FLIP TANEDO: FEYNMAN IN SUPERSPACE    6 FEB 2010

CORNELL BSM JOURNAL CLUB; SPRING 2010

SUPERGRAPH TECHNIQUES (FEYNMAN RULES ON $\mathbb{R}^{4|4}$ SUPERSPACE) ARE AN ANCIENT TOOL (70s-80s) ANALOGOUS TO BLOOD LETTING. MODERN TECHNIQUES (mostly by Seiberg) HAVE MADE THEM UNIMPORTANT & THEY ARE OMITTED IN TEXTBOOKS POST Y2K. WEINBERG'S QFT SERIES, WHICH IS NORMALLY VERY COMPREHENSIVE, ONLY GIVES A NOD TO THE SUBJECT. WHY SHOULD WE CARE?

1. CURIOSITY - my primary motivation
2. COMPLETENESS - Feynman: GOOD PHYSICISTS CAN DERIVE RESULTS IN MULTIPLE WAYS
3. ELEGANCE - GEOMETRY IS BEAUTIFUL
4. REVIEW QFT - 'relearn' key methods by working with a more general system with rich structure.
5. APPLICATIONS - EFFECTIVE KÄHLER POT, SUPERGRAVITY, ...

BIG PICTURE: WE KNOW THAT SYMMETRY SIMPLIFIES OUR LIVES BY GROUPING TOGETHER THINGS WHICH ARE REALLY DIFFERENT MANIFESTATIONS OF A SINGLE "THING".

eg. POINCARÉ : PARTICLE IN DIFF FRAME IS THE SAME
$SU(3)_c$ : $u_R, u_G, u_B$ BEHAVE 'IDENTICALLY'
$SU(3)_F$ : $e, \mu, \tau$ ARE THE SAME (UP TO FLAVOR EFFECTS)

ie WE WORK W/ ENTIRE MULTIPLETS UNDER THE SYM. MUCH EASIER THAN WORKING W/ COMPONENT FIELDS, often EVEN WHEN SYMMETRY IS BROKEN! (cf FLAVOR & SPURION ANALYSIS; see also RECENT PAPER ON COVARIANT FORMULATION.)

THIS IS WHAT WE OBSERVE (4D REPS) + SUSY IS BROKEN

BUT IN SUSY, WE ALWAYS WORK IN COMPONENTS!

A GOOD ANALOGY: 5D THEORIES [5D: extend SM w/ BOSONIC XD; SUSY: extend w/ FERMIONIC XD]
JUST LIKE SUSY, WE TYPICALLY WORK WITH 4D REPS: KK REPS = 5D MULTIPLET.
5D POINCARÉ CAN EVEN BE BROKEN: BRANES, WARPING, SOLITONS
HOWEVER: THERE ARE CASES WHERE A FULLY 5D FORMALISM IS BENEFICIAL
→ Just ask Flip & Yuhsin! CAN FIND FINITENESS ARGUMENTS IN 5D WHICH ARE SUBTLE IN 4D. WE WILL DO AN ANALOGOUS THING IN SUPERSPACE!

REMARK: one 'clear' shortfall: CANNOT (so far) CANONICALLY NORMALIZE IN SUPERSPACE (Buchbinder p. 186)

THIS BOOK IS REALLY JUST A PURE JOY TO READ. VERY GEOMETRIC INTUITION W/ RIGOROUS FORMALISM.

REFS

- ANY REALLY OLD SUSY BOOK
- West, Wess+Bagger, Misra ARE ESPECIALLY GOOD
- THE BIBLE OF MATHEMATICAL SUPERSPACE: Buchbinder & KUZENKO
- LECTURE NOTES BY C. Sämann

SERIOUS CAVEAT: I HAVE BEEN VERY SLOPPY WITH MINUS SIGNS! DO NOT TRUST THEM! (fortunately they don't really matter for our purposes.)

**Review**: STUFF YOU ALREADY KNOW*

IF YOU DON'T ALREADY KNOW THIS STUFF THEN YOU SHOULD LEAVE + LEARN IT ASAP!

# Path Integral Formalism

$$d^4\theta = d^2\theta\, d^2\bar\theta$$

$$\underset{\text{ACTION}}{S} = \int \underset{\substack{\text{SPACETIME}\\\text{VOLUME FORM}}}{d^4x}\ \underset{\substack{\text{LAGRANGIAN}\\\text{DENSITY}\\\text{OVER SPACETIME}}}{\mathcal{L}} \xrightarrow{\text{SUPERSPACE}} S = \int \underbrace{d^4x\, d^4\theta}_{\text{VOLUME OVER } \mathbb{R}^{4|4}} \mathcal{L}$$

**BUT**: YOU ALREADY KNOW $\mathcal{L}^{d^4x} \sim \int d^4\theta\, K + \left[\int d^2\theta\, W + h.c.\right]$
so this will require some finesse

$$\boxed{S = \underset{\text{FREE PART OF ACTION: QUADRATIC, EXPLICITLY SOLVABLE}}{S_0} + S_{int}}$$

INTERACTION PART. HAVE TO SOLVE VIA PERTURBATION THY.

- $S_0$ GIVES OUR FIELD PROPAGATORS.
  '**EASY**': JUST INVERT THE QUADRATIC OPERATOR ('kinetic + Mass')

  You already know:

  $$\langle 0|T\,\varphi(x)\varphi^\dagger(y)|0\rangle = i\Delta(x-y) \qquad \frac{1}{p^2-m^2} \sim \frac{1}{-\Box^2-m^2}$$

  $$\langle 0|T\,\psi_\alpha(x)\bar\psi_{\dot\beta}(y)|0\rangle = \underbrace{\sigma^\mu_{\alpha\dot\beta}\partial_\mu}\Delta(x-y)$$

  † $\alpha\alpha$, $\dot\alpha\dot\alpha$ PROPS HAVE MASS INSERTIONS. eg $i\delta_\alpha^\beta m\Delta(x-y)$

  Dirac operator acting on KG green's func!
  ⇒ HINT THAT WE SHOULD BE ABLE TO TREAT DIFFERENT COMPONENTS OF A SUPERMULTIPLET IN A UNIFIED WAY!

