REFS. Review: Strassler hep-th/0309149
Original: Aharony et al. Nucl. Phys. B 499 67 (1997)

MOTIVATION: WHO CARES ABOUT $d=3$?

- I WAS ORIGINALLY INTERESTED IN SUSY IN HIGHER DIMENSIONS, WHICH HAS BEEN STUDIED FOR MANY REASONS (eg AdS/CFT, STRING THY, ...).

  HOWEVER, FOR $d \neq 4$ THE STRUCTURE OF QFTs CHANGES IN MANY INTERESTING WAYS THAT, IN TURN, SHED LIGHT ON THE "SPECIAL" CASE $d=4$. [in particular: PHASE STRUCTURE OF GAUGE THEORIES]

  IT IS MUCH EASIER TO DIMENSIONALLY REDUCE TO $d=3$ TO STUDY THESE PHENOMENA. (& WE WILL SEE THAT $d<4$ IS VERY NEAT.)

- THIS TURNS OUT TO GIVE SOME INSIGHT ON MANY GENERAL CONCEPTS
  - RENORMALIZATION GROUP FIXED POINTS
  - DUALITY IN QFT ("mirror sym.")
  - NON-PERTURBATIVE OBJECTS LIKE SOLITONS
  - etc!

$\Rightarrow$ $d=3$ is surprisingly rich & can teach us a lot!
... also "prep" for RS lectures (Flip, Oct) + Seiberg-Witten review (DC, Nov.)

DIMENSIONAL REDUCTION

RECALL IN $d=4$: # SUSY GENERATORS = $\boxed{\mathcal{N} \times (2 + 2)}$

$$\underbrace{Q_\alpha \quad \bar{Q}^{\dot{\alpha}}}$$

Weyl is $\mathbb{C}$ in $d=4$

= 4 comp. MAJORANA SPINOR
(eg. Weinberg Vol 3 uses this)

ON THE OTHER HAND, FOR $d=3$ NO WEYL REP (ODD DIM, NO PARITY OP)

SUSY GENERATORS = $\mathcal{N} \times (2)$
← dimension of Majorana rep.

$\Rightarrow$

# SUSY GENS	$d=3$	$d=4$
4	$\mathcal{N}=2$	$\mathcal{N}=1$
8	$\mathcal{N}=4$	$\mathcal{N}=2$
16	$\mathcal{N}=8$	$\mathcal{N}=4$

{ FOR MORE ABOUT SPINORS IN d DIM SEE POLCHINSKI II (APPENDIX) }

Q. IF WE WANTED A SIMPLE TOY THEORY, WHY NOT CONSIDER $d=3$ $\mathcal{N}=1$?

ANSWER: $\mathcal{N}=1$ IS MORE COMPLICATED!
WHY? NO HOLOMORPHY TO CONTROL NON-PERTURBATIVE DYNAMICS!

Recall: SEIBERG USED THE HOLOMORPHY OF THE $d=4$ SUPERPOTENTIAL TO SEVERELY CONSTRAIN THE FORM OF BOTH PERTURBATIVE + NONPERT. CORRECTIONS!

Why not? IN $d=4$ WE HAD CHIRAL SUPERSPACE COORDINATES θ_α AND $\bar{\theta}^{\dot\alpha}$. IN $d=3$ WE ONLY HAVE MAJORANA COORDINATES θ^α $(\alpha=1,2)$.

$\rightarrow \mathcal{N}=2$ $d=3$ INHERITS THESE PROPERTIES FROM $\mathcal{N}=1$ $d=4$ COMPACTIFICATION.

WHAT ABOUT THE ALGEBRA? (side remark only)

IN GENERAL WE KNOW THAT THE ALGEBRA OF EXTENDED SUSY ($\mathcal{N}>1$) GENERICALLY PERMITS CENTRAL CHARGES WHICH, IN TURN, PLAY A ROLE IN DETERMINING THE BPS BOUND ON SHORT MULTIPLETS.

FOR DIMENSIONALLY REDUCED ($d=4$ $\mathcal{N}=1 \rightarrow d=3$ $\mathcal{N}=2$) THE ALGEBRA IS

$$\{Q_\alpha, \bar{Q}_\beta\} = 2\sigma^\mu_{\alpha\beta} P_\mu + 2i\epsilon_{\alpha\beta} Z$$

$\mathbb{C}$ GENERATORS
(= 2 × $\mathbb{R}$ MAJORANA GENERATORS)
⇒ we can use 4D SUPERSPACE NOTATION!!

$\mu = 0,1,2$

usual σ_i's w/ factors of i to get $d=3$ Clifford algebra.

$\mathbb{R}$ CENTRAL CHARGE CORRESPONDS TO P_3

... BUT WE WON'T WORRY ABOUT THIS IN THIS TALK.

(This is a separate & very interesting subject!
SEE eg Witten + Olive's SEMINAL WORK ON CENTRAL CHARGES & SUSY SOLITONS)

& or the equally famous 'ELECTROMAGNETIC DUALITY FOR CHILDREN' review by J.F.'d F.

