromy at ∞

↳ large $|a|$ → $u = \frac{1}{2}a^2$ is valid, $\mathcal{F} = \mathcal{F}_{1\text{-loop}} \Rightarrow a_D = \frac{\partial \mathcal{F}}{\partial a} = \frac{2ia}{\pi} \log\frac{a}{\Lambda} + \frac{ia}{\pi}$

↳ loop around $u = 0$: $u \to e^{2\pi i} u$

$$\ln u \to \ln u + 2\pi i$$

$$\ln a \to \ln a + i\pi$$

$$\Rightarrow v = \begin{pmatrix} a_D \\ a \end{pmatrix} \to \begin{pmatrix} -a_D + 2a \\ -a \end{pmatrix} \Rightarrow \boxed{M_\infty = \begin{pmatrix} -1 & 2 \\ 0 & -1 \end{pmatrix}}$$

- Show there are at least two singularities at finite u

$\hookrightarrow M_\infty \neq \mathbb{1} \Rightarrow$ at least one singularity of MS in u-plane

$\hookrightarrow$ Say $\exists$ only one singularity $\rightarrow \forall$ paths, $M = M_\infty$ or $\mathbb{1}$

$\rightarrow a^2$ is invariant $\rightarrow$ good global coord

$\rightarrow \tau(a)$ is globally defined $\times$.

$\rightarrow$ AT LEAST 2 SINGULARITIES!

- Origin of Singularities : Massless Dyons

$\hookrightarrow$ integrate out heavy particles. their loops give non trivial Kähler metric.
singularity of the coordinate on MS $=$ Particle becomes MASSLESS!
$\hookrightarrow$ (u ceases to be a good coordinate)

$\hookrightarrow$ Nature of singularity depends on massless particle.

$\hookrightarrow$ Massless gauge bosons are not consistent!

$U(1)$ w/o matter is CFT

$\hookrightarrow$ CFT + SUSY = SCFT $\rightarrow$ vanishing instanton anomaly?

$\Rightarrow \boxed{\text{MASSLESS DYONS}}$

- What is the monodromy associated with a massless dyon?

$\hookrightarrow$ Find M for massless electric hyper, then use duality to get $M(n_m, n_e)$ for general dyons

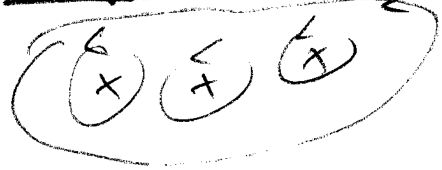
$\hookrightarrow$ massless electric hyper: low-E theory is $U(1)$ + hyper

- near $u = u_q$, $a(u) \approx c_1 (u - u_q)$ since $a \rightarrow 0$ as $u \rightarrow u_q$ to satisfy $M = M_{BPS} = 2|Z|$ with $Z = n_e a + n_m a_D$
- RGE gives $\tau \approx -\frac{i}{\pi} \ln \frac{a}{\Lambda} = \frac{\partial a_D}{\partial a} \Rightarrow a_D = -\frac{i}{\pi} a \ln\frac{a}{\Lambda} + \frac{i}{\pi}$
- go around path $(u - u_q) \rightarrow e^{2\pi i}(u - u_q)$: $v = \begin{pmatrix} a_D \\ a \end{pmatrix} \rightarrow \begin{pmatrix} a_D + 2a \\ a \end{pmatrix} \Rightarrow M_{(0,1)} = \begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix}$

$\hookrightarrow$ Do duality transformation $(0,1) \rightarrow (n_m, n_e)$: $\boxed{M_{(n_m, n_e)} = \begin{pmatrix} 1 + 2n_e n_m & 2n_e^2 \\ 2n_m^2 & 1 - 2n_e n_m \end{pmatrix}}$

- What are the individual singularities?

- $M_\infty = \prod M_i(n_m^i, n_e^i)$



M_{cb}

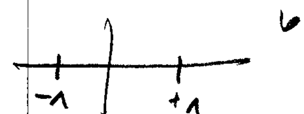
NO SOLUTION FOR MORE THAN TWO MASSLESS DYONS

$\boxed{\Rightarrow \text{EXACTLY TWO}}$

- solve $M_\infty = M_{(m,n)} M_{(m',n')}$

  $\hookrightarrow$ $\boxed{\text{get massless dyon, monopole}}$

  $\hookrightarrow$ Hypothesis: $u_{sing} = \pm\Lambda$ (confirmed by exact SW calculation)



$\underline{u = -\Lambda}$

massless dyon $(m', n') = (1, -1)$

$a_D = a$

$\underline{u = +\Lambda}$

massless monopole $(m, n) = (1, 0)$

$a_D = 0$

as required by

$M_{BPS} = 2|Z| \to 0$

$Z = a n_e + a_D n_m$

3.8 Monopole Condensation & Confinement

- we can softly break $N=2$ SUSY to $N=1$ to understand gaugino condensation in $N=1$ SYM as a dual Meissner effect from magnetic higgs mechanism due to monopoles getting a VEV.

- $N=2 \to N=1$: add $W = m A^2$ mass to χSF in $N=2$ vector

  $\to$ low-E theory will be pure $N=1$ SYM below scale m.

- near point where monopoles are massless:

$$W = \underbrace{\sqrt{2} A_D M \tilde{M}}_{\text{from } N=2 \text{ SUSY}} + m U(A_D) \longrightarrow \frac{\partial W}{\partial A_D} = 0 = \sqrt{2} M \tilde{M} + m \frac{\partial U}{\partial A_D}$$

$$\to \boxed{\langle M \tilde{M} \rangle \neq 0}$$

$$\frac{\partial W}{\partial M} = 0 = \sqrt{2} A_D \tilde{M} \to \boxed{\langle A_D \rangle = 0}$$