- $S_{int}$ GIVES OUR INTERACTIONS.
  IN THE PATH INTEGRAL FORMALISM, WE CONSTRUCT THE GENERATING FUNCTIONAL

  $$Z[J,\dots] = \underset{\text{NORMALIZ.}}{(Z_0^{-1})} \int \underset{\text{PATH INTEGRAL}}{\mathcal{D}[\text{fields}]} \exp\left[\underset{\text{free}}{S_0} + \underset{\text{interaction}}{S_{int}} + \underset{\text{source} \sim \int d^4x\, J\varphi}{S_{source}}\right]$$

  not really necessary in this object.

  THEN WE CAN READ OFF INTERACTIONS BY TAKING FUNCTIONAL DERIVATIVES wrt THE SOURCE(s)

  $$G(x_1,\dots,x_n) = \frac{1}{Z[0]}\frac{\delta^n Z[J]}{\delta J(x_1)\cdots\delta J(x_n)}\bigg|_{J=0}$$

  POSITION SP. BUT IT IS 'trivial' TO FOURIER TRANSFORM.

  PERT THY (ie SINCE WE CAN SOLVE $Q_2$)

  $$Z[J,\dots] = \exp\left(iS_{int}\left[\tfrac{\delta}{\delta J}\right]\right) Z_0[J]$$

  $$Z_0[J,\dots] = \int \mathcal{D}(\text{fields}) \exp\left(iS_0 + \underset{S_{source}}{\int d^4x\, J\varphi}\right)$$

THIS IS ALL VERY EASY TO SEE DIAGRAMMATICALLY & YOU ARE ALL VERY FAMILIAR W/ IT. YOU ALSO KNOW THAT FROM $Z$ WE CAN DEFINE MORE SOPHISTICATED OBJECTS LIKE $W$ (generating functional of connected diagrams) & $\Gamma$ (effective action = generating functional of 1PI diagrams). THE USEFUL THING TO KEEP IN MIND IS THAT WE CAN ACTUALLY USE $\Gamma$ (calculated @ some loop order) TO WRITE A TREE-LEVEL "QUANTUM RESUMMED" ACTION. WE'LL GET TO THIS @ THE END OF MY TALK.

GOAL: SUPERSYMMETRIZE ALL THIS! (& treat in manifestly susy'ic way)

$$\int d^4x \longrightarrow \int d^4x\, d^4\theta = \int d^4x\, d^2\theta\, d^2\bar\theta \equiv \int d^8 z$$

this is the volume form for SUPERSPACE

REMARK: IT IS EVEN MORE ELEGANT TO WORK IN A COMPLEXIFIED SUPERSPACE $\mathbb{C}^{4|2}$ & TREAT "OUR" SUPERSPACE AS A SUBSPACE.

cf: STRING THY: WORLDSHEET COORDS ARE $(z,\bar z) \sim \mathbb{C}^2$, BUT PHYSICAL WORLDSHEET IS $\mathbb{R}$ SUBSPACE.

$\mathbb{R}^{4|4}$ ← 4 ANTICOMMUTING DIRECTIONS $\sim \theta^\alpha, \bar\theta_{\dot\alpha}$

4 COMMUTING DIRECTIONS $\sim \mathbb{R}^{3,1}$

note: SUPERSPACE HAS A VERY RICH GEOMETRIC STRUCTURE, eg. SUSY COVARIANT DERIVATIVE AS A HORIZONTAL LIFT, KSF AS A PROJECTION, SUPERGRAVITY, ...

I EXPECT YOU ALL TO BE FAMILIAR W/ GRASSMANNIAN COORDINATES! [eg from SUSY, PATH INTEGRAL FOR FERMIONS, BRST METHOD, ... ]

IF NOT:
1. GET YOUR SHIT TOGETHER.
2. JUST RECALL THAT THEY HAVE FUNNY PROPERTIES
   - ANTICOMMUTING
   - INTEGRATION & DIFFERENTIATION ARE THE SAME

SUPERSPACE IS A MAGICAL LAND WHERE FERMIONS LIVE AS HIGHER-DIMENSIONAL "SHADOWS" OF SCALARS. LET US CONSIDER ONLY THE SIMPLEST CHIRAL SUPERMULTIPLETS. ON THE $\theta = \bar\theta = 0$ SLICE OF SUPERSPACE ONE HAS A THEORY OF COMPLEX SCALARS.

↳ FOR YUHSIN: THIS IS LIKE CSABA'S BRANE EFFECTIVE ACTION!

BUT AS ONE OF THESE COMPLEX SCALARS 'PROPAGATES' (via interaction) INTO THE BULK [eg "KICKED INTO THE BULK" VIA INT. W/ OTHER SCALARS], IT MAGICALLY BECOMES A FERMION (Χral) AND REVEALS ITS TRUE NATURE AS A SUPERFIELD.

"Propagation" in GRASSMANNIAN DIRECTIONS = SUSY TRANSFORMATION.

cf: "PROPAGATION" IN $SU(2)_L$ (gauged) IS A GAUGE TRANSFORM or "PROP" IN $SU(3)_f$ IS A FLAVOR TRANSFORM.

WHAT WE EXPECT: SUPERGRAPHS GIVE PROPAGATION THROUGH SUPERSPACE
WE CAN PICK OUT COMPONENTS TO GET INDIVIDUAL MINKOWSKI SPACE DIAGRAMS.

↳ cf. VECTOR BOSON SCATTERING OR SPINOR SCATTERING
[LORENTZ MULTIPLETS] WE PICK EXTERNAL STATES
(ie $\epsilon_\mu$ or $u(p)$'s) AND GRAPH TELLS US AMPLITUDE.

THAT'S KIND OF BORING. THE REAL POWER IS @ LOOP LEVEL.
cf 5D ANALOGY: USING 5D METHODS LOOP DIAGRAMS AUTOMATICALLY
'RESUM' KK TOWER.

IN SUSY, SUPERGRAPHS WILL AUTOMATICALLY SUM OVER INTERNAL FERMION
+ BOSON STATES. IN PARTICULAR, FERMION-BOSON CANCELLATIONS ARE
MANIFEST & 'TRUE' DIVERGENCE STRUCTURE CAN BE SEEN DIRECTLY.

allons-y! LET'S GET TO WORK.

