KINKY KINEMATICS WITH MT2

REF: hep-ph/0304226 "Truth behind the glamour"
SUMMARY OF THE ORIGINAL MT2 ANALYSIS
arXiv: 0810.5178 "Minimal kinematic constraints"
arXiv: 0908.3779 "From the boundary to the crease"

MANY OTHER PAPERS
- GENERALIZED / APPLIED TRANSVERSE VARIABLES
- USING OTHER KINEMATIC INFORMATION
- See FLIP FOR MORE SPECIFIC REFS

MAIN IDEA: LOOKING FOR NEW PHYSICS @ LHC
NP = (EFFECTIVE) LAGRANGIAN TO AUGMENT SM
→ NEW PARTICLES (MASS SPECTRUM, SPIN)
→ INTERACTIONS (COUPLINGS, ...) ⇔ quantum #'s

WHAT WE MEASURE: σ ; function of masses & couplings

IN ORDER TO MEASURE m_i NEED g_i
IN ORDER TO MEASURE g_i NEED m_i

Chicken & Egg problem!

SOLUTION: USE KINEMATICS OF FINAL STATE PARTICLES
eg. $p_i^2 = m_i^2$, INDEPENDENT OF COUPLINGS

BUT: KINEMATICS ARE NON-TRIVIAL
→ INVISIBLE PARTICLES (ν, LSP)
→ + PDF ⇒ CAN'T BALANCE p^z

BEST WE COULD DO (SSC EDITION): MEASURE Δm^2
collider physics motto:
- HADRON MACHINES ARE GREAT FOR REACHING HIGH ENERGIES, PRODUCING MANY NEW PARTICLES, AND MEASURING MASS DIFFERENCES
- LEPTON MACHINES ARE LIMITED TO LOWER ENERGY SPECTRA, BUT CAN MEASURE ABSOLUTE MASSES.

QUESTION: CAN WE DO BETTER W/ KINEMATICS??

WANT: MODEL INDEPENDENT (but MODEL-MOTIVATED → LSP)
LEARN AS MUCH ABOUT SPECTRUM

⇝ interface of thy & exp.

[ASIDE]: MAP OF MAJOR PLAYERS

CAMBRIDGE OXFORD (LAUSANNE)	LESTER, BARR, GRIPAIOS — many excellent big picture papers ↑ father of MT2	
KOREA	Cho et al @ KAIST, demonstrated new features (eg KINKS) in model-specific applications; YG KIM, ...	
JAPAN	Nojiri et al	
DAVIS WISCONSIN	Cheng & Han, Tao Han — beyond MT2, kinematic picture	
STANFORD ?	DIMOPOULOS (via OXFORD CONNECTION)	
FLORIDA	Matchev, Park, et al. — SUBSYSTEM MT2	
ITHACA/IRVINE	Jim & Nic ; Flip & Felix	

REVIEW: THE W MASS MEASUREMENT

← along w/ Dalitz plot, striking example of spectrum measurement @ hadron colliders

OUR TEXTBOOK EXAMPLE OF HOW TO DEAL WITH MISSING ENERGY. RECALL THAT @ THE TEVATRON, LARGEST NON-TRIVIAL (ie not pure jets) CROSS SECTION IS SINGLE GAUGE BOSON PRODUCTION.

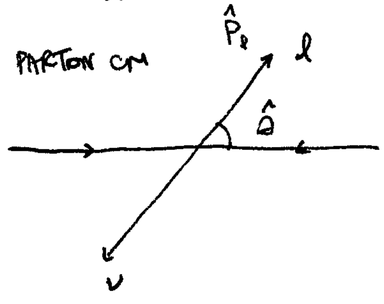
$$\sigma(q\bar{q} \to Z) = \frac{4}{3}\pi^2 \frac{\Gamma(Z\to q\bar{q})}{M_Z}\,\delta(s - M_Z^2)$$

$$\sigma(q\bar{q} \to Z+\text{jets}) = \frac{4}{3}\pi^2 \frac{\Gamma(Z\to q\bar{q})}{M_Z}\left(\ldots\right) \quad \leftarrow \int \text{over pdfs}$$

FOR W, WE NEVER DIRECTLY MEASURE THE ν MOMENTUM. BECAUSE OF PARTON-PARTON INTERACTION, P_Z INFORMATION IS LOST.

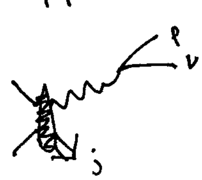
BEST WE CAN DO IS IMPOSE TRANSVERSE MOMENTUM CONSERVATION ~~SO WE TRY TO MANAGE EVERYTHING INTO TRANSVERSE VARIABLES~~.

PARTON CM



$$P_T = |\hat{P}_\ell| \sin\hat{\theta} = \frac{M_W}{2}\sin\hat{\theta}$$

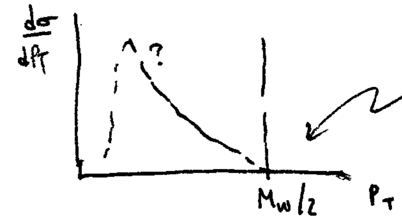
$$P_T^{\,MAX} = \frac{1}{2}M_W$$



SO, THE IDEA IS THAT IF WE LOOK AT <u>LOTS</u> OF EVENTS, THE DISTRIBUTION WOULD HAVE AN <u>ENDPOINT</u> @ $\frac{1}{2}M_W$. (*)

THIS ONLY WORKS IF WE HAVE A FAVORABLE DISTRIBUTION

eg.