Dimensional Analysis

$$S = \int d^dx \, (\partial\Phi)^2 + \cdots \quad \Rightarrow \quad [\partial\Phi] = d/2 \quad \Rightarrow \quad \boxed{[\Phi] = d/2 - 1}$$

THE DIMENSIONS OF OUR FIELDS ALL ~~...~~ CHANGE!

$$S = \int d^dx \left\{ \int d^4\theta \, K(\Phi, \Phi^\dagger) + \left(\int d^2\theta \, W(\Phi) + h.c. \right) \right\}$$

note: be sure you understand that the $d=4$ $\mathcal{N}=1$ SUPERSPACE NOTATION IS PRECISELY WHAT WE WANT TO DESCRIBE $d=3$ $\mathcal{N}=2$!

further: since $Q \sim \partial/\partial\theta + \cdots$ } $\{Q, \bar{Q}\} \sim P$

$[\theta] = -1/2$ AS USUAL.

then: $[W(\Phi)]_{d=3} = 2$ cf. $[W(\Phi)]_{d=4} = 3$

MARGINAL OPERATOR LOOKS SOMETHING LIKE $W \sim \Phi^4$ since $[\Phi]_{d=3} = 1/2$

MARGINAL OPERATOR LOOKS SOMETHING LIKE $W \sim \Phi^3$ since $[\Phi]_{d=3} = 1$

SO: the $d=4$ MARGINAL WESS-ZUMINO SUPERPOTENTIAL is DIM REDUCED TO A $d=3$ RELEVANT SUPERPOTENTIAL!

(if these words don't mean anything to you, then see, eg., HOLLOWOOD'S LECTURES ON RG + SUSY.)

Punchline: $W = \hat{y} X^3$ HAS DIMENSIONLESS COUPLING y IN 4D

BUT IN $d=3$, $[\hat{y}] = 1/2$

$\Rightarrow$ scales w/ ENERGY, CLASSICALLY

(why hat on $\hat{y}$? TO REMIND US IT IS THE HOLOMORPHIC COUPLING)

"CLASSICAL" RG FLOW

DIMENSIONFUL COUPLINGS DON'T MAKE MUCH SENSE. IT'S ALWAYS BETTER TO RECAST THEM IN TERMS OF DIM-<u>LESS</u> COUPLINGS TIMES THE SCALE AT WHICH ONE PROBES THE THEORY.

<u>TRIVIAL EXAMPLE</u>:

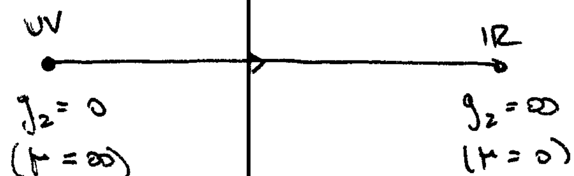
PROBE SCALE

SCALAR W/ MASS $m = g_2 \mu$

$g_2 \equiv \frac{m}{\mu}$ dimless

@ ENERGY SCALES $\mu \gg m$, $g_2 \ll 1$
WE SEE THAT MASS TERM DOESN'T AFFECT AMPLITUDES.
THY IS EFFECTIVELY MASSLESS ~~[illegible]~~

@ ENERGIES $\mu \ll m$, $g_2 \gg 1$
MASS TERM IS HUGE, THE FIELD FREEZES OUT
($O(\mu)$ SCATTERING CANNOT EXCITE THE FIELD)
→ THE FIELD DECOUPLES.



(trivial) "PHASE DIAGRAM"

USEFUL TO DEFINE A [<u>CLASSICAL</u>] BETA FUNCTION TO CHARACTERIZE μ-DEPENDENCE

$$\beta_{g_2} = \mu \frac{\partial g_2}{\partial \mu} = - g_2$$

"RELEVANT" OPERATOR, GROWS AS $\mu \to 0$

this is of <u>course</u> just the definition of the usual β function, BUT WE'RE <u>NOT</u> DOING ANY QUANTUM MECHANICS!

EVERYTHING IS TREE-LEVEL (CLASSICAL)

[in $d=4$ mass parameter is the only dim-ful physical coupling @ tree level. OTHER PARAMS ARE EITHER IRRELEVANT ($\beta > 0$) OR MARGINAL s.t. β ARISES ONLY @ LOOP LEVEL.]

SO FOR WZ_3 (Wess-Zumino in $d=3$), WE HAVE

$$\beta_{\hat{\omega}} \equiv \mu \frac{\partial}{\partial\mu} \underbrace{\left(\frac{\hat{y}}{\sqrt{\mu}}\right)}_{\equiv \hat{\omega}} = -\frac{1}{2}\omega$$ CLASSICALLY.

So far this is all trivial!

$\boxed{\text{BIG PICTURE}}$ $$\beta_{\hat{\omega}} = -\frac{1}{2}\hat{\omega} + \underbrace{\mathcal{O}(\hat{\omega}^3)}_{}$$

from quantum corrections
ie SCALE SYM IS BROKEN @ QUANTUM LEVEL
(BECAUSE QFT'S ARE DEFINED W/ A CUTOFF)
SO THAT THE DIMENSION OF Y CHANGES.

IN THE PERTURBATIVE REGIME $\mathcal{O}(\omega^3) \ll \mathcal{O}(\omega)$
SO SCALING IS DOMINATED BY TREE-LEVEL TERM.

HOWEVER, NON-PERTURBATIVELY ALL BETS ARE OFF!

OF COURSE, IT'S HARD FOR US TO SAY ANYTHING INTELLIGENT ABOUT GENERIC NONPERTURBATIVE QFTS.

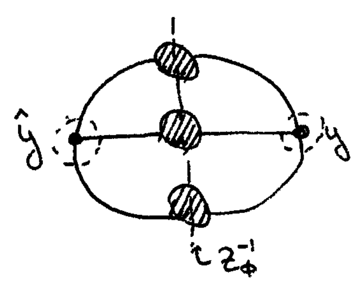
→ SUSY to the rescue!