WE WILL RESTRICT OURSELVES TO THEORIES OF CHIRAL SUPERFIELDS
(ie IGNORE VSF, SUPERGRAVITY, etc.) THIS TURNS OUT TO CAPTURE MOST
OF THE FORMALISM.

↳ in fact, it is more complex than VSF (fun!), THOUGH
VSF HAS ADDITIONAL SUBTLETIES ABOUT GAUGE FIXING.

SO: IF WE COULD WRITE DOWN $S_{SUSY} \sim \int d^4x\, d^4\theta\, \mathcal{L}$
THEN WE'RE ALMOST DONE.

BUT YOU KNOW THAT LIFE ISN'T THAT KIND.

$$S_{\chi SF} = \underbrace{\int d^4x\, d^4\theta\, K(\Phi, \Phi^\dagger)}_{\text{KÄHLER TERM ALREADY SET} \to \text{KINETIC TERMS}} + \underbrace{\left[\int d^4x\, d^2\theta\, W + h.c.\right]}_{\text{SUPERPOTENTIAL: needs work!}}$$

STRATEGY: USE SOME SUSY VOO-DOO IDENTITIES
TO CONVERT $d^4x\, d^2\theta \longrightarrow d^4x\, d^4\theta$

CAN IT BE DONE? YES!
JUST NEED TO CONSTRUCT AN OPERATOR THAT
PROJECTS ONTO THE χSF "HALF SUPERSPACE"

BY THE WAY, PHYSICAL MEANING OF THIS "HALF SUPERSPACE"
SHOULD BE CLEAR: χSF DOESN'T HAVE $\bar{\theta}$ COMPONENTS, CAN'T
PROPAGATE THERE. A FANTASTIC & ELEGANT (though mathematical)
PRESENTATION CAN BE FOUND IN Buchbinder + Kuzenko §2.5.

OBSERVE

1. $\int d^4x\, d^2\bar\theta\, f(x,\theta,\bar\theta) = \int d^4x\, d^2\bar\theta\, f_{\bar\theta\bar\theta}\,\bar\theta\bar\theta = \int d^4x\, f_{\bar\theta\bar\theta}$

extra superspace integral that we want to include

expression without the $d^2\bar\theta$ integral.

$$f = f + f_\theta \theta + f_{\bar\theta}\bar\theta + f_{\theta\theta}\theta^2 + \cdots$$

DROP TOTAL $d^4x$ DERIVATIVES ie only true under the $d^4x$ ~~[illegible]~~ INTEGRAL.

2. $\int d^4x\, -\frac{1}{4}\bar D^2 f(x,\theta,\bar\theta) = \int d^4x\, -\frac{1}{4}\bar D^2 f_{\bar\theta\bar\theta}\,\bar\theta\bar\theta = \int d^4x\, f_{\bar\theta\bar\theta}$

(EXPAND, KEEP ONLY SURVIVING TERM)

$$\left.\begin{aligned} D_\alpha &= \frac{\partial}{\partial\theta^\alpha} + i\sigma_{\alpha\dot\alpha}{}^\mu\bar\theta^{\dot\alpha}\partial_\mu \\ \bar D_{\dot\alpha} &= -\frac{\partial}{\partial\bar\theta^{\dot\alpha}} - i\theta^\alpha\sigma^\mu{}_{\alpha\dot\alpha}\partial_\mu \end{aligned}\right\}$$

SUSY COVARIANT DERIVATIVES

factor of $-4$ comes from $\int d^2\theta = -\frac{1}{4}\frac{\partial}{\partial\theta^\alpha}\frac{\partial}{\partial\theta_\alpha}$

ASIDE: THIS IS ACTUALLY A GEOMETRIC OBJECT (along w/ $Q$'s)
- LEFT/RIGHT TRANSLATION IN SUPERSPACE (see Wess + Bagger)
- PUSH FORWARD OF VECTOR ($\in$ Algebra) OF A LIE GROUP MANIFOLD
  → see, eg, Buchbinder & Kuzenko p. 170
- PROJECTION OPERATOR ONTO CHIRAL SUBSPACE
  → eg. $\bar D\Phi = 0$ ; we'll see this more formally
- HORIZONTAL LIFT OF 'SUPER' FIBER BUNDLE
  → see eg. Göckeler & Schücker, Azcárraga & Izquierdo

SO: ① + ②: $\boxed{\int d^4x \int d^2\bar\theta \equiv +\frac{1}{4}\int d^4x\, \bar D^2}$ (up to sign)

THIS GIVES US A GUIDING PRINCIPLE:

IF YOU CAN PULL OUT A $\bar D^2$ FROM AN INTEGRAND (a chiral integrand ($d^2\theta$))
THEN YOU CAN CONVERT IT INTO A $d^2\bar\theta$.

IT IS 'OBVIOUS' THAT SIMILARLY PULLING OUT A $D^2 \to d^2\theta$.

OK, GOOD. BUT HOW DO WE GO ABOUT PULLING OUT $\bar D^2$s OUT OF OUR ASSES?

SOLUTION: ANOTHER MAGICAL IDENTITY.

MAGIC : $\boxed{\mathbb{1} = + \frac{\bar{D}^2 D^2}{16\,\Box}}$ $\qquad \Box = \partial^2 = \partial_t^2 - \vec{\partial}^2$

caveat: I'M NOT ~~BEING~~ CAREFUL WITH SIGNS (& all these ~~old~~ references use GR metric!). BUT SIGNS WON'T BE TOO IMPORTANT FOR WHAT WE'RE INTERESTED IN.