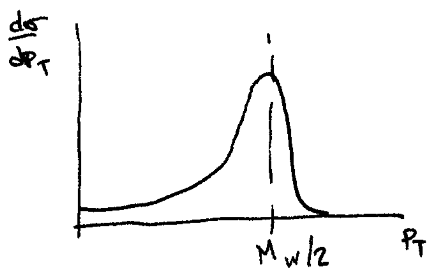


THIS WOULDN'T BE HELPFUL SINCE IT WOULD BE HARD TO IDENTIFY THE ENDPOINT.

FORTUNATELY, WE HAVE A <u>JACOBIAN PEAK</u>

$$\frac{d\sigma}{dP_T} = \underbrace{\frac{d\cos\hat{\theta}}{dP_T}}_{\text{jacobian}} \underbrace{\frac{d\sigma}{d\cos\hat{\theta}}}_{\text{Peskin \& Sch.}} = \frac{P_T}{\sqrt{(M_W/2)^2 - P_T^2}} \frac{d\sigma}{d\cos\hat{\theta}}$$

BIG PEAK!
(smoothed out by W width)



<u>BUT</u>: ONLY WORKS IF $P_T^{\,W} = 0$
ie NO EXTRA JETS ($q\bar{q} \to W + X$)
ie NO <u>ISR</u>

modern parlance in this field: <u>UPSTREAM MOMENTUM</u> (UTM)

(this is a general issue that will carry over to M_{T2})

We need a better transverse variables.

namely, A VARIABLE THAT TAKES INTO ACCOUNT <u>UTM</u>

(UTM ASSUMED TO BE VISIBLE)

TRANSVERSE MASS

WE KNOW MORE : $\vec{p}_T^{\,\nu} = -\vec{p}_T^{\,\ell} - \underbrace{\sum \vec{p}_T^{\,j}}_{\text{OBVIOUS THING TO INCLUDE}}$

KINEMATICS :

$$E_T \equiv \sqrt{(\vec{p}_T)^2 + m^2}$$

$$
\begin{aligned}
M_W^{\,2} &= (p^\ell + p^\nu)^2 \\
&= \underbrace{m_\ell^{\,2} + m_\nu^{\,2}}_{\text{usually ignore}} + 2\big(E_\ell E_\nu - \underbrace{\vec{p}_\ell \cdot \vec{p}_\nu}_{\vec{p}_T^{\,\ell}\cdot\vec{p}_T^{\,\nu} \;+\; p_z^\ell p_z^\nu}\big)
\end{aligned}
$$

NICE VARIABLES :

$$\eta = \tfrac{1}{2}\log\frac{E+p_z}{E-p_z} \left\{ \begin{aligned} \sinh\eta &= p_z/E_T \\ \cosh\eta &= E/E_T \end{aligned} \right\} \cosh\Delta\eta = \cosh\eta^\ell\cosh\eta^\nu - \sinh\eta^\ell\sinh\eta^\nu$$

$$= \boxed{m_\ell^{\,2} + m_\nu^{\,2} + 2\big(E_T^\ell E_T^\nu \underbrace{\cosh\Delta\eta}_{\cosh x \,\geq\, 1} - \vec{p}_T^{\,\ell}\cdot\vec{p}_T^{\,\nu}\big)}$$

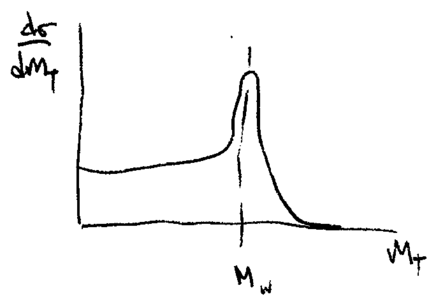
$$\boxed{M_W^{\,2} \geq m_\ell^2 + m_\nu^{\,2} + 2\big(E_T^\ell E_T^\nu - \vec{p}_T^{\,\ell}\cdot\vec{p}_T^{\,\nu}\big) \equiv m_T(\vec{p}_T^{\,\ell}, \vec{p}_T^{\,\nu}; m^\nu)^2}$$

of course $\vec{p}_T^{\,\ell} = -\vec{p}_T^{\,\nu}$

AND FOR THIS CASE $m_\nu = 0$
m_ℓ CAN BE 'MEASURED') OR
MORE PRACTICALLY IT IS
A SM PARTICLE OF
KNOWN MASS .

bare w/ me as I
write things
excessively pedagogically

THIS DISTRIBUTION IS NOW INSENSITIVE TO UCM of W
(included in definition already!)



AND THAT'S HOW THEY MEASURED M_W.
IT WOULD BE GREAT IF WE COULD DO THE SAME ANALYSIS TO WEIGH NEW PARTICLES @ THE LHC
... in a *MODEL - INDEPENDENT* WAY.