THE POWER OF HOLOMORPHY: SUPERPOTENTIAL IS HOLOMORPHIC & HOLOMORPHIC COUPLINGS DO NOT RENORMALIZE!

$$\beta_{\hat{\omega}} = -\frac{1}{2}\hat{\omega}$$ EXACT

... BUT, NOBODY GIVES A FUCK ABOUT THE HOLOMORPHIC COUPLING. THIS IS NOT THE COUPLING THAT IS PHYSICALLY MEASURED.

THE PHYSICAL COUPLING IS

$$|y|^2 \equiv \frac{\hat{y}^\dagger \hat{y}}{Z_\Phi^3} \sim$$ 

CONSIDER $d=4$ ($[y]=0$ classically)

$$\beta_{|y|^2} = -3|y|^2 \boxed{\frac{\partial \ln Z_\phi}{\partial \ln \mu}}$$

$|y|^2 \equiv \frac{y^\dagger y}{Z_\phi^3}$

$\equiv -\gamma$, the ANOMALOUS DIMENSION

Why is it called an anomalous dimension?

LOOP (QUANTUM) EFFECTS GENERATE CORRECTIONS TO THE KINETIC TERMS

$$K = z\,\phi^\dagger\phi \qquad (\text{or } \mathcal{L} = z(\partial\phi)^2)$$

DIMENSIONFUL (!!) WITH DIMENSION γ

DEFINE DIMENSIONLESS $Z = z/\mu^\gamma \rightsquigarrow \beta_Z = -\gamma Z \Rightarrow \frac{\partial \ln Z}{\partial \ln \mu} = -\gamma$

$$K = Z\,\mu^\gamma\,\phi^\dagger\phi \xrightarrow[\text{NORMALIZE}]{\text{CANONICAL}} K = \phi'^\dagger\phi'$$

where ϕ now absorbs a dimensionful dependence, ie

$$\boxed{\dim \phi' = 1 + \tfrac{1}{2}\gamma}$$

tree level dimension (ENGINEERING DIM)

QUANTUM EFFECT ("ANOMALOUS DIMENSION")

THUS FROM $|y|^2$ WE CAN DEFINE THE PHYSICAL COUPLING y

$$\beta_{|y|^2} = y^\dagger\beta_y + y\,\beta_{y^\dagger} \Rightarrow \boxed{\beta_y = \frac{3}{2}y\gamma}$$

$$\gamma \sim \frac{|y|^2}{16\pi^2}$$

THEOREM (see Mnwalla hep-th/9712074)

NEAR A CONFORMAL FIXED POINT, ALL GAUGE INVT OPS MUST HAVE DIM $\geq (d-2)/2$
(THIS COMES FROM UNITARITY + IS INDEPENDENT OF SUSY)

$\Rightarrow \boxed{\gamma \geq 0}$ & $\boxed{\gamma = 0 \Leftrightarrow y = 0} \Rightarrow \beta_y > 0$, y irrelevant

FROM THIS POWERFUL THEOREM WE SEE THAT $d=4$ WESS ZUMINO FLOWS TO A GAUSSIAN FIXED POINT, ie non-interacting theory.

↳ BORING!

What about $d=3$?

Holomorphy carries over (as we already argued).
INSTEAD OF PHYSICAL COUPLING y, LET US WRITE DIMENSIONLESS PHYSICAL COUPLING:

$$|\omega|^2 = \mu \frac{\hat{y}^\dagger y}{Z_\phi^3}$$

CLASSICAL SCALING

$$\boxed{\beta_\omega = \omega\left[-\frac{1}{2} + \frac{3}{2}\gamma(\omega) \right]}$$

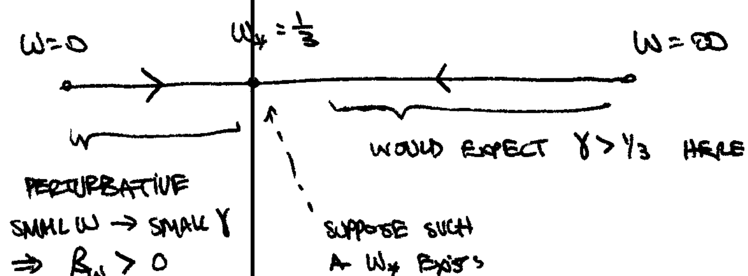
tree-level   quantum (SAME AS $d=4$ CASE)

What would be very interesting: $\exists\ \omega_*$ s.t. $\beta_{\omega_*} = 0$?

↳ COULD THERE BE A RENORMALIZATION GROUP FIXED POINT ?

Plausible : UNITARITY THEOREM $\Rightarrow \gamma > 0$
SO SIGN IS CORRECT TO CANCEL TREE-LEVEL

↳ note: this wouldn't happen in $d>4$!



↳ attractive IR fixed point! (if it exists)

Some "philosophy"

IN $d=4$ OUR THEORIES TYPICALLY HAVE ONLY GAUSSIAN (FREE) FIXED POINTS. AS WE FLOW FURTHER + FURTHER INTO THE IR, FIELDS w/ NONZERO MASS "FREEZE OUT" (OR DECOUPLE) AND WE ARE LEFT WITH A THEORY OF MASSLESS MODES.

THIS THEORY IS ALWAYS (we think) NON-INTERACTING. (*)

boring.