Pf/
$$\bar{D}^2 D^2 \Phi \equiv \bar{D}_{\dot\alpha} \bar{D}^{\dot\alpha} D^\beta D_\beta \Phi$$
$$= \bar{D}_{\dot\alpha} \left( \{\bar{D}^{\dot\alpha}, D^\beta\} - D^\beta \bar{D}^{\dot\alpha} \right) D_\beta \Phi$$
$$= \left[ \{\bar{D}^{\dot\alpha}, D^\beta\}\{\bar{D}_{\dot\alpha}, D_\beta\} - \bar{D}_{\dot\alpha} D^\beta \{\bar{D}^{\dot\alpha}, D_\beta\} \right] \Phi$$

$\{\bar{D}_{\dot\alpha}, D^\beta\}\{\bar{D}^{\dot\alpha}, D_\beta\}\Phi$ since extra term vanishes via $\bar{D}\Phi = 0$

USING: $[D, \{D, \bar{D}\}] = 0$

$[\bar{D}, \{D, \bar{D}\}] = 0$

$\leftarrow \{D, \bar{D}\} = -2\sigma^\mu \partial_\mu$

& $\partial_\mu$ COMMUTES

$$= +4 \left[ (\bar\sigma^\mu)^{\dot\alpha\beta} (\sigma^\nu)_{\beta\dot\alpha} + (\bar\sigma^\nu)^{\dot\alpha\beta} (\sigma^\mu)_{\beta\dot\alpha} \right] \partial_\mu \partial_\nu \Phi$$
$$= +4 \, \mathrm{Tr}\, \mathbb{1}_2 \, (+2\eta^{\mu\nu}) \, \partial_\mu \partial_\nu \Phi$$
$$= +16 \, \Box \Phi$$

LET'S PUT THIS TO WORK - QUADRATIC PART ($S_{free}$), FIX MASS TERM IN $W$

CONSIDER $W_2 = \frac{1}{2} m \Phi^2$

$$\int d^4x \, d^2\theta \; \Phi \underset{\mathbb{1}}{\,} \Phi = \int d^4x \, d^2\theta \; \Phi \left( \frac{+\bar{D}^2 D^2}{16\,\Box} \right) \Phi$$
$$= \int d^4x \, d^2\theta \left( -\tfrac{1}{4} \bar{D}^2 \right) \left\{ \Phi \frac{+D^2}{4\Box} \Phi \right\}$$

$\leftarrow$ CAN PULL THIS OUT SINCE $\bar{D}\Phi = 0$

$$= \int d^4x \, d^4\theta \; \Phi \frac{+D^2}{4\Box} \Phi$$

SO INCLUDING THE CANONICAL KÄHLER POTENTIAL

$$S_{free} = \int d^4x \, d^4\theta \; \underbrace{\Phi^\dagger \Phi}_{K} + \underbrace{\tfrac{1}{2} m \Phi \left( \frac{+D^2}{4\Box} \right) \Phi + \tfrac{1}{2} m \Phi^\dagger \left( \frac{+\bar{D}^2}{4\Box} \right) \Phi^\dagger}_{W_2}$$

NOW, AS WE SAID BEFORE THE FREE ACTION GIVES US OUR PROPAGATORS

$$Z_0[J, \bar{J}] = \int \mathcal{D}\Phi \, e^{iS_{free} + iS_{source}}$$

$\uparrow$ $i \int d^4x \, d^2\theta \, J\Phi + \text{h.c.}$

SINCE $J$ IS ALSO A χSF.

$$= i \int d^4x \, d^4\theta \; J \left( \frac{D^2}{4\Box} \right) \Phi + \text{h.c.}$$

WE NEED TO KNOW HOW TO TAKE FUNCTIONAL DERIVATIVES
AGAIN, THERE IS A SUBTLETY FOR XSF BECAUSE THEY ARE CONSTRAINED

FOR AN UNCONSTRAINED SUPERFIELD S , $\frac{\delta S(x',\theta',\bar\theta')}{\delta S(x,\theta,\bar\theta)} = \delta^4(x-x')\,\delta^2(\theta-\theta')\,\delta^2(\bar\theta-\bar\theta')$

NOTE THAT $\delta$ FUNCTIONS IN SUPERSPACE ARE ALSO VERY SIMPLE : $\delta^2(\theta) = \theta^2$

WHAT ABOUT CHIRAL SUPERFIELD? (cf Buchbinder & Kuzenko p.207)

$$\delta\Phi(x',\theta',\bar\theta') = \int d^4x\, d^2\theta\, d^2\bar\theta\; \delta\Phi(x,\theta,\bar\theta)\; \delta^8(z-z') \quad \leftarrow = \delta^4(x-x')\,\delta^2(\theta-\theta')\,\delta^2(\bar\theta-\bar\theta')$$

$$= \int d^4x\, d^2\theta\, \frac{\bar D^2}{4}\,\delta^8(z-z')\;\delta\Phi$$

$$\Rightarrow \boxed{\frac{\delta\Phi(z')}{\delta\Phi(z)} = \frac{1}{4}\bar D^2\,\delta^8(z-z')}$$

(AGAIN MAYBE OVERALL SIGN DEPENDING ON CONVENTIONS)

SANITY CHECK :

HOLOMORPHIC

$$\frac{\delta}{\delta\Phi(x,\theta)}\int d^4x'\int d^2\theta'\, f(\Phi(x',\theta')) = \int d^4x'\, d^2\theta'\, f'(\Phi)\,\frac{\delta\Phi(x',\theta')}{\delta\Phi(x,\theta)}$$

XRAL

$$= \int d^4x\, d^2\theta'\, f'(\Phi)\cdot\frac{\bar D^2}{4}\,\delta^8(z-z')$$

THIS IS THE KEY TECHNIQUE, CAN INTEGRATE BY PARTS SINCE f HOLOM.

$$= \int d^4x\, d^2\theta'\,\frac{\bar D^2}{4}\left[f'(\Phi)\,\delta^8(z-z')\right]$$

$$= f'(\Phi) \quad \checkmark$$

WE WANT TO SEE THIS IN ACTION IN THE PARTITION FUNCTION
eg IN THE SOURCE TERM [cf $W_2$ term]

$$\int d^4x'\, d^2\theta'\,\Phi(x',\theta')\,J(x',\theta') = +\int d^4x'\, d^4\theta'\;\Phi\,\frac{D^2}{4\Box}\,J \quad \checkmark$$