~~[illegible]~~
<u>MOTIVATION</u>: DARK MATTER (WIMP) MOTIVATES $\exists$ OF BSM PARITY
eg. R PARITY IN MSSM

($\Rightarrow$ ① NP IS PAIR PRODUCED)
$\Rightarrow$ ② $\exists$ 'LSP' $\rightsquigarrow$ missing energy

<u>PROBLEMS</u>

- NP IS PAIR PRODUCED $\Rightarrow$ ANALOG IS DIBOSON PRODUCTION
  we now have <u>two</u> particles contributing to MISSING E

  $\rightarrow$ bigger problem: each chain may be different
  THIS IS A CURRENT RESEARCH TOPIC (eg. w/ Felix)
  WE WILL FOCUS ON THE SIMPLE CASE.
  (IT IS WELL MOTIVATED: ~~[illegible]~~

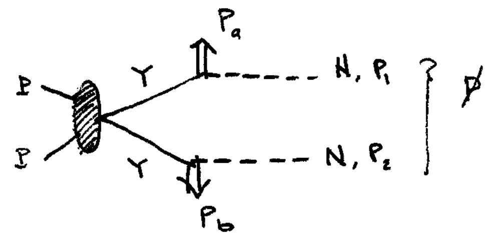
  $$\tilde{g}\tilde{g} \to qq\chi^0 + qq\chi^0 \qquad 0709.0288$$)

- WE WANT TO MEASURE MASS OF INTERMEDIATE PARTICLES BUT WE ALSO DON'T KNOW (? WOULD LIKE TO MEASURE) THE MASS OF THE FINAL STATE LSP.

  $\rightsquigarrow$ WE CANNOT IGNORE ANALOG OF m_ν

<u>naively</u>: no hope to measure anything meaningful in principle
[THIS MIGHT STILL BE THE PRACTICAL TRUTH, BUT THAT'S AN "EXPERIMENTAL ISSUE", FOR NOW CARE ABOUT 'IN PRINCIPLE']

WE 'SATURATED' OUR INFORMATION WITH THE W MEASUREMENT, AND NOW WE'RE MAKING IT 2x AS COMPLICATED.



P_a & P_b ARE KNOWN (OBSERVABLE) COMBINATIONS OF 4 MOMENTA. WE CAN TREAT THEM AS A SINGLE PARTICLE.

WHAT WE WROTE DOWN BEFORE:

$$m_T{}^2(\vec{p}_T^{\,a}, \vec{p}_T^{\,1}; m_N) \equiv m_a{}^2 + m_N^2 + 2(E_T^a E_T^1 - \vec{p}_T^{\,a}\cdot\vec{p}_T^{\,1})$$

$$\qquad\qquad\;\uparrow\quad\uparrow\qquad\qquad\quad\uparrow\qquad\quad\uparrow\qquad\uparrow$$

$$\qquad\qquad\;?\quad\;?\qquad\qquad\quad?\qquad\quad?\qquad?$$

and the assignment of $\vec{p}^{\,1}$ w/ $\vec{p}^{\,a}$ vs $\vec{p}^{\,b}$

IF WE KNEW $\vec{p}_T^{\,1}$, $\vec{p}_T^{\,2}$, AND m_N, THEN WE HAVE PREVIOUS CASE TWICE:

$$m_Y{}^2 \geq m_T{}^2(\vec{p}_T^{\,a}, \vec{p}_T^{\,1}; m_N)$$
$$m_Y{}^2 \geq m_T{}^2(\vec{p}_T^{\,b}, \vec{p}_T^{\,2}; m_N)$$

WE CAN SUMMARIZE THIS BY DISCARDING THE WEAKER (REDUNDANT) STATEMENT

$$m_Y{}^2 \geq \max\left\{ m_T{}^2(\vec{p}_T^{\,a}, \vec{p}_T^{\,1}; m_N)\ ,\ m_T{}^2(\vec{p}_T^{\,b}, \vec{p}_T^{\,2}; m_N) \right\}$$

OK. NOW LET'S DEAL WITH OUR IGNORANCE OF $\vec{p}_T{}^1, \vec{p}_T^{\,2}$. WHAT WE KNOW:

$$\vec{p}_T{}^1 + \vec{p}_T{}^2 = \not{p}_T$$

SO ALL WE REALLY DON'T KNOW IS HOW $\not{p}_T$ IS PARTITIONED BETWEEN $\vec{p}_T^{\,1}$ AND $\vec{p}^{\,2}_T$.

<u>TRICK</u>: <u>SCAN</u> OVER PARTITIONS.