IN $d=3$ THIS IS NOT THE CASE. WE'VE SHOWN FOR WZ_3 THAT IT IS PLAUSIBLE FOR A NONTRIVIAL FIXED POINT TO EXIST, BUT WELL DOCUMENTED EXAMPLES ARE KNOWN:

WILSON-FISHER FIXED POINT FOR $d=4-\epsilon$

O(N) MODEL IN $d=3$, where $1/N$ expansion gives control of loop-level effects.

WE CAN SEE GENERICALLY WHY THIS HAPPENS:

IN $d<4$ WE HAVE MORE RELEVANT OPERATORS (ie $\beta_{classical} < 0$) s.t. we can hope $\beta_{quantum}$ might enter w/ opposite sign.

$\Rightarrow$ WE ALSO SEE WHY SUCH FEATURES ARE DIFFICULT IN $d>4$

(there was recently a strange paper by Vekusa 1008.0487 about relevant ops in RS... but I don't necessarily agree with its application in this context.)

by the way: there isn't much we can say @ the FP. the theory would look very different.
→ DIFFERENT PERTURBATIVE DESCRIPTION? (eg "confinement")
→ non local CFT ? ("unparticle")

(*) THERE ARE PERHAPS A FEW EXCEPTIONS IN GAUGE THEORY, e.g. THE FAMOUS BANK-ZAKS FIXED POINT.

See PESKIN'S TALK @ UCSC Banks+Fischler symposium 2009

A result from **SUPER**-CFT:

THE SUPERCONFORMAL ALGEBRA CONNECTS THE ANOMALY-FREE (PREFERRED) R-SYMMETRY WITH ~~THE~~ SCALE TRANSFORMATIONS.

↳ the generators are both part of the SUPERCURRENT

[lots of recent work on the SUPERCURRENT; see KOMARGODSKI, SEIBERG, BUTTER, DIENES, THOMAS, KUZENKO, ...]

@ A SCFT FIXED POINT

$$\dim(\text{xral op}) = \frac{d-1}{2} R(\text{xral op})$$

↑

CLASSICAL + ANOMALOUS DIMENSION!

thus for $W \sim \Phi^3$ (eg dim reduction from $d=4$)

@ FIXED POINT, $[\Phi] = \begin{cases} 1 & d=4 \\ 2/3 & d=3 \\ 1/3 & d=2 \end{cases}$

BUT: UNITARITY THEOREM $\Rightarrow$ $\dim(\text{GAUGE INV } \mathcal{O}) \geq \frac{d-2}{2}$

ie $[\Phi] \geq \begin{cases} 1 & d=4 \\ 1/2 & d=3 \\ 0 & d=2 \end{cases}$ ← but EQUALITY ⇔ NONINTERACTING.

SO: $\boxed{\text{ONLY EXPECT NONTRIVIAL FIXED POINTS IN SUSY FOR } d<4!}$

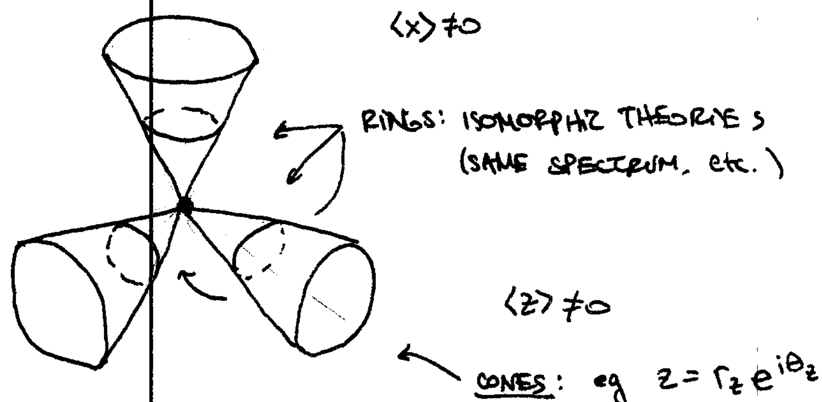
FOR REASONS THAT WILL BECOME CLEAR MUCH LATER, LET'S STUDY A COUSIN OF THE WESS-ZUMINO MODEL:

XYZ model

$$W = hXYZ \longrightarrow V_{scalar} = |h|^2\left(|xy|^2 + |xz|^2 + |yz|^2\right)$$

coupling; XSFs

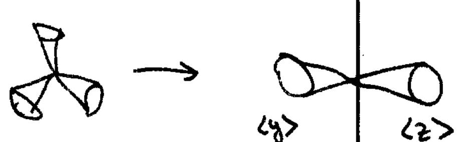
WHAT DOES THE MODULI SPACE LOOK LIKE?
MIN WHEN ANY 2 SCALARS $\equiv 0$, THIRD ARBITRARY.



All: this theory is conformal @ the origin (somewhat trivially)

<u>EXTRA STUFF</u> (would be nice to discuss if time)

MASS PERTURBATION: $\Delta W = \frac{1}{2} m X^2 \longrightarrow$ kills $\langle x \rangle \neq 0$ branch



WHAT ABOUT RG?

EOM: $\nabla^2 X^+ = m^+ (mX + hYZ)$

IN FAR IR, $\nabla^2 X^+ \ll |m|^2 X \longrightarrow \boxed{mX = -hYZ}$ EOM.