VARIATION & D COMMUTE

$$\frac{\delta}{\delta J(z)}\int d^4x'\, d^2\theta'\;\Phi J \quad = +\int d^8z'\;\Phi(z')\,\frac{D^2}{4\Box}\cdot\frac{\delta}{\delta J(z)}\,J(z')$$

$$= +\int d^8z'\;\Phi(z')\,\frac{D^2}{4\Box}\cdot\frac{1}{4}\bar D^2\,\delta^8(z-z')$$

$$= \int d^8z'\,\underbrace{\left(\frac{+D^2\bar D^2}{16\Box}\right)}_{\equiv\,\mathbb{1}}\Phi(z')\;\delta^8(z-z')$$

$$= \Phi(z) \quad \checkmark$$

SO LET'S SOLVE THE FREE ACTION → PROPAGATOR FOR XSF

WRITE:
$$X = \begin{pmatrix} \Phi \\ \bar\Phi \end{pmatrix} = \begin{pmatrix} \Phi \\ \Phi^+ \end{pmatrix}$$ field (XSF + $\bar{X}$SF)

$$K = \frac{1}{4\Box}\begin{pmatrix} D^2 J \\ \bar D^2 \bar J \end{pmatrix}$$ source

$$Z_0[K] = \int \mathcal{D}^2 X \; e^{i\int d^8z \, \frac{1}{2}\bar X A X \; + \bar X K \; + \bar K X}$$

we take
$\bar X = (\bar\Phi \;\; \Phi)$

OTHER REFS DO NOT DO THIS, SO A TAKES A ROTATED FORM.

$$A = \begin{pmatrix} 1 & \frac{m}{4}\bar D^2/\Box \\ \frac{m}{4} D^2/\Box & 1 \end{pmatrix}$$

THERE ARE UGLIER WAYS OF WRITING THIS THAT SOME BOOKS PREFER. *

$$A^{-1} = \frac{\Box}{\Box + m^2}\begin{pmatrix} 1 & \frac{m}{4}D^2/\Box \\ \frac{m}{4}\bar D^2/\Box & 1 \end{pmatrix}$$

easy to check $AA^{-1} = 1$ using $\mathbb{1} = \bar D^2 D^2/16\Box$.

NOW WE CAN USE OUR USUAL GAUSSIAN INTEGRAL TRICK

$$\int \prod_k d^2 z_k \exp(-\bar z_k A_{k\ell} z_\ell + \bar u_i z_i + \bar z_i u_i) \propto \exp(\bar u_i A^{-1}_{ij} u_j)$$

$$\Rightarrow Z_0[K] = e^{iW_0[K]}$$

$$W_0[K] = \int d^8 z \; \bar J \frac{1}{\Box^2 + m^2} J + \frac{1}{2}\int d^8 z \; J \frac{\frac{m}{4\Box} D^2}{\Box^2 + m^2} J + \bar J \frac{\frac{m}{4\Box}\bar D^2}{\Box^2 + m^2}\bar J$$

---

* : cf Misra p. 72

USE:
$$D^2 \bar D^2 D^2 = 16\Box^2 D^2$$
$$\bar D^2 D^2 \bar D^2 = 16\Box^2 \bar D^2$$

$$\frac{1}{1 - m^2 \, D^2\bar D^2/16\Box^2} = 1 - \frac{m^2 D^2 \bar D^2}{16\Box^2}\left[1 - \frac{m^2}{\Box^2} + \cdots\right]$$
$$= 1 + \frac{m^2 D^2 \bar D^2}{16\Box(\Box + m^2)}$$

BUT SIGNS ARE ALL AMBIGUOUS

WE END UP WITH A SET OF PROPAGATORS (FOURIER TRANSFORMING MINK. COORDS)
( $\langle \cdots \rangle = \langle 0 | T \cdots | 0 \rangle$ ) [GRISARU - ROČEK - SIEGEL]

$$\langle \Phi(z) \bar\Phi(z') \rangle = \frac{1}{p^2 - m^2} \delta^4(\theta - \theta') \equiv \Delta_F$$

$$\langle \Phi(z) \Phi(z') \rangle = \frac{(m/4) D^2}{p^2(p^2 - m^2)} \delta^4(\theta - \theta')$$

$$\langle \bar\Phi(z) \bar\Phi(z') \rangle = \frac{(m/4) \bar D^2}{p^2(p^2 - m^2)} \delta^4(\theta - \theta')$$

$D$ acts on $z$ (not $z'$) w/ $\partial_\mu \to i p_\mu$

THESE ARE OUR PROPAGATORS!

some people write

$\Phi$ ———— $\bar\Phi$

$\bar\Phi$ —+——+— $\bar\Phi$

$\Phi$ ———+— $\Phi$

- THERE ARE 3 VARIETIES $\Phi\bar\Phi$, $\Phi\Phi$, $\bar\Phi\bar\Phi$
  of FERMION PROPAGATORS: $\chi\psi$, $\chi\chi$, $\psi\psi$
  in fact, can see the mass terms.

$\Phi$ ———— $\Phi, \bar\Phi$ $\quad$ $\bar\Phi$ ———— $\Phi, \bar\Phi$

- THE WEIRD $\frac{m}{4} D^2 \cdot \frac{1}{p^2}$ IS JUST A PROJECTION ONTO A CHIRAL SUBSPACE, eg ENFORCES CHIRAL SUPERFIELD CONDITION.

- CAN WE SEE THIS IN COMPONENTS?
  eg from Wess + Bagger P.63 ← WSB MOTIVATES USING COMPONENTS.
  WE KNOW WE CAN WRITE $\Phi = \Phi(y, \theta)$, $y = x + i\theta\sigma\bar\theta$

$$\langle \Phi(y,\theta) \Phi(y',\theta') \rangle = \langle (\varphi(y) + \sqrt{2}\theta\psi(y) + \theta^2 F(y))(\varphi(y') + \sqrt{2}\theta'\psi(y') + \theta'^2 F(y')) \rangle$$

$$= \theta'\theta' \langle \varphi(y) F(y') \rangle + \theta\theta \langle F(y) \varphi(y') \rangle + 2\theta'^\alpha \theta^\beta \langle \psi_\alpha(y) \psi_\beta(y') \rangle$$

$$= -im (\theta - \theta')^2 \Delta_F(y - y')$$

$$= -im\, \delta^2(\theta - \theta') \Delta_F(y - y')$$

( then $\frac{D^2}{4\Box} \delta^2(\bar\theta - \bar\theta')$ for full superspace + Xral PROJECTOR