@ THE CORRECT PARTITION, WE GET A TRUE INEQUALITY.
@ INCORRECT PARTITIONS
- RHS TOO BIG, INEQUALITY IS FALSE (BAD)
- RHS TOO SMALL, INEQUALITY STILL TRUE, BUT WEAKER

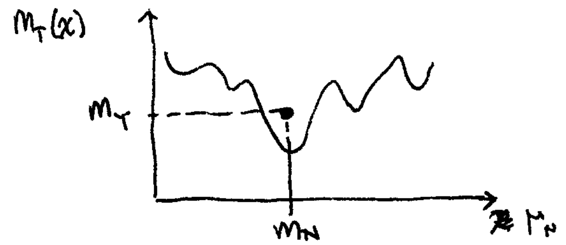
SO AS LONG AS WE JUST WANT A TRUE STATEMENT, JUST TAKE THE MOST CONSERVATIVE VALUE.

$$m_Y{}^2 \geq \left[M_{T2}(m_N)\right]^2 \equiv \min_{\not{p}_T^1 + \not{p}_T^2 = \not{p}_T} \max\left\{ m_T^2(\vec{p}_T^{\,a}, \vec{p}_T^{\,1}; m_N),\ m_T^2(\vec{p}_T^{\,b}, \vec{p}_T^{\,2}; m_N) \right\}$$

IDEA: WE GET A VALUE OF M_{T2} FOR EACH EVENT. AFTER MANY EVENTS WE'LL SATURATE THE BOUND & CAN DETERMINE ENDPOINT.

THE ELEPHANT IN THE ROOM: WTF is m_N?

M_{T2} IS A <u>FUNCTION</u>: $M_{T2} = M_{T2}(\mu_N)$
SUCH THAT AT $\mu_N = M_N$, WE GET A TRUE BOUND.
AWAY FROM $\mu_N = M_N$, ALL BETS ARE OFF.



@ THIS POINT ($\mu_N = M_N$)

$$M_Y^2 \geq M_{T_2}(M_N)$$

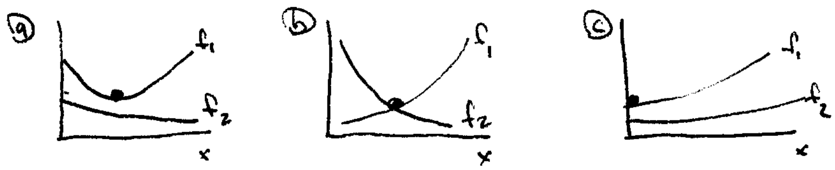
monte carlo: can depend sensitively on x!

BUT UNLESS WE KNOW M_N ALREADY, THIS IS USELESS.
SITUATION IS EVEN WORSE: FOR $x \neq M_N$, M_{T2} ISN'T EVEN INVARIANT UNDER TRANSV. BOOSTS!

WHAT I WILL NOT DISCUSS: PF THAT WE SATURATE THE BOUNDS. SEE, eg, hep-ph/0304226 §3.3. WE'LL DISCUSS THIS FROM THE KINEMATIC VARIABLE POINT OF VIEW.

aside on the methods of hep-ph/0304226 §3.3

WE ARE MINIMIZING THE MAX OF 2 FUNCTIONS OVER A PARAM (PARTITION)
POSSIBILITIES:



min of f_1 when $f_1 > f_2$

$f_1 = f_2$

boundary
not relevant for M_{T2} since $M_T(\pm\infty) = \pm\infty$

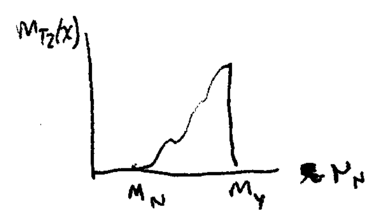
MINIMIZING UNCONSTRAINED M_T: $M_T^{min} = M_a + M_N$
SINCE BOTH BRANCHES ARE IDENTICAL, THE GLOBAL MIN OF $M_{T1}^{min} = M_{T2}^{min}$

$\Rightarrow$ WHEN f_1 IS MIN, IT IS NOT $> f_2$. SO M_{T_2} LOOKS LIKE b

can show M_{T2} saturates a <u>lower bound</u> as well,

$$\underbrace{M_N + M_a \leq M_{T2}}_{} \leq M_Y$$

CAN IDENTIFY M_N FROM LOWER ENDPOINT?
NOT REALLY: SHALLOW ENDPOINT.
CAN BE CLOSE TO ZERO, etc.
cuts?



is this true?

SO THIS WAS THE STATUS OF $M_{T2}(\overset{\mu_p}{\chi})$ FROM 99 - 2007
IT'S NOT A KINEMATIC VARIABLE, IT'S A KINEMATIC FUNCTION
WHOSE ARGUMENT IS UNKNOWN.

mention this later

BUT THEN IN 2007 SOMETHING HAPPENED.
A KAIST (KOREA) COLLABORATION FOUND THAT THE $M_{T2}(\chi)$ 0709.0288
DISTRIBUTION EXHIBITED A KINK @ $\mu_N = m_N$.

THEN SOMEONE ELSE (Gripaios) ALSO FOUND KINKS OF A DIFFERENT NATURE TO FIND m_N.

KINKS CORRESPONDED TO DIFFERENT SITUATIONS THAT DEPEND ON THE # FINAL STATE PARTICLES.

THIS LED TO THE CURRENT RENEWED INTEREST IN M_{T2}.