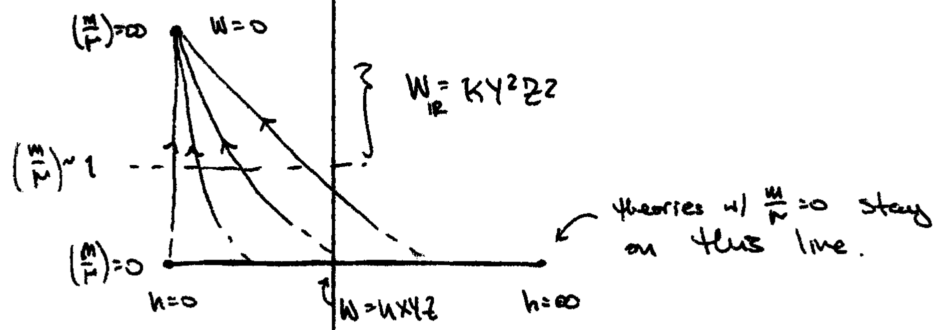
$$\Rightarrow W_{IR}(Y,Z) = -\frac{h^2}{2m} Y^2 Z^2$$

$$\equiv \kappa Y^2 Z^2$$

EXTRA STUFF, continued

K IS AN IRRELEVANT COUPLING : $[K] < 0$ $(= -1$ for $d=4)$

let's focus on $d=4$. CLASSICAL RG FLOW:



certainly makes sense @ ~~[illegible]~~ $\langle y\rangle = \langle z\rangle = 0$.

BUT: if $\langle y\rangle \neq 0$, then $V_{IR} \supset \underbrace{K^2\langle y\rangle^2 |z|^2 + K\langle y^2\rangle \psi_z\psi_z}_{\text{FROM } W_{IR}(Y,Z)}$

ie Z GETS A MASS $\sim \mathcal{O}(K\langle y^2\rangle)$

E

X, Y, Z w/ $W(X,Y,Z)$

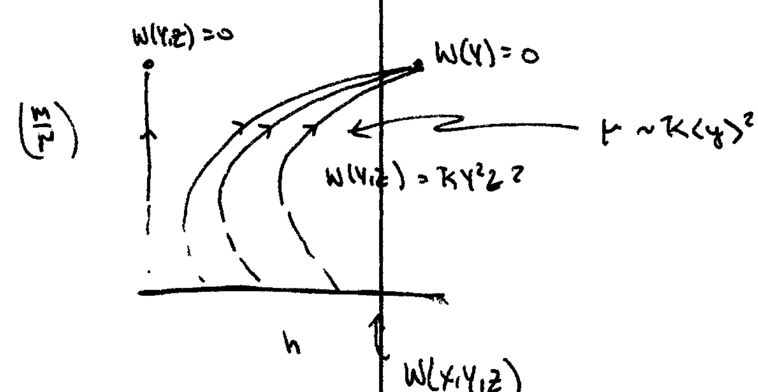
— — — m

Y, Z w/ $W_{IR}(Y,Z)$

— — — $K\langle y\rangle^2$ ⟵ Z's decouple & theory changes!

Y w/ $W_{IR'}(Y)$

SO FOR $\langle y\rangle \neq 0$, FLOW CHANGES.



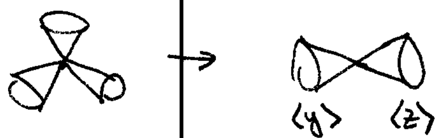
(lesson: $\langle y\rangle \to 0$ & $\mu \to 0$ limits don't commute)

//

Vortices in XYZ_3

HERE'S AN INTERESTING FEATURE OF PERTURBED XYZ IN $d=3$ (XYZ_3)

$\Delta W = \xi X$; kills $\langle x \rangle$ BRANCH OF MODULI SPACE.

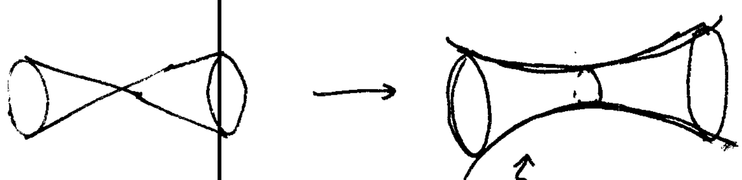


EOM:

$$\left.\begin{aligned} W_X &= hYZ + \xi = 0 \\ W_Y &= hXZ = 0 \\ W_Z &= hXY = 0 \end{aligned}\right\} \quad \text{SOLS:} \quad \begin{aligned} &X = 0 \\ &\boxed{YZ = \xi/h} \end{aligned}$$

EQUATION FOR $\mathbb{C}$ PARABOLA

SO WHAT'S ACTUALLY HAPPENED IS



PARAMETERIZED BY $\frac{Y}{\sqrt{\xi/h}} \in \mathbb{C}/\{0\}$

RECALL MEANING OF MODULI SPACE: MINIMUM OF POTENTIAL
(IN SUSY: ZERO OF POTENTIAL)

WE CAN IMAGINE time-indep. TOPOLOGICAL (SOLITONIC) FIELD CONFIGURATIONS THAT TWIST IN FIELD SPACE s.t. ASYMPTOTICALLY THEY LIVE ON MODULI SPACE $\longrightarrow$ FINITE ENERGY.

((GAUGE THEORY ANALOG: INSTEAD OF WRAPPING MODULI SPACE, FIELDS WRAP CONFIGURATIONS IN GAUGE SPACE WHICH ARE ON THE SAME ORBIT AS THE ORIGIN. SEE FLIP'S INSTANTON NOTES FOR A VERY NICE DISCUSSION.))