WHERE $-F^\dagger = \frac{\partial W_0}{\partial \Phi} = m\varphi$

HAVING DONE THE HEAVY LIFTING, LET'S CONSIDER THE $\frac{1}{3!}\lambda\Phi$ VERTEX.
[IGNORING THE TADPOLE TERM WHICH CAN BE ABSORBED INTO A FIELD REDEFINITION, THIS GIVES US THE MOST GENERAL RENORMALIZABLE THEORY OF XSF IN $\mathbb{R}^{4|4}$.] $Z[J] = e^{iS_{int}[\delta J]} Z_0[J]$

$$S_{int}\left[\frac{\delta}{\delta J}\right] J(z_1)J(z_2)J(z_3) = \frac{\lambda}{3!}\int d^4x \underbrace{d^2\theta}_{\text{SUPERPOT.}} \frac{\delta^3}{\delta J^3(z)} J(z_1)J(z_2)J(z_3)$$

$$= \lambda \int d^4x\, d^2\theta \left[+\tfrac{1}{4}\bar{D}^2\delta^8(z-z_1)\right]\left[+\tfrac{1}{4}\bar{D}^2\delta^8(z-z_2)\right]\left[+\tfrac{1}{4}\bar{D}^2\delta^8(z-z_3)\right]$$

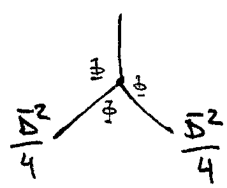
sanity check: func. derivative for XSF

TRICK: NOW PULL OUT ONE OF THE $\bar{D}^2$'s (integrate by parts & use $\bar{D}^2(\text{rest of terms}) = 0$) TO CONVERT INTO A FULL SUPERSPACE INTEGRAL.

$$= \lambda \underbrace{\int d^4x\, d^2\theta \frac{\bar{D}^2}{4}}\left[\delta^8(z-z_1)[\cdots][\cdots]\right]$$

$$= \lambda \int d^8z\ \delta^8(z-z_1)\left[\frac{\bar{D}^2}{4}\delta^8(z-z_2)\right]\left[\frac{\bar{D}^2}{4}\delta^8(z-z_3)\right]$$

GENERAL LESSON: 2 PROPAGATOR LEGS COME W/ $\bar{D}^2/4$ IN 3 PT INT.
cf MASS TERM: 1 PROPAGATOR LEG COMES W/ $\bar{D}^2/4$ IN 2 PT INT.




recall these are Xral projectors!
WE CAN ALREADY GUESS WHAT THEY DO:
PICK OUT FERMIONS IN YUKAWA INTERACTION

Q: what about scalar potential quad term?
RECALL $V(\varphi) \supset \lambda^2\phi^4$; ie FORM OF 4 PT SCALAR IS CONSTRAINED BY SUSY TO RELATE TO YUKAWA.
SUPERFIELD:

[$\bar{D}$: gives $\theta$, takes $\bar\theta$]
($\delta^8 \sim \bar\theta^2\theta^2$
the $\bar{D}$'s DO A $\bar\theta$ INTEGRAL: $\bar{D} \sim \partial/\partial\bar\theta \sim \int d\bar\theta$

$\bar{D}$'s ACT ON THE ATTACHED PROPAGATORS

but tree-level diagrams are boring anyway
TRIVIAL TO DO IN COMPONENTS, EASIER SINCE WE USUALLY ONLY CARE ABOUT SPECIFIC EXTERNAL STATES.

## (3) FEYNMAN RULES FOR $\chi$SF

- PROPAGATORS ARE ON P.9
- VERTICES FROM THE SUPERPOTENTIAL

  ↳ $n$ POINT VERTEX ($\chi$ral)
  GETS $(n-1)$ FACTORS OF $\bar{D}^2/4$ ON ATTACHED PROPAGATORS

- INTEGRATE OVER LOOP MOMENTA & $\int d^4\theta$ @ VERTICES
- (USUAL FACTORS OF $-1$ FROM GHOSTS IN GAUGE THY)

### VECTOR SUPERFIELD REMARK

VSF's ARE EASIER SINCE THEIR INTERACTIONS w/ $\chi$SF LIVE IN THE KÄHLER POTENTIAL → DON'T NEED TO FINAGLE $\chi$SF → UNCONSTR. SF

BUT: KINETIC TERMS LIVE IN FIELDSTRENGTH $\chi$SF.

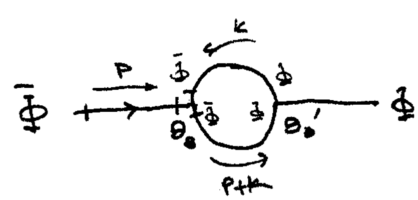
THIS ALL FOLLOWS THE ABOVE TRICKS, EXCEPT THAT ONE HAS TO DO ALL THE USUAL GAUGE FIXING. WE WON'T BOTHER.

## Raison d'être : LOOPS!

WE ARE INTERESTED IN CONSTRUCTING THE EFFECTIVE ACTION $\Gamma$ IN A LOOP EXPANSION (LEADING ORDER).

HOW TO: AMPUTATE EXTERNAL LEGS
MULTIPLY BY SOURCES $\Phi, \bar{\Phi}$ (omit $D^2/4$, $\bar{D}^2/4$)
↳ this will allow us to integrate by parts.

eg. 1-loop correction to 2 point function w/ $m=0$




USEFUL IDENTITIES:
$$\delta^4(\theta_1-\theta_2)\overleftarrow{\bar{D}}_1(-p) = -D_2(-p)\,\delta^4(\theta_1-\theta_2)$$
$$\bar{D}^2 D^2\,\delta^4(\theta'-\theta^2) = 16$$

[pf: write $\delta^2(\theta-\theta') = (\theta-\theta')^\alpha(\theta-\theta')_\alpha$ ]

$$\Gamma[\Phi,\bar\Phi] = \frac{\lambda^2}{2}\int d^4p \int d^4\theta\, d^4\theta'\ \bar\Phi(-p,\theta)\,\Phi(p,\theta')$$

sym. factor

$$\times \int d^4k\ \delta^4(\theta-\theta')\,\frac{1}{(p+k)^2}\left\{ \frac{(\bar D'(k))^2}{4}\,\frac{1}{k^2}\,\delta^4(\theta-\theta')\,\frac{(D(-k))^2}{4}\right\}$$

$$= \frac{\lambda^2}{2}\int d^4p \int d^4\theta\ \bar\Phi(-p,\theta)\,\Phi(p,\theta)$$

$$\int d^4k\ \frac{1}{k^2(p+k)^2}\ \underbrace{\frac{1}{16}\, D^2(k)\,\bar D^2(k)\,\delta^4(\theta-\theta')\Big|_{\theta=\theta'}}_{=1}$$

$$= \frac{\lambda^2}{2}\int d^4p \int d^4\theta\ \bar\Phi\Phi \cdot \int d^4k\ \frac{1}{k^2(p^\bullet+k^\bullet)^2} \sim \frac{\log\Lambda}{16\pi^2}$$

What about the other diagrams? (in the $m\neq 0$ limit)

where we've written chiral $\theta$

these are proportional to $\delta^2(\theta-\theta') = \delta^2(\bar\theta-\bar\theta') = \delta^2(0) = 0$

$\Rightarrow$ NO MASS RENORMALIZATION

sanity check: $\Phi\Phi$, $\bar\Phi\bar\Phi$ INTERNAL PROPAGATORS HAVE FUNNY FACTORS OF $\bar D^2/\Box$, BUT THESE JUST ENFORCE CHIRALITY SO THAT WE DO INDEED GET $\delta^2(\theta-\theta')$.

WHY IS $\delta^2(0) = 0$? FOR GRASSMANN VAR $\delta^2(\theta) = \theta^2$.

$\hookrightarrow$ Xral CANCELLATION THM.

$\hookrightarrow$ no miraculous cancellations between fermions + bosons.

## NOW SOME INTERESTING REMARKS

1. REGULARIZATION OF SUPERSYMMETRIC THEORIES

WE SAW THAT $\Phi\bar{\Phi}$ @ 1 LOOP IS LOG DIVERGENT
ONE OF THE BENEFITS OF THE SUPERFIELD FORMALISM IS THAT SUSY IS MANIFEST @ EVERY STAGE, SO IT WOULD BE A SHAME TO MESS IT ALL UP WHEN WE REGULATE OUR INTEGRALS.

IN OTHER WORDS, CHOOSING A BAD REGULATOR WILL VIOLATE SYMMETRY IN PERTURBATIVE CALCULATIONS. (This is why we always use dim reg – won't screw up Lorentz or internal GAUGE SYM.) WE WANT TO PRESERVE OUR WARD IDENTITIES.

IN SUSY, dim reg doesn't work!
CHANGING d MAKES IT HARD TO KEEP # BOSONS & # FERMIONS EQUAL. THE CURRENT BEST STRATEGY IS DIMENSIONAL REDUCTION (DRED) BY SIEGEL. ('79)

DO ALL $\gamma$ ALGEBRA (& HENCE $D, \bar{D}$ ALGEBRA) IN 4D
MOMENTUM INTEGRALS IN $(4-2\epsilon)$ DIM.

This is not perfect! SUSY NOT MANIFEST (still!)
HAVE TO SACRIFICE FIERZ IDENTITY + $\exists$ ANOMALIES
BUT HOLDS AT LEAST UP TO 2 LOOPS.
CURRENT STATUS: hep-ph/9707278

Remark: CAN DO NDA ON SUPER GRAPHS
see: West § 17.5
Gates § 6.6 (p. 393) hep-th/0108200

L LOOP GRAPH, V VERTICES, P PROPAGATORS : $L + V = P + 1$
C XRAL PROP ($\Phi\Phi, \bar{\Phi}\bar{\Phi}$), E EXT LINES (χral)

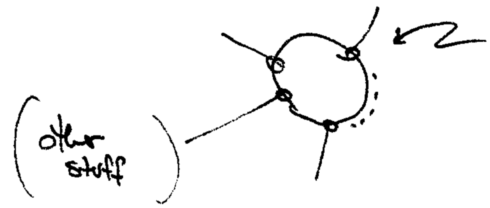
VERTICES: V factors of $D^2\bar{D}^2 \sim k^2$
PROPAGATORS: P factors of $k^{-2}$ w/ C ADDITIONAL FACTORS OF $D^2 k^{-2} \sim k^{-1}$
LOOPS: $d^4k \sim k^4$ BUT USES UP A $D^2\bar{D}^2 \sim k^2$ FACTOR
EXT LINE: EATS UP ONE $\bar{D}^2 \sim k$ @ A VERTEX (no $\bar{D}$'s on ext line)

superficial: $D = 4L - 2L - 2P + 2V - C - E = \boxed{2 - C - E}$

## 2. LOCALIZATION THM

THE EFFECTIVE ACTION IS SUPPORTED ON A <u>SINGLE</u> $d^4\theta$ INTEGRAL.
WE SAW THIS IN OUR SAMPLE CALCULATION.

CONSIDER ANY LOOP IN A DIAGRAM




ea vertex has a $\delta^4(\theta_i - \theta_j)$

$\sim \delta^4(\theta_1 - \theta_2)\,\delta^4(\theta_2 - \theta_3) \cdots \delta^4(\theta_n - \theta_1)$

THESE HAVE A BUNCH OF D'S ACTING ON THEM
CAN REDUCE TO

$$(D_1^2)^{l_1} (\bar{D}_1^2)^{k_1} \delta^4(\theta_1 - \theta_2) (D_2^2)^{l_2} (\bar{D}_2^2)^{k_2} \delta^4(\theta_2 - \theta_3) \cdots$$

using
$$\bar{D}^2 D^2 \bar{D}^2 = 16 \Box D^2$$
$$D^2 \bar{D}^2 D^2 = 16 \Box \bar{D}^2$$

CAN SIMPLIFY TO $l, k \in \{0, 1\}$

NOW DO INTEGRATION BY PARTS TO GET RID OF ALL DERIVATIVES ACTING ON, eg, $\delta^4(\theta_n - \theta_1)$. (recall total derivatives of Grassmannian quantities vanish upon integration

THEN USE $\delta^4(\theta_n - \theta_1)$ TO DO $\int d^4\theta_n$ INTEGRAL.