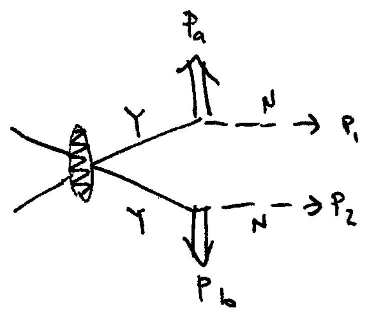
TO THE BEST OF MY KNOWLEDGE, THIS IS DISCUSSED IN 0711.4008
WHICH SORTS OUT THE NATURE OF THE VARIOUS KINKS.
(LAZY ASSES LIKE ME CAN ALSO SEE CHRIS LESTER'S IPMU 2008 TALK)

BEFORE WE GET TO THAT, THERE IS A MORE INTUITIVE WAY TO UNDERSTAND M_{T2} DUE TO CHENG (Yuhsin's friend) & Z. HAN, 0810.5178.
THIS LED TO
① DEEPER UNDERSTANDING OF M_{T2}
② QUICKER + MORE ACCURATE ALGORITHMS
③ MORE NATURAL EXPLANATION OF KINKS
→ very recent! 0908.3779

MT2 FROM MINIMAL KINEMATIC CONSTRAINTS

ad hoc DEFINITION OF $M_{T2}(\mu_N)$ ACTUALLY HAS A VERY NATURAL INTERPRETATION THAT WASN'T REALIZED BY THE ORIGINAL AUTHORS, BUT BY CHENG & HAN 0810.5178 & LED TO A RELATED INDUSTRY OF COLLIDER KINEMATICS ; eg reviewed by Tao Han WHEN HE WAS VISITING.

↳ CURRENT PICTURE: US GROUPS WORK ON 'MINIMAL KINEMATIC CONSTRAINTS'
JAPAN, KOREA, SWITZERLAND, UK WORK ON TRANSVERSE VARIABLES (& +FLORIDA)



$$p_1^2 = p_2^2 = \mu_N^2 \quad \text{test N mass}$$
$$(p_1+p_a)^2 = (p_2+p_b)^2 = \mu_Y^2 \quad \text{test Y mass}$$

$$p_1^x + p_2^x = \not{p}^x$$
$$p_1^Y + p_2^Y = \not{p}^Y$$

6 EQNS (eg if i fixed μ_N, μ_Y, wouldn't be able to reconstr. p's.)

minimal constraint

↑ SHORTEST DECAY CHAIN ?
ADDITIONAL INFO IMPOSES MASS-SHELL CONDITIONS THAT GIVE MORE CONSTRAINTS

CHENG + HAN 0802.4290 + follow ups

CLAIM: CONSTRAINTS ~~...~~ SATISFIED FOR PHYSICAL MOMENTA IFF $\mu_Y \geq M_{T2}(\mu_N)$

PHYSICAL: $\begin{cases} p_\mu \in \mathbb{R} \\ p_\bullet^0 > 0 \end{cases}$

"$M_{T2}(\mu_N)$ is the BOUNDARY ON THE (μ_N, μ_Y) PLANE SATISFYING THE MINIMAL CONSTRAINTS W/ PHYSICAL MOMENTA"

Pf. 1. $\exists$ PHYSICAL MOMENTA $\Rightarrow$ $\mu_Y \geq M_{T2}(\mu_N)$

TRIVIAL FROM OUR ORIGINAL DEF OF M_{T2} & "PHYSICAL."

2. $(\mu_N, M_{T2}(\mu_N)) \in$ PHYSICAL REGION

ALSO TRIVIAL FROM OUR DEF OF M_{T2}. $\exists$ PHYSICAL MOMENTA SUCH THAT

$$(p_T^1)^2 = (p_T^2)^2 = \mu_N^2$$
$$m_{T2}^2 = (\vec{p}_{T1} + \vec{p}_{Ta})^2 \geq (\vec{p}_{T2} + \vec{p}_{Tb})^2$$
$$\vec{p}_{T1} + \vec{p}_{T2} = \not{p}_T$$

eg. WHEN $\eta_1 = \eta_a$, $\cosh \Delta\eta_{1a} = 1$

(pf cont'd)

THEN: $(p_1+p_a)^2 = m_{T2}^2$

OTHER CHAIN:

if $(\vec{p}_{Tb}+\vec{p}_{T2})^2 = m_{T2}^2$ CHOOSE $\eta_2=\eta_b$ & $(p_2+p_b)^2 = m_{T2}^2$ ✓

if $(\vec{p}_{Tb}+\vec{p}_{T2})^2 < m_{T2}^2$

$(p_2+p_b)^2 < m_{T2}^2$ when $\eta_2=\eta_b$

$(p_2+p_b)^2 \to \infty$ WHEN $\eta_2 \to \pm\infty$

$\Rightarrow \exists\, \eta_2$ s.t. $(p_2+p_b)^2 = m_{T2}^2$

THUS: WE HAVE PHYSICAL MOMENTA p_1 & p_2 w/ $\mu_Y = m_{T2}(\mu_N)$ □

CALCULATING MT2 W/ MINIMAL KINEMATIC CONSTRAINTS

MORE EFFECTIVE (EFFICIENT + ACCURATE) ALGORITHM THAN ORIGINAL MT2 PARTITIONING SCAN. NOW PART OF 'OFFICIAL' OXBRIDGE MT2 LIBRARY

~~ALGEBRA~~

FIRST: HOW TO DETERMINE IF $(\mu_N,\mu_Y) \in$ PHYSICAL REGION

[assume $m_a>0$, $m_b>0$]