IT'S EASY TO MAKE AN ANSATZ FOR SUCH A SOLUTION IN $d=3$

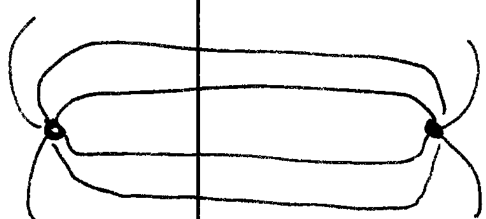
$$\left.\begin{aligned} Y &= \sqrt{\xi/h}\, f(r)\, e^{i\theta} \\ Z &= \sqrt{\xi/h}\, f(r)\, e^{-i\theta} \end{aligned}\right\} \quad \text{w/} \quad \begin{aligned} & f(r\to 0) = 0 \quad \text{(single-valuedness)} \\ & f(\underbrace{r \gg r_0}_{\text{some characteristic scale}}) = 1 \quad \text{s.t. } YZ \to \xi/h \text{ on mod. space} \end{aligned}$$

QUALITATIVE REMARKS

- TOTAL ENERGY OF A SINGLE VORTEX IS <u>LOG DIVERGENT</u>

  well known result; <u>DERRICK'S THM</u>: SCALAR FIELDS CANNOT HAVE SOLITONIC SOLUTIONS IN $d>1$. (see, eg. Weinberg II)
  ... SEE FLIP'S INSTANTON NOTES FOR HOW GAUGE SOLITONS AVOID DERRICK'S THEOREM!

- <u>BUT</u>, VORTEX - ANTIVORTEX PAIR HAS FINITE ENERGY
  CAN EVEN PULL THEM APART A BIT



"confinement"

- <u>ANALOG</u>: $d=3$ QED w/ $m_e \neq 0$

  ↳ electrons are log confined!

  We will explore this shortly.

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by the way: at this point we haven't done much, but we have seen surprisingly rich structure where one would not have expected it.

[eg: DISCOVERING DEEP SEA LIFE FAR FROM THE PENETRATION DEPTH OF SUNLIGHT!]

<u>QUANTUM XYZ$_3$</u> (extra topic, but very cute)

ANOMALOUS DIMS

$$W = \underbrace{\sqrt{F}\,\eta}_{\hat{h}}\,(XYZ) \rightsquigarrow \beta_\eta = \tfrac{1}{2}\eta\,(-1 + \overbrace{\gamma_X + \gamma_Y + \gamma_Z})$$

dim-less coupling

EASY TO WRITE DOWN BY ANALOGY TO WESS-ZUMINO

FIXED POINT IF $\exists\ \eta_*$ s.t. $\underbrace{\gamma_X(\eta_*) = \gamma_Y(\eta_*) = \gamma_Z(\eta_*)} = 1/3$

by the way: $\exists\ \{X,Y,Z\}$ permutation sym $\Rightarrow \gamma_X = \gamma_Y = \gamma_Z \equiv \gamma$

SO FAR THE SAME.

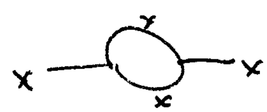
$$\beta_{\omega_x} = \tfrac{1}{2}\omega_x\,(-1 + 3\gamma_x) \qquad \text{(WZ)}$$

<u>PERTURBATION</u>: $\omega_x \sqrt{F}\, X^3$ (+ similar w/ y, z)

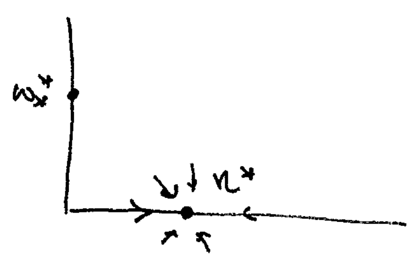
<u>CLAIM</u>: ω_x IS A <u>MARGINALLY IRRELEVANT</u> COUPLING @ η_*.

marginal: @ η_* $(\omega_x = 0)$ $\gamma_x = \gamma_y = \gamma_z = \frac{1}{3}$; $\beta_{\omega_x} = 0$

irrelevant: for small perturbations $\omega_x = \epsilon$ we generate additional contributions to γ_x from



these are <u>positive</u> s.t. $\beta_{\omega_x} > 0$ (if $\epsilon > 0$).



SO: ω_x PERTURBATION DIES IN THE IR (logarithmically)

SO: $(\eta, \omega_x, \omega_y, \omega_z)$ "theory space" has fixed point @ $(\eta_*, 0,0,0)$.

THERE'S MORE! FP $\Leftrightarrow$ $\beta_i = 0 \ \forall i$

$$\left.\begin{aligned} \beta_\eta &= \tfrac{1}{2}\eta(-1 + \gamma_x + \gamma_y + \gamma_z) \\ \beta_{\omega_i} &= \tfrac{1}{2}\omega_x(-1 + \gamma_i) \qquad i = x, y, z \end{aligned}\right\} \begin{array}{l} \text{4 EQS} \\ \text{4 UNKNOWNS?} \end{array}$$