ITERATE THIS PROCESS UNTIL THERE'S ONLY ONE INTEGRAL LEFT : $\int d^4\theta\, (D^2)^l (\bar{D}^2)^k \ldots$ THIS IS ONLY NONZERO WHEN $l = k = 1$.

terms w/ wrong D structure vanish!
some contributions to $\Gamma = 0$ $\Rightarrow$ nonrenormalization.

eg. SUPERPOTENTIAL COUNTER TERMS CANNOT ARISE b/c THEY ARE SUPERSPACE SUB INTEGRALS
$\Rightarrow$ no $\infty$'s in renormaliz of these quantities

cf. $\Gamma \to e\gamma$ one-loop finiteness

$\Rightarrow$ NEVER MENTIONED <u>HOLOMORPHY</u>!

3. <u>CANCELLATION THM</u> ( from #2)

TO UNDERSTAND THIS BETTER, LET'S SEE IT EXPLICITLY.
<u>CLAIM</u>: CLOSED CHIRAL LOOPS VANISH.
ie $(\Phi\Phi)\cdots(\Phi\Phi)$ LOOPS $=0$.

<u>STRATEGY</u>: RETURN TO Xral COORDS

$$\frac{1}{2}\int d^8z\, J\, \frac{\frac{+m}{4\Box}D^2}{D^2+m^2}\, J = \frac{1}{2}\int d^4x\, d^2\theta\, J\, \frac{m}{\Box + m^2}\, J$$

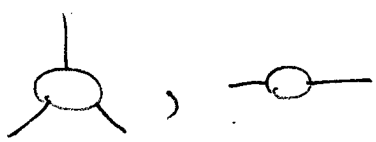
$$\Rightarrow \langle\Phi\Phi\rangle = \frac{m}{p^2+m^2}\,\delta^2(\theta-\theta')$$

then chiral loop $\sim \int \prod_j d^2\theta_j\; \delta^2(\theta_1-\theta_2)\cdots\delta^2(\theta_n-\theta_1)$

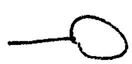
$$= \int \prod_j d^2\theta_j\; \delta^2(\theta_1-\theta_2)\,\delta^2(\theta_1-\theta_3)\cdots\delta^2(\theta_1-\theta_n)\,\delta^2(\theta_n-\theta_1)$$

[ $\delta(z_1-z_2)f(z_2) = \delta(z_1-z_2)f(z_1)$

note: $\delta^2(\theta)\delta^2(\theta) = (\theta\theta)(\theta\theta) = 0$
THUS THIS DIAGRAM VANISHES.
↳ CANCELLATION BTWN BOSONS + FERMIONS



(chiral) $\equiv 0$ !

also:  $=0$, no tadpoles!

<u>COR</u>: EFFECTIVE POTENTIAL VANISHES FOR CLASSICALLY SUSY THY
EFFECTIVE POTENTIAL is X-ind PART OF $\Gamma$

$$V = \int d^4\theta\, d^4x\;\; f(\langle\Phi\rangle, D\langle\Phi\rangle, \cdots)$$

BUT IF SUSY PRESERVED CLASSICALLY, VEVS HAVE NO $\theta$ DEPENDENCE $\Rightarrow \int d^4\theta = 0$.

(we, of course, knew this from susy algebra already!)

CAN WE STILL DO SUPERFIELDS w/ BROKEN SUSY?
YES! ~~BACKGROUND FIELD METHODS (cf BROKEN GAUGE SYM)~~
see eg. SCHOLL, Z. Phys C. 28, 545-553 (1985)

ONE FINAL REMARK

THE USEFULNESS OF SUPERGRAPH TECHNIQUES ARE EXEMPLIFIED IN THE CALCULATION OF THE EFFECTIVE KÄHLER POTENTIAL.

Recall $W$ NOT RENORMALIZED (via many arguments)
$\Rightarrow$ BUT $\exists$ WAVEFUNCTION RENORMALIZATION

↳ eg. cf NSVZ $\beta$ func vs. HOLOMORPHIC $\beta$ FUNC.

GENERALLY ONE LOOP CORRECTIONS CAN BE VERY IMPORTANT IN SUSY: eg STABILIZING MODULI SPACES.
FOR EXAMPLE, IN METASTABLE ~~SUSY~~ing MODELS ONE HAS TO TYPICALLY CALCULATE THE COLEMAN-WEINBERG EFFECTIVE POTENTIAL TO DETERMINE THE STABILITY OF ONE'S METASTABLE STATE. THIS CALC IS DONE IN COMPONENTS.
$\Rightarrow$ & IS TEDIOUS

ONE CAN ALSO DO THIS w/ SUPERGRAPHS.
$\Rightarrow$ EFFECTIVE KÄHLER POTENTIAL

↳ IT IS NOW OBVIOUS WHY THIS SHOULD BE A K CORRECTION
($\delta W = 0$, $\int d^4\theta$ ONLY PICKS UP $\theta^4$ TERM)

$\exists$ CLOSED FORM 1-LOOP GENERAL FORMULA (hep-th/9605149)

$$K_{eff} = -\frac{1}{32\pi^2} \mathrm{Tr}\left[\mathcal{M}^\dagger \mathcal{M} \ln\left(\frac{\mathcal{M}^\dagger \mathcal{M}}{\Lambda^2}\right)\right]$$

SUPERFLD. EFF MASS, eg. $\mathcal{M} \sim M + \lambda X$

trace, not SUPERTRACE!
(DOES not USE)

CAN TREAT THIS AS AN ALTERNATIVE to $V_{CW}$!

$$V_{SCALAR} = K_{eff}^{X\bar{X}} \, |W_X|^2 \qquad \text{(see Argyres)}$$

ONE MAJOR DRAWBACK: MANIFESTLY SUPERSYMMETRIC
SO FORMULA IS ONLY USEFUL IN SMALL ~~SUSY~~ LIMIT.
(BUT see eg. ORIGINAL ISS PAPER, APP 5 or INTRILIGATOR-SEIBERG NOTES on ~~SUSY~~ing FOR DISCUSSION)

[could BG field methods help this?]

see also: UPCOMING PAPER BY BEN KAIN + UNDERGRADS FOR KÄHLER ANALYSIS OF ISS MODELS & ~~R~~.