1-LOOP TECHNIQUES FOR SCATTERING AMP (Dean) 15 Apr.

1. a) SPINOR - HELICITY REVIEW
   b) TREE-LEVEL BCFW REVIEW
2. INTEGRAL REDUCTION
3. UNITARITY METHOD

SEE ZVI BERN LECTURES @ JERUSALEM WINTER SCHOOL

Tree Stuff - review of last fall

COLOR ORDERING :

$$\mathcal{A}_n^{\text{tree}}(\{k_i, \lambda_i, a_i\}) = \sum_{\sigma \in S_n/\mathbb{Z}_n} \mathrm{Tr}\left[T^{a_{\sigma(1)}} \cdots T^{a_{\sigma(n)}}\right] \times \underbrace{A_n^{\text{tree}}\left(k_{\sigma(1)}^{\lambda_{\sigma(1)}}, \ldots, k_{\sigma(n)}^{\lambda_{\sigma(n)}}\right)}_{\text{COLOR ORDERED}}$$

Parke-Taylor

$$A_3^{\text{tree}}(1^-, 2^-, 3^+) = \frac{i\langle 12\rangle^3}{\langle 23\rangle\langle 31\rangle}$$

$$A_4^{\text{tree}}(1^-, 2^-, 3^+, 4^+) = \frac{i\langle 12\rangle^4}{\langle 12\rangle\langle 23\rangle\langle 34\rangle\langle 41\rangle}$$

REMINDERS: spinor helicity

EMBED EVERYTHING IN $SL(2,\mathbb{C})$ RATHER THAN $SO(3,1)$
FOR SOME k^μ w/ $k^2=0$, WRITE $\lambda \in SL(2,\mathbb{C})$

$$\underbrace{\lambda_k^a \lambda_k^{\dot a}}_{RH} = k^\mu \sigma_\mu{}^{a\dot a}$$

$\lambda^{\dot a} = (\lambda^a)^\dagger$; λ not Grassmannian

$$k^\mu = \frac{1}{2}\lambda_k^a \, \sigma^\mu{}_{a\dot a} \, \lambda_k^{\dot a}$$

AN ALTERNATE, CONVENIENT NOTATION:

$$\langle ij \rangle = \lambda_i^a \lambda_{aj}$$
$$[ij] = \lambda_{i\dot a} \lambda_j{}^{\dot a} \qquad \langle ij\rangle^* = [ji]$$

USEFUL for SIMPLE CONTRACTIONS.

ANOTHER NOTATION FOR MORE COMPLICATED CONTR. (DIXON)

$$|i^+\rangle \equiv \lambda_{ia} \qquad \langle i^-| \equiv \lambda_i{}^a$$

$$|i^-\rangle \equiv \lambda_i{}^{\dot a} \qquad \langle i^+| \equiv \lambda_{i\dot a}$$

then: $k^\mu = \frac{1}{2}\langle k^- | \sigma^\mu | k^- \rangle$
$\langle ij \rangle = \langle i^- | j^+ \rangle$

Real power: can also write polarization in the same way:

$$\varepsilon_\mu^\pm(k) = \frac{\pm\langle q^\mp|\sigma^\mu|k^\mp\rangle}{\sqrt{2}\,\langle q^\mp|k^\pm\rangle}$$

where q is a reference momentum, $q^2=0$

IF YOU'RE CLEVER ENOUGH TO CHOOSE q INTELLIGENTLY (GAUGE CHOICE), THINGS SIMPLIFY.

eg. $A(1^-, 2^+, 3^+, \cdots, n^+) = 0$

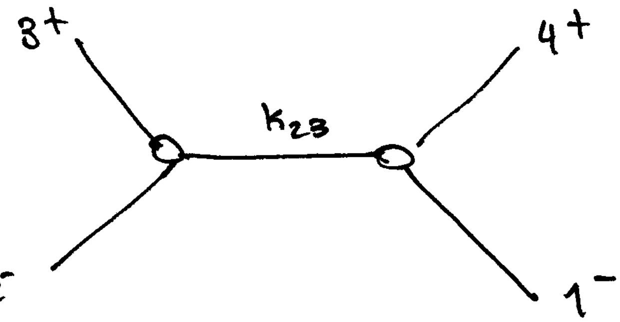
LOTS of IDENTITIES (Shouten $\leftrightarrow$ Jacobi)

eg. IN ANY FRAME, PICK $\varepsilon \perp k$
BUT THEN BOOST FRAME & ε PICKS UP A t COMPONENT. CHOICE OF q RELATED TO THIS UNPHYSICAL PIECE.

RECALL: COLOR ORDERING (PVS PAGE)

Parke-Taylor (MHV)

$A^{tree}(1^-, 2^-, 3^+, 4^+) =$



$= A_3 \frac{1}{k_{23}^2} A_3$

Feynman: $k_{23}^2 \neq 0$ (OFF SHELL)

BCFW: SHIFT k_1 & k_2 INTO $\mathbb{C}$ PLANE, KEEPING THEM ON-SHELL: $k_1^2 = k_2^2 = 0$

eg
$$k_1^\mu \to k_1^\mu + \frac{z}{2}\langle 1^- | \sigma^\mu | 2^- \rangle$$
$$k_2^\mu \to \underbrace{k_2^\mu - \frac{z}{2}\langle 1^- | \sigma^\mu | 2^- \rangle}_{\text{still square to zero (CAN CHECK)}}$$

WHAT DOES THIS DO? k_{23} ALSO SHIFTED.

$$0 = \oint \frac{A(z)}{z}\, dz \qquad (\text{BNDY OF } \mathbb{C})$$

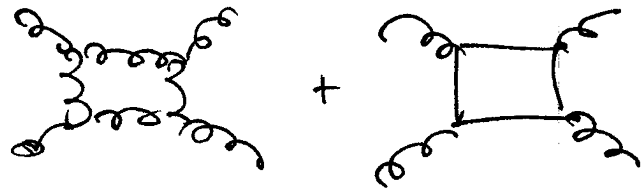
$$= \boxed{A(0)} + \underbrace{\frac{A}{z}\left(\text{POLE } k_{23}^2 = 0\right)}_{\text{ON SHELL DECOMP.}}$$

what we want

Moral: CLEVER ANALYTICITY TRICK | term by term you satisfy Ward Identity (don't req. large cancellations like Feynman method)

LOOP LEVEL

(VERY ABBREVIATED)

Think about: 

FIRST HAVE TO COLOR ORDER. [WE'LL SKIP]

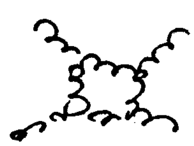
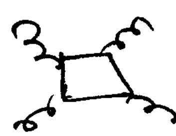
ANSWER: GOES LIKE TREE LEVEL "SQUARED"

$$A_n^{1\text{-loop}}\{k_i, \lambda_i, a_i\} = \sum_c \sum_\sigma Gr_{n,c}(\sigma)\, A^{1\text{-loop}}_{n,c,\sigma}(\varepsilon)$$

$c=1$: tree

$c\geq 2$: $Tr(1 \cdots c)\, Tr(c \cdots n)$

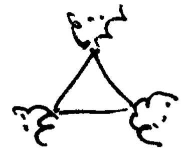
GIVES GAUGE STRUCTURE @ LOOP LEVEL

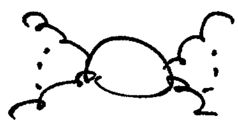
$A_{4,1} =$  $+$  $\times \dfrac{N_f}{N_c}$ (FACT)

Integral Reduction

n-point Gluon amplitude
WHICH LOOPS ARE IMPORTANT & WHICH AREN'T?

$$A_n^{1\text{-loop}} = \sum_{i_1<i_2<i_3<i_4} d^0_{i_1 \cdots i_4}$$  I_4

$$+ \sum_{i_1<i_2<i_3} c^0_{i_1,\ldots,i_3}$$  $\Big\} I_3$

$$+ \sum_{i_1<i_2} b^0_{i_1 i_2}$$  $\Big\} I_2$

I's ENCODE LOOP FUNCTIONS

$+$ higher Θ, non div.

<u>b, c, d</u>: RATIONAL FUNC of EXTERNAL MOMENTA
(WE DON'T KNOW THEM)
↑
want to find them

PREVIOUSLY: USED RES. THM TO DETERMINE UNSHIFTED POLE FROM SHIFTED ONE

<u>NOW</u>: USE OPTICAL THM.

WE WILL CUT THESE & IMPOSE THAT DISCONTINUITY MATCHES; this will fix b, c, d

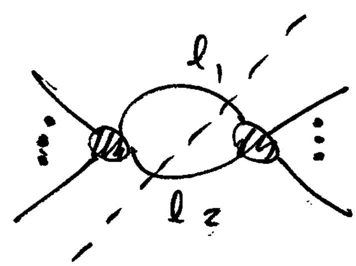
↳ we will get recursion relations, cut-by-cut

3. UNITARITY METHOD ("analyticity method")

@ the INTEGRAND LEVEL:

$$A_n^{1\text{-loop}}(\ell) = \sum \frac{\bar{d}(\ell)}{D_1 D_2 D_3 D_4} + \sum \frac{\bar{c}(\ell)}{D_1 D_2 D_3} + \sum \frac{\bar{b}(\ell)}{D_1 D_2}$$

OPTICAL THM:



CAN EXTEND THE OPTICAL THM TO n-CUTS:

$$\mathrm{Disc}_{s_{i_1} \cdots s_{i_k}} A_n^{1\text{-loop}} = \underbrace{\sum_{\text{states}}}_{\text{ON SHELL INTERNAL}} \underbrace{d\Phi}_{\text{PHASE SPACE}} A_{n_1}^{\text{tree}} \cdots A_{n_k}^{\text{tree}}$$

end up w/ recursion relations; eg.

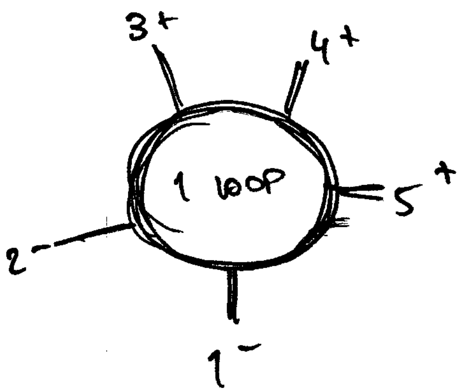
$$\bar{d}_{i_1 \cdots i_4}(\ell) = \sum_{\text{state}} A^{\text{tree}}_{n_1}(\ell) \cdots A^{\text{tree}}_{n_4}(\ell)$$

$$\bar{c}_{i_1 i_2 i_3}(\ell) = (\cdots)_3 - \sum_{j \neq \cdots} \frac{d_{i_1 i_2 i_3 j}}{D_j}$$

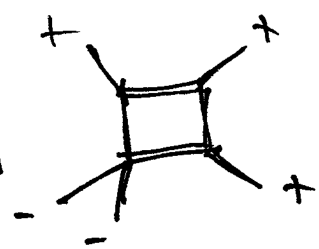
<u>EXAMPLE - concretely</u>

eg. $A_5^{1\text{-loop}}(1^-, 2^-, 3^+, 4^+, 5^+)$

@ tree level we know, Parke-Taylor

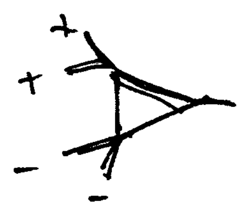


$=\ d_1$



+ cyclic PERM.

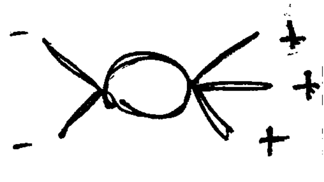
$+\ c_1$



$+ \cdots$

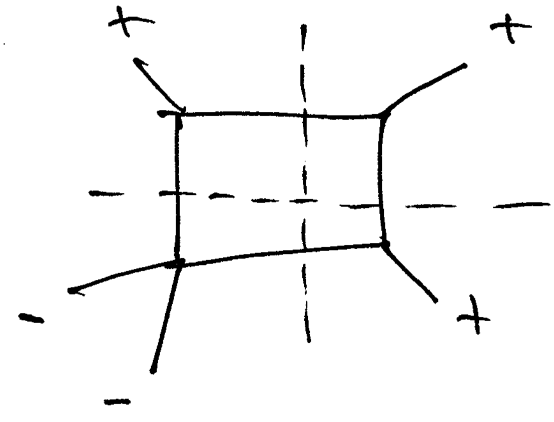
USING MPU TO WRITE EACH DIAGRAM

$+\ b_1$

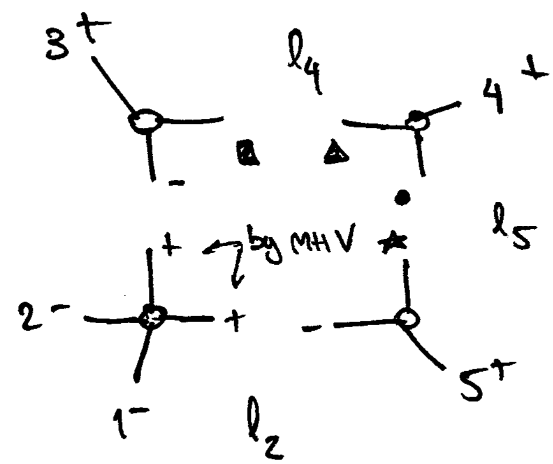


$+ \cdots$

getting d_1:

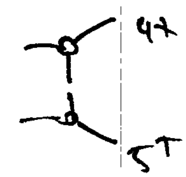


PUT EVERYTHING ON SHELL



■	+	±	some choices (check these)
▲	−	∓	
●	±	−	
★	∓	+	

only one survives by MHV rules on one tree graph



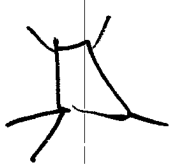
once again shift individual, eg

$$l_4 - \frac{z}{2} \langle 3^+ | \sigma^\mu | 4^+ \rangle$$

$\vdots$

same trick as tree.

computing this is the discontinuity
@ integrand level

 $= \dfrac{-\frac{1}{2}\langle 12\rangle^3 s_{34} s_{45}}{\langle 23\rangle\langle 34\rangle\langle 45\rangle\langle 51\rangle}$ eg $(k_4+k_5)^2$