BY DEFINITION/CONSTRUCTION, M_T IS INVARIANT UNDER LONGITUDINAL BOOSTS (indep of p_z completely)

$\Rightarrow M_{T2}$ is INV UNDER ~~INDEP OF~~ ~~each boosts~~ INDEPENDENT LONGITUDINAL BOOSTS OF EACH BRANCH.

w/o loss, SET $\boxed{p_a^z = p_b^z = 0}$

[algebra]

$$(p_1+p_a)^2 = \mu_Y^2 = p_1^2 + p_a^2 + 2E_1E_a - 2\vec{p}_1\cdot\vec{p}_a = \mu_N^2 + m_a^2 + 2E_1E_a - 2\vec{p}_T^{\,1}\cdot\vec{p}_T^{\,a}$$

$$E_1 = \frac{1}{2E_a}\Big(\underbrace{2p_a^x p_1^x + 2p_a^y p_1^y}_{2\vec{p}_T^{\,1}\cdot\vec{p}_T^{\,a}} + \mu^2 - \mu_N - m_a^2\Big)$$

"PHYSICAL" CONSTRAINT: plug

$$\boxed{(p_1^z)^2 = E_1^2 - (p_1^x)^2 - (p_1^y)^2 - \mu_N^2 \geq 0}$$

TO GET SIGNS RIGHT MULT LHS & RHS BY -1 & TURN $\geq$ TO $\leq$ s.t. PHYS. REGION IS INSIDE ELLIPSE

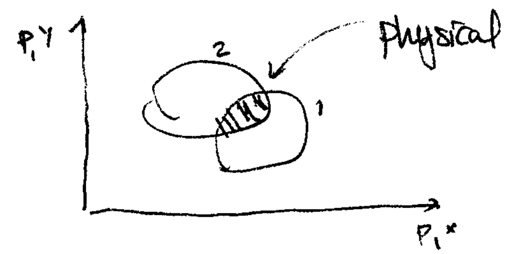
IN (p_1^x, p_1^y) SPACE, PHYSICAL REGION IS AN ELLIPSE. FURTHER, SIZE OF ELLIPSE CONTROLLED BY μ_Y → SHRINKS TO POINT @ $\boxed{\mu_Y = \mu_N + m_a}$

↑ all PARTICLES HAVE SAME VELOCITY

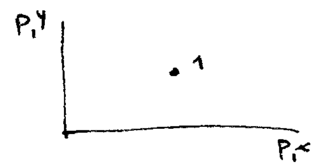
BETTER TO SAY: SIZE OF ELLIPSE DEPENDS ON μ_Y MONOTONICALLY

OTHER DECAY CHAIN: ANALOGOUS ELLIPSE IN p_2^x, p_2^y PLANE.
BUT: $\not{p}_T = \vec{p}_T^{\,1} + \vec{p}_T^{\,2}$ CONVERTS $(p_2^x, p_2^y) \to (p_1^x, p_1^y)$
SO CAN PLOT ON JUST THE (p_1^x, p_1^y) PLANE.

ELLIPSE REPRESENTS PHYSICAL REGION FOR EACH CHAIN.
PHYSICAL REGION OF WHOLE SYSTEM IS THE OVERLAP.



SO THE ALGORITHM
START WITH $M_Y = M_N + m_a$
→ 1ST ELLIPSE IS A POINT

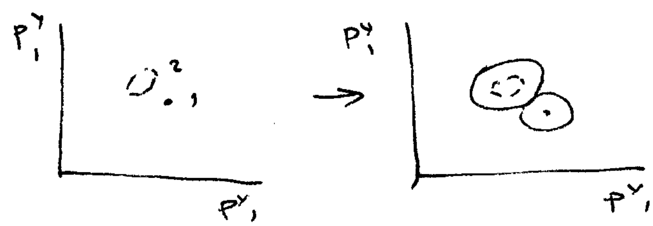


SECOND ELLIPSE HAS FINITE SIZE (OR POINT IF $m_a = m_b$)

CASE 1

THEN DONE

CASE 2



INCREASE M_Y UNTIL BOTH ELLIPSES KISS. THIS IS THE KINEMATIC BOUNDARY, GIVES $M_Y(M_N)$ VIA MT2.

NOW WE HAVE TO ADDRESS M_N DEPENDENCE OF $M_{T2}(\chi)$

1. CONTINUE W/ KINEMATIC CONSTRAINTS (Cheng + Han, Tao Han)
   ["AMERICAN" APPROACH]
   on-shell intermediate particles give additional constraints.
   EVENTUALLY OVERCONSTRAIN SYS.
   ea vertex is new on shell constr.
   (still wm even if int. mass unknown → 2 vert per unknown particle mass)

2. CONTINUE TO USE M_{T2}, BUT FIND A WAY TO IDENTIFY M_N.

   see p. 8 for my historical summary.

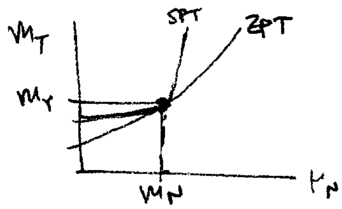
THE KINKs – BROAD PICTURE; 0711.4008 (OLD POV) + 0908.3779 (Modern POV)

KINKS HAVE DIFFERENT ORIGINS, BUT THEY TELL US ABOUT μ_N
("dimensional analysis" – μ_N IS THE ONLY OTHER MASS SCALE THAT'S RELEVANT)

CATEGORIZED BY
- # FINAL STATE PARTICLES
- UPSTREAM MOMENTUM $(\underbrace{\text{UCM} = 0}_{\text{ZPT}}, \underbrace{\neq 0}_{\text{SPT}})$

"zero P_T" "some P_T"

↑ stupid notation, if you ask me.



eg: 2 BODY FINAL STATE (JUST m_T, NOT m_{T2}); Gripaios 0709.2740

$m_T(\mu_N)$ IS NOT INVARIANT UNDER TRANSVERSE BOOSTS (SPT)
EXCEPT @ $\mu_N = m_N$. ⇒ get a kink when SPT ↑

we'll discuss this from a kinematic POV.

SEPT '07: MONTH OF KINKS, REBIRTH OF m_{T2} VARIABLE

eg. 3 BODY FINAL STATE; KAIST 0709.0288

EXTRA DEGREE OF FREEDOM: m_{vis} can vary in an event
GRADIENT OF $m_{T2}(\mu_N)$ DEPENDS ON m_{vis}
LOW m_{vis} ARE FLATTER } even @ ZPT
Hi m_{vis} ARE STEEPER } exaggerated by SPT

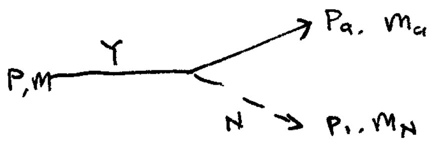
So: m_T w/ SPT KINK ≠ m_{vis} KINK
BUT SPT ALWAYS ENHANCES KINK.

GENERICALLY: HARD TO MEASURE!

BUT IN PRINCIPLE IT PROVIDES A WAY TO DETERMINE SPECTRUM, @ least in certain MODEL-INDEP. SCENARIOS.

SIMPLE KINK IN M_T (not M_{T2}!) 0709.2740 Gripaios

CLAIM: COULD HAVE DISCOVERED M_W & M_ν EVEN IF $M_\nu \neq 0$.



(to keep w/ our old notation)
COMPRESS TRANSVERSE PLANE TO 1D FOR SIMPLICITY

$$M_T(\mu_N) = m_a^2 + \mu_N^2 + 2\sqrt{\underbrace{[(p_T^a)^2 + m_a^2]}_{E_T^a}\underbrace{[(p_T^1)^2 + \mu_N^2]}_{E_T^1}} - 2p_T^a p_T^1$$

PHYSICAL CONSTRAINTS

$$\left.\begin{aligned} E - E_a - E_1 &= 0 \\ P_T - P_T^a - P_T^1 &= 0 \\ P_z - P_z^a - P_z^1 &= 0 \end{aligned}\right\} \quad p^\mu - p_a^\mu - p_1^\mu = 0$$

(need $P_T \neq 0$)

$$\left.\begin{aligned} E^2 - P_T^2 - P_z^2 - M^2 &= 0 \\ E_1^2 - (P_T^1)^2 - (P_z^1)^2 - M_N^2 &= 0 \\ E_a^2 - (P_T^a)^2 - (P_z^a)^2 - M_a^2 &= 0 \end{aligned}\right\} \quad M^2 = E^2 - P_T^2 - P_z^2$$

for each particle
note: M_N is TRUE MASS

6 EQNS, 6 UNKNOWNS: P_T^1, P_T^a; E^1, E^a; P_z^1, P_z^a
sanity check: M's ARE CONSTANT! (even μ_N; fix this)

SOLVE USING LAGRANGE MULTIPLIERS:
MINIMIZE $f + \Sigma \lambda_i g_i$ WHERE g_i's ARE LHS OF CONSTRAINT EQS.
SUBJECT TO $\{g_i = 0\}$.

[algebra]

MAX wrt E_1, E_a, P_z^1, P_z^a: $E_1 P_z^a = E_a P_z^1$
NOW COMBINE w/ CONSTRAINT EQNS, GET:

eqn for p's.

$$\boxed{M^2 = M_a^2 + M_N^2 - 2P_T^a P_T^1 + E_T^a E_T^1 = f(\mu_N = M_N)}$$

this is just the eqn we get for $\Delta\eta = 0$
KIND OF OBVIOUS! JUST SAYING M_T IS MAX @ M
NON TRIVIAL: EVEN FOR TEST MASS μ_N, MAXIMIZATION GIVES ME THE $\Delta\eta = 0$ CONFIGURATION.

skip algebra. this is the 'physics' statement

$$\underbrace{\max_{\text{KINEM.}} f(\mu_N)}_{\equiv f^{\max}(\mu_N)} = m^2 + (\mu_N^2 - M_N^2) + 2\underbrace{\sqrt{(p_T^a)^2 + m_a^2}}_{E_T^a}\left(\underbrace{\sqrt{(p_T^1)^2 + \mu_N^2}}_{E_T^1(\mu_N)} - \underbrace{\sqrt{(p_T^1)^2 + M_N^2}}_{E_T^1(M_N)}\right)$$

this is what we measure with endpoints (for μ_N fixed)

NOW TAYLOR EXPAND

$$f(\mu_N) = f(M_N) + f'(M_N)\,(\mu_N^2 - M_N^2)$$

$$= m^2 + \underbrace{(\mu_N^2 - M_N^2)\left(1 + E_T^a/E_T^1\right)}$$

THIS IS THE POINT

TO MAXIMIZE f WHEN
$\mu_N < M_N$, WANT TO MINIMIZE $(1 + E_T^a/E_T^1)$
$\mu_N > M_N$, WANT TO MAXIMIZE $(1 + E_T^a/E_T^1)$

~~[illegible]~~

THUS $f^{\max}(\mu_N)$ BEHAVES DIFFERENTLY @ $\mu_N = M_N \pm \varepsilon$!
$\Rightarrow$ THIS IS THE ORIGIN OF THE KINK.

[some algebra]

EXTREMIZE $(1 + E_T^a/E_T^1)$ SUBJECT TO PHYSICAL CONSTRAINTS.
END UP w/

$$\left(1 + \frac{E_T^a}{E_T^1}\right)_{\text{ext.}} = 1 + \frac{M^2 \pm \sqrt{M^4 - m_1^2 m_a^2}}{m_1^2},$$

$$M^2 = m^2 - m_a^2 - m_1^2$$

$p = (E, p_T, p_z)$

<u>INTERPRETATION</u>: $1 \to 2$ DECAY; KINEMATICS CONSTRAIN $1+2$ dim $\to$ $1+1$ dim w/ E_T = "true" ENERGY

KINEMATICS OF SUCH A DECAY IS FIXED IN CM FRAME.
ONLY FREEDOM IS BOOSTING DECAYING PARTICLE

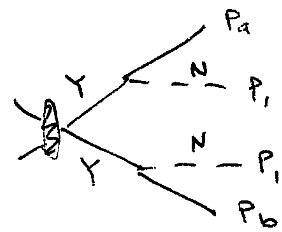
$(1 + E_T^a/E_T^1)$ MAX: ASYMP. LARGE BOOST IN p_a DIR
MIN: ASYMP LARGE BOOST IN p_1 DIR

(confirms previous statement: SPT kink)

so : MEASURE $m_T(\mu_N < m_N)$ functional form (eg gradient)
vs. $m_T(\mu_N > m_N)$ $\Rightarrow$ determine m_N

CLAIM - EASY TO GENERALIZE TO M_{T2} SCENARIO.

eg. IDENTICAL PAIR DECAY KINK w/ M_{T2} 0908.3779
BACK TO KINEMATIC BOUNDARY DEFINITION



$$p_1^2 = p_2^2 = m_N^2$$
$$(p_1+p_a)^2 = (p_2+p_b)^2 = m_Y^2$$
$$\vec{p}_T^{\,1} + \vec{p}_T^{\,2} = \not{p}_T \qquad (\star)$$

extremal BOUNDARY GIVEN BY
INTERSECTION OF ALLOWED REGIONS
FOR ALL POSSIBLE EVENT CONFIGURATIONS.
~~(intersection of all ellipses)~~
~~(2D ELLIPSE ALGORITHM~~

WHEN WE CONSIDER SPACE OF ALL EVENTS
WE ARE REALLY SCANNING OVER $\not{p}_T$ s.t. $(\star)$ CAN ALWAYS
BE SATISFIED & IT IS TRIVIAL. (ONLY WHEN CONSIDERING ALL EVENTS)

THEN THE EQUATIONS DECOUPLE
AND WE HAVE TWO COPIES OF THE M_T KINK ARGUMENT.

PROOF FOR MORE COMPLICATED SCENARIOS (eg MULTI FINAL STATE KINK) FOLLOWS SIMILAR ARGUMENTS.

- see 0908.3779 or 0711.4008

SUMMARY & A GLIMPSE OF FURTHER DIRECTIONS

WANT: MEASURE SPECTRUM OF NEW PARTICLES IN A MODEL-INDEP. WAY.

DALITZ PLOTS

M_T ← edges + endpoints (HISTORICALLY: hep-ph/9907518)

GENERALIZE TO PAIR PRODUCED NP, WIMP/LSP INVISIBLE

$M_{T2}(\mu_N)$ ⇒ determines M_Y IF WE KNOW $\mu_N = M_N$

KINK ANALYSIS ← - - - → INTERPRETATION AS MINIMAL KINEMATIC CONSTRAINT

natural language

NONMINIMAL KINEMATIC CONSTRAINTS

- CHENG & HAN 0905.1344

ANTLERS & CUSPS

- TAO HAN et al 0906.5009

ASSOCIATED TRANSVERSE VARIABLES

- MTGEN ("general # final particles") 0708.1028
- MCT "CONTRANSVERSE MASS" 0802.2879, 0803.3344 MT2 ($\mu_N = 0$) & ZPT
- M2C & M2CUB: 0712.0943, 0806.3224 SPT ANALYSIS
- M_{T2}^{-1}: 0908.3779

... DIFFERENT DECAY CHAINS many people; eg. FT.