$\downarrow$

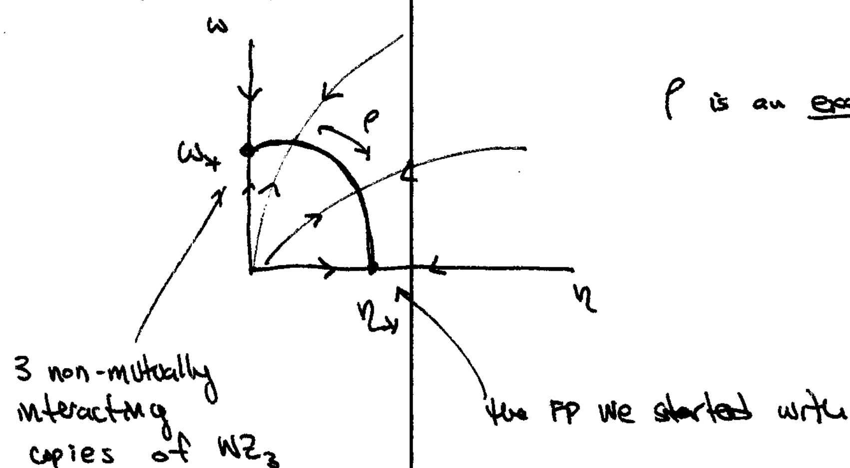
$\beta_\eta = \beta_{\omega_i} = 0$ is ONLY THREE CONSTRAINTS ON COUPLINGS!!

$$\boxed{\gamma_x = \gamma_y = \gamma_z = \tfrac{1}{3}}$$

$\Rightarrow$ EXPECT 1-PARAMETER FAMILY OF FIXED POINTS!

$\exists$ EXACTLY MARGINAL DIRECTION

BY $\{x, y, z\}$ PERMUTATION SYMMETRY, ASSUME $\omega_x = \omega_y = \omega_z \equiv \omega$ ON THIS SUBSPACE.



ρ is an exactly marginal op

d=3 SUPER QED (SQED$_3$) ... $\boxed{\mathcal{N}=2}$

TO REPEAT WHAT WE SAID @ BEGINNING:
$\mathcal{N}=2$ $d=3$ SUSY $\Leftrightarrow$ $\mathcal{N}=1$ $d=4$ COMPACTIFIED SUSY

So WE CAN THINK OF THE DIMENSIONAL REDUCTION OF SQED$_4$

A_μ $\mathbb{R}$ vector 4$\mathbb{R}$ DOF $\longrightarrow$ $A_i + \varphi$

λ MAJ. FERMION 2$\mathbb{C}$ DOF $= 4\mathbb{R}$ DOF $\longrightarrow$ $\lambda_1 + \lambda_2$

$\updownarrow$

in $d=4$ Q IS A 4 COMP $\mathbb{R}$ MAJ. SPINOR

in $d=3$ Q is a 2 COMP $\mathbb{R}$ MAJ. SP
SO FOR $\mathcal{N}=2$ WE HAVE 2 GAUGINOS $\underset{4D}{\lambda} = \underbrace{\lambda_1 \oplus \lambda_2}_{3D}$

MODULI SPACE : SPACE OF SCALAR VEVS w/ $V=0$

SO: CLASSICALLY : $\mathcal{N}=2$ SQED$_3$ HAS A $\mathbb{R}$ SCALAR

$\Rightarrow$ $\boxed{\text{MODULI SPACE} = \mathbb{R}}$?!

this will NOT BE TRUE QUANTUM'LY!
HINT: HAVE YOU EVER HEARD OF $\mathbb{R}$ SCALARS IN SUSY?

Reminder: [CLASSICAL] S-DUALITY (ELECTROMAGNETIC DUALITY)

4D $\tau_{QED} = 4\pi^2/e^2$ w/ $\tau \to -1/\tau$ $(\vec{E} \leftrightarrow \vec{B})$

[see Itay's BSM JOURNAL CLUB TALK LAST SPRING]

FANCY WAY OF WRITING:

1 FORM ("dual vector") POTENTIAL A w/ FIELDSTRENGTH $F = dA$ (2 FORM)

~~HODGE~~ HODGE DUAL is $*F = \underbrace{*dA}_{} \equiv \underbrace{dC}_{*F}$, WHERE C is THE DUAL POTENTIAL

IN COORDINATES:

$F_{\mu\nu} = \partial_{[\mu}A_{\nu]}$
$\updownarrow$ S-DUALITY
$\tilde{F}_{\mu\nu} = \epsilon_{\mu\nu}{}^{\rho\sigma}\partial_{[\rho}C_{\sigma]}$

ALSO A 2-FORM IN $d=4$ ONLY.

In d DIM FOR p-FORM P, $*P$ is A $(d-p)$ FORM

(3D) $\underbrace{*_3\, dA}_{\text{1 FORM !!}} = d\,C$

C MUST BE A SCALAR, CALL IT σ

IN 3D $\vec{E}$ IS A 2 VECTOR $\xrightarrow{\;s\;} \nabla\sigma$
B IS A SPACE SCALAR (time deriv.) $\longrightarrow \partial_t \sigma$

$\boxed{F_{\mu\nu} = \partial_{[\mu}A_{\nu]} = \epsilon_{\mu\nu\rho}\partial^\rho\sigma}$

Think: how iiiinteresting. Another real scalar...

- FROM FORM OF $F_{\mu\nu}$ WE SEE THAT σ HAS A GLOBAL SHIFT SYMMETRY!

$\boxed{\sigma \to \sigma + c}$

A VEV FOR σ BREAKS THIS GLOBAL SYM $\to$ $\partial\sigma$ IS THE GOLDSTONE

THIS IS JUST THE PHOTON THAT WE STARTED WITH!

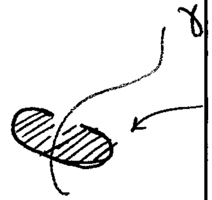
- FACT: σ IS PERIODIC (U(1) - VALUED)

Pf/ CONSIDER THE DUAL FIELD STRENGTH (1-FORM!)

$\tilde{F} = *F$ ~~$= d\sigma$~~ | IN VACUUM, MAXWELL $\Rightarrow$ $d\tilde{F} = 0$, so LOCALLY $\tilde{F} = d\sigma$ (Poincaré)

SUPPOSE $\exists$ ELECTRIC CHARGE w/ WORLDLINE γ

γ



SURFACE Σ

FORMALLY: 2-FORM THAT IS δ-FUNCTION LOCALIZED ALONG γ

MAXWELL EQUATION: $d\tilde{F} = 2\pi\,\delta_{\gamma(x)} \neq 0$, not closed!

BUT $\Rightarrow \int_{\partial\Sigma} \tilde{F} = \int_\Sigma d\tilde{F} = 2\pi$

MORE GENERALLY, $\int_{\partial\Sigma}\tilde{F}$ IS DEF UP TO INTEGER MULTIPLES OF $(2\pi e)$ (we set $e=1$) ACCORDING TO MULTIPLE SOURCE WORLDLINES.

$\Rightarrow$ σ IS ONLY WELL DEFINED MODULO $2\pi e$!

"IN PRESENCE OF CHARGES, HOLONOMY OF $\tilde{F}$ IS U(1)-VALUED, SO MAGNETIC DUAL THEORY MUST BE U(1) ~~[illegible]~~ GAUGE THY (as opposed to $\mathbb{R}$)."

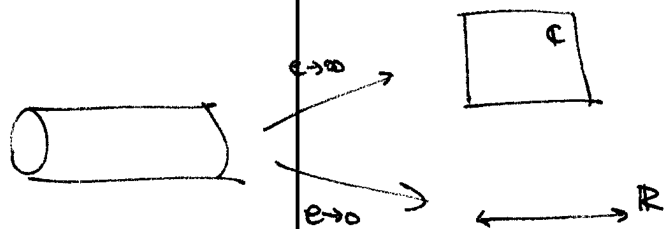
FLIP THANKS DAVID SIMMONS-DUFFIN FOR EXPLAINING THIS.

SO THE MODULI SPACE OF $\mathcal{N}=2$ $SQED_3$ SHOULD ALSO INCLUDE σ!
IN FACT, WE CAN COMBINE φ AND σ INTO A SINGLE COMPLEX FIELD

$$\boxed{\Sigma \equiv \varphi + i\sigma}$$

MODULI SPACE IS A CYLINDER BECAUSE OF $2\pi e$ PERIODICITY OF σ!

AS $e \to \infty$ IT BECOMES $\mathbb{C}$ PLANE
$e \to 0$ $\mathbb{R}$ LINE



@ CLASSICAL LEVEL

SO: WE'VE DISCOVERED THAT IN $d=3$ EM-DUALITY (S-DUALITY) TEACHES US SOMETHING NEAT ABOUT THE SQED MODULI SPACE!

EXTRA TOPIC DETOUR (mention this if time permits)

CLASSICAL $d=4$ $\mathcal{N}=1$ SQED w/ some # of e^+, e^- flavors

$$V \sim D^2 + \sum_{\text{FLAVORS}} |F_i|^2 + |\tilde{F}_i|^2$$

$$D = \sum q_i |e_i|^2 - \sum q_i |\tilde{e}_i|^2$$

we really care about simple case, $q_i = 1$ $\forall i$

CHARGE FIELD 'POSITRON'

$V = 0$ (mod. space) $\Rightarrow$ $\left.\begin{array}{l} D=0 \\ F=0 \\ \tilde{F}=0 \end{array}\right\}$ separately.

[EXTRA TOPIC CONTINUED]

INTERESTING OBSERVATION:

AS FAR AS THE MODULI SPACE IS CONCERNED, IT APPEARS AS IF **GAUGE INVARIANCE HAS BEEN COMPLEXIFIED!**

Whaaaat?

$\boxed{D=0} \Rightarrow |e|^2 - |\tilde{e}|^2 = 0$

IN WORDS: the MAGNITUDE OF THE e AND $\tilde{e}$ VEVS MUST BE EQUAL ON THE MODULI SPACE.

GAUGE INVARIANCE ($U(1)$) WAS ALREADY TELLING US THAT WE COULD FIX THE PHASES s.t. $\arg\langle e\rangle = \arg\langle\tilde{e}\rangle$.

THE PHYSICAL MODULI SPACE IS MODDED OUT BY GAUGE ORBITS. THIS IS EQUIVALENT TO LOOKING AT THE MODULI SPACE AFTER GAUGE FIXING, IN PARTICULAR PICKING A GAUGE WHERE $\arg\langle e\rangle = \arg\langle\tilde{e}\rangle$.

$\Rightarrow$ the physical! MODULI SPACE IS PARAMETERIZED BY A SINGLE COMPLEX PARAMETER: $v = \langle e\rangle = \langle\tilde{e}\rangle$

this should look very familiar from SQCD!

HINT: WE COULD EQUIVALENTLY PARAMETERIZE THE MODULI SPACE BY "GAUGE INVARIANT POLYNOMIALS," NAMELY THE "MESON"

$$M = v^2 = \langle e\tilde{e}\rangle$$

WE STARTED W/ 2 (× # flavors) CHIRAL MULTIPLETS.
WE ONLY NEED 1 (× # flavors) TO PARAMETERIZE MODULI SPACE!

WHAT HAPPENED? ℂ'ified GAUGE GROUP GOT HIGGSED WHEN $\langle M\rangle \neq 0$ (fixes mag.)
PHOTON GETS MