hep-ph/9601359

 : SCATTERING

10 SEPT

<u>RAISON D'ÊTRE:</u> eg $2 \to 3$ g SCATTERING IS COMPLICATED DUE TO REDUNDANCIES

<u>IDEA:</u> DISCARD FEYNMAN
RE-ORGANIZE COLOR + HELICITY INFO

TODAY
1. COLOR ORDERING
2. SPINOR HELICITY
3. TREE: ON SHELL RECURSION

COLOR ORDERING

 $\sim T^a$  $\sim f^{abc}$

BUT WE KNOW:

$$f^{abc} = \frac{-i}{\sqrt{2}} \mathrm{Tr}\left(T^a [T^b, T^c]\right)$$

$$= \frac{-i}{\sqrt{2}} \left[\mathrm{Tr}\left(T^a T^b T^c\right) - \mathrm{Tr}\left(T^a T^c T^b\right)\right]$$

SUGGESTIVE OF REWRITING DIAGRAM IN TERMS OF FERMION LOOPS

SUGGESTS USE of TWO IDENTITIES:

$$\boxed{T^a_{i_1 \bar{j}_1} T^a_{i_2 \bar{j}_2} = \delta_{i_1}^{j_2}\delta_{i_2}^{j_1} - \frac{1}{N}\delta_{i_1}^{j_1}\delta_{i_2}^{j_2}}$$ COMPLETENESS

n-POINT

SO WE CAN TAKE SOME AMPLITUDE & EXPAND

$$A_n^{tree}(\{k_i, \lambda_i, a_i\}) = g^{n-2} \sum_\sigma \mathrm{Tr}(T^{a_{\sigma(1)}} \cdots T^{a_{\sigma(n)}}) \times A_n(\sigma(1^{\lambda_1}) \cdots \sigma(n^{\lambda_n}))$$

SUM OVER DISTINCT ORDERINGS MOD CYCLIC

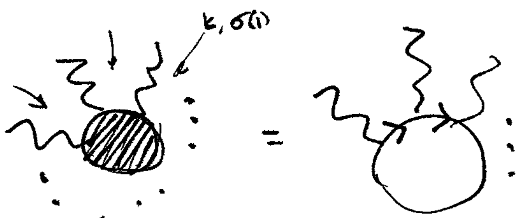
(k_n, λ_1)

PARTIAL AMP.
MANIFEST ANALYTICAL STRUC.

This is called <u>COLOR STRIPPING</u>

$$A^{tree} = (\text{color})(\text{partial amplitude})$$

<u>DIAGRAMMATICALLY</u>:



+ permutations

statement is true for color structure & ANALYTIC STRUCTURE (POLES)

SO, eg for 5 PT AMP, $\exists$ LARGE # of DIST. ORDERINGS IN COLOR TRACE + PARTIAL AMP.

$A_5(k_1, \ldots, k_5)$:

$\exists$ dependence identities

TURNS OUT $\exists$ 4 INDEPENDENT PARTIAL AMP

$A_5^{tree}(1^+, 2^+, 3^+, 4^+, 5^+) \longrightarrow = 0$

$A_5^{tree}(1^-, 2^+, 3^+, 4^+, 5^+) \longrightarrow = 0$

$\left.\begin{array}{l} A_5^{tree}(1^-, 2^-, 3^+, 4^+, 5^+) \\ A_5^{tree}(1^-, 2^+, 3^-, 4^+, 5^+) \end{array}\right\}$ RELATED BY "γ DECOUPLING THM"

SPINOR HELICITY

GIVE A NATURAL WAY TO WRITE PARTIAL AMPLITUDES.

IDEA: WRITE p IN TERMS of $SL(2, \mathbb{C})$ INSTEAD of $SO(3,1)$.

larger space.

$$\boxed{k^\mu = \frac{1}{2}\lambda^a \sigma^\mu_{a\dot{a}} \bar{\lambda}^{\dot{a}}} \qquad \bar{\lambda} = \lambda^+$$

CAN DO THIS IF $k^2 = 0$ (not obvious why)

this generates a bunch of identities (eg from FIERZ)

eg: $\lambda^a \bar{\lambda}^{\dot{a}} = k^a \sigma_\mu^{a\dot{a}} = \not{k}$

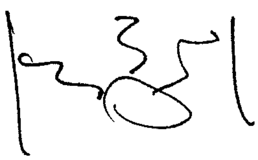
SO CAN WRITE P-CONTRACTIONS IN TERMS OF SPINOR CONTR.

↳ note: σ DEP ON s, t, u

BUT: $\mathcal{M}$ DEP ON "SQRT" of THESE

↳ THE SPINORS ARE PRECISELY THIS

eg WE CAN GUESS POLE STRUCTURE AFTER COLOR STRIPPING



$$\to \frac{1}{s_{\sigma(1)\sigma(2)}\,,\ s_{\sigma(2)\sigma(3)}\ \cdots}$$

↳ 

$$\to \frac{1}{\sqrt{s_{\cdots}}}$$

NOTE: λ ARE $\mathbb{R}$, NOT GRASSMANNIAN

$$\boxed{\begin{aligned} \lambda_i^{a}\lambda_{ja} &= \langle ij\rangle \\ \lambda_{i\dot a}\lambda_j^{\dot a} &= [ij] \end{aligned}}$$

w/ $\langle ii\rangle = [ii] = 0$

ALT. NOTATION:

$$|i^+\rangle = \lambda_{ia} \qquad \langle i^-| = \lambda_i^{a}$$
$$|i^-\rangle = \lambda_i^{\dot a} \qquad \langle i^+| = \lambda_{i\dot a}$$

$\pm$ HELICITY
(not the same as the given helicity!!)

$$\Rightarrow \boxed{\begin{aligned} \langle ij\rangle &= \langle i^-|j^+\rangle \\ [ij] &= \langle i^+|j^-\rangle \end{aligned}}$$

USEFUL IDENTITIES

$$\langle i^{\pm}|\gamma^{\mu}|j^{\pm}\rangle\langle k^{\pm}|\gamma_{\mu}|l^{\pm}\rangle = 2\langle ik\rangle[jl]$$

Fierz identity (do you see it? $\sigma^{\mu}\bar{\sigma}_{\mu} \sim \cdots$)

COMPLETENESS: $\langle ij\rangle[jl] = \langle i^-|\not{k}_j|l^-\rangle$

basically from def $\lambda^a\bar{\lambda}^{\dot{a}} = \not{k}$

NOTE: $\langle ij\rangle[ji] = \mathrm{Tr}(\not{k}_i \not{k}_j)$
$= 2k_i \cdot k_j$
$= s_{ij}$ (massless)

$\Rightarrow$ CRUDELY: $\langle ij\rangle = e^{i\phi_{ij}} s_{ij}^{1/2}$ (as we suggested)
a posteriori motivation

MOMENTUM CONSERVATION

$$\sum_{j=0}^{n} \langle ij\rangle[jl] = 0$$

$\sum_j \langle i^-|\not{k}_j|l^-\rangle$

ALSO: $\langle i^-|\not{k}_i|l^-\rangle = \langle ii\rangle[il]$, $\langle ii\rangle = 0$

so this is null.

SUGGESTIVE of POLZ!

NOW CAN THINK of POLARIZATION W/RT THIS!

btw: no unique way of defining polz, can always shift by momenta by Ward Identity.

SO LET'S DEFINE:

$$\boxed{\varepsilon_\mu^\pm(k,q) = \frac{\pm\langle q^\mp|\gamma^\mu|k^\mp\rangle}{\sqrt{2}\,\langle q^\pm|k^\pm\rangle}}$$

ref momentum (OUR CHOICE)

OBVIOUS: $\varepsilon^\pm_\mu k^\mu = 0$ from COMPLETENESS RELATION.

ALSO WANT: $\varepsilon^+\cdot(\varepsilon^+)^* = -1$

$$\varepsilon^+_\mu \varepsilon^{-\mu} = \frac{-\langle q^-|\gamma^\mu|k^-\rangle\langle q^+|\gamma_\mu|k^+\rangle}{2\langle qk\rangle[qk]}$$

$$= \frac{-\langle qk\rangle[qk]}{\langle qk\rangle[qk]}$$

similarly: $\varepsilon^+\cdot(\varepsilon^-)^* = 0$, etc.

NOW HAVE SOME FUN: CALCULATE TREE STUFF

FIRST: SOMETHING IMPORTANT.

A JUDICIOUS CHOICE of REF MOMENTUM q WILL SAVE US SOME TROUBLE.

eg PICK q TO BE SAME FOR TWO POLZ VECTOR, GET THINGS TO VANISH.

$$\varepsilon_1^+ \cdot \varepsilon_2^+ = 0 \quad \propto \langle q^- | \gamma^\mu | k_1^- \rangle \langle q^- | \gamma_\mu | k_2^- \rangle \overset{\text{FIERZ}}{\propto} \underbrace{\langle q q \rangle}_{0} [k_1 k_2]$$

CONSIDER:

$$A_n^{\text{tree}}(1^+, 2^+, \ldots, n^+) \propto (\varepsilon_i^+ \cdot \varepsilon_j^+) = 0$$

$$\Rightarrow \boxed{\text{AMP w/ ALL HELICITIES IN } = 0 \,!}$$

Maximal Helicity Violating

NEXT MOST HELICITY VIOLATING:

$$A_n^{\text{tree}}(1^-, 2^+, \ldots, n^+) \propto \varepsilon_1^- \cdot \varepsilon_2^+ \propto \langle q_1 k_2 \rangle [q_2 k_1]$$

PICK: $q_1 = k_n$ & $q_n = k_1$ $\Big\}$ $A_n^{\text{tree}}(1^-, 2^+, \ldots n^+) = 0$

$q_2, \ldots q_n$

Parke-Taylor

$A_n^{tree}(1^-, 2^-, 3^+, \cdots, n^+) \neq 0$!

MHV : largest hel. viol amp that is nonzero

$\overline{MHV}$: $(++-----)$

CAN SHOW: $\boxed{= \frac{\langle 12\rangle^4}{\langle 12\rangle\langle 23\rangle \cdots \langle n1\rangle}}$ $\qquad \overline{MHV}: \langle\,\rangle \to [\,]$

only poles in adjacent momenta comb.

(why $\neq 0$? b/c cannot pick REF MOMENTA s.t. they all vanish)

Q: HOW DO WE DEAL W/ INTERNAL LINES?

FEYN: P CONS. @ EA VERTEX, SHIFT P ~~off~~ SHELL.

NEW: CONS. MOMENTA.
KEEP MOMENTA ON-SHELL
$\longrightarrow$ SHIFT INTO COMPLEX PLANE. $\Big\}$ genus.

$\hookrightarrow$ this is why i needed $k^2 = 0$
i needed extra IR dof in x for accomodating this shift!

Then: LARGE DIAGRAMS ARE RECURSIONS OF SMALLER ON-SHELL DIAGRAMS.

CONSIDER: k_1^μ, k_2^μ FOR TWO PARTICLES

$$k_1^\mu(z) = k_1^\mu - \frac{z}{2}\langle 1^- | \gamma^\mu | 2^- \rangle \leftarrow \tfrac{1}{2}\lambda_1^a \sigma^\mu_{a\dot a}(\lambda_1^{\dot a} - z\lambda_2^{\dot a})$$

$$k_2^\mu(z) = k_2^\mu + \frac{z}{2}\langle 1^- | \gamma^\mu | 2^-]$$

$$\uparrow \; (\lambda_2^a + z\lambda_1^a)\sigma^\mu_{a\dot a}\lambda_2^{\dot a}$$

NULL VECTORS. THE SHIFT KEEPS THEM NULL.

$$\left[\begin{array}{l} k_1(z)^2 = k_2(z)^2 = 0 \\ ; \; k_1(z) + k_2(z) = k_1 + k_2 \end{array}\right]$$

CAN WRITE SHIFT IN TERMS OF SPINORS:

$$k_1^\mu(z) \Rightarrow \lambda_1^{\dot a} \to \lambda_1^{\dot a} - z\lambda_2^{\dot a}$$
$$k_2^\mu(z) \Rightarrow \lambda_2^a \to \lambda_2^a + z\lambda_1^a$$

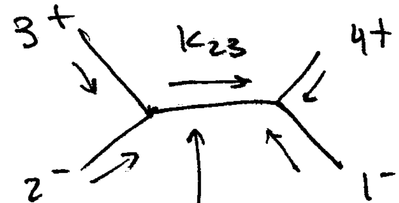
if I shifted $\lambda_2^{\dot a}$, I'd get $\langle 2^- | \gamma^\mu | 1^- \rangle$

CLAIM: 'OBVIOUS' FORM:

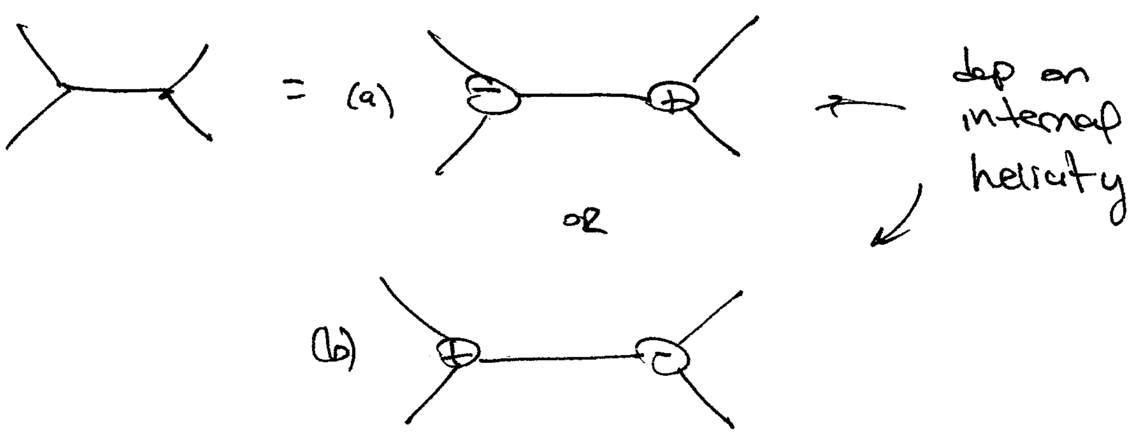
$$A_3^{tree}(1^-,2^-,3^+) = \frac{i\langle 12\rangle^3}{\langle 12\rangle\langle 23\rangle}$$

$$A_3^{tree}(1^+,2^+,3^-) = \frac{-i[12]^3}{[12][23]}$$

WANT TO FIND: $A_4^{tree}(1^-,2^-,3^+,4^+)$



SHIFT THIS TO BE ON SHELL BY SHIFTING EXTERNAL LEGS: $2 \to \hat{2}$, $1 \to \hat{1}$ ①



$$0 = \hat{K}_{23}^2 = \left(k_3 + k_2 + \tfrac{z}{2}\langle 1^- | \gamma^\mu | 2^- \rangle\right)^2$$

$$= s_{23} + z\langle 1^- | \not{k}_3 | 2^- \rangle$$

$$= \langle 23\rangle[32] + z\langle 13\rangle[32]$$

TO SHIFT ON-SHELL: $z = \dfrac{-\langle 23\rangle}{\langle 13\rangle} = z_{23}$

NOW WE COMPUTE DIAGRAMS (a) & (b) W/ SPINOR ACROBATICS.

$$(a) = A_3(\hat{2}^-, 3^+, \hat{K}_{23}^-)\,\frac{-i}{K_{23}^2}\,A_3(4^+, \hat{1}^-, \hat{K}_{23}^+)$$

$-$ & $+$ b/c helicity always into vertex.

$$\frac{i\,[4\,\hat{K}_{23}]^3}{[41][\hat{1}\,\hat{K}_{23}]}$$

$$[a\hat{k}] = \frac{[a\hat{k}]\langle\hat{k}1\rangle}{\langle\hat{k}1\rangle} = \langle 1^- | \not{k} | a^- \rangle = \frac{\langle 1^- | k | a^- \rangle}{\langle \hat{k} 1\rangle}$$

$$\Rightarrow [4\hat{K}_{23}] = \langle 1^- | \not{K}_{23} | 4^- \rangle = 0$$

$-\not{k}_1 - \not{k}_4$

$$\Rightarrow \boxed{(a) = 0}$$

(b) $A_3(\hat{2}^-, 3^+, \hat{k}_{23}^+)\ \frac{1}{k_{23}^2}\ A_3(4^+, 1^-, \hat{k}_{23})$

$\langle 23\rangle[32]$

answer

$$= \frac{-i\langle 12\rangle^4}{\langle 12\rangle\langle 23\rangle\langle 34\rangle\langle 41\rangle}$$

Moral: demonstration of on-shell approach using recursion w/ lower-point func.

from here: 1-loop stuff (unitarity cut method)
twistors from spinor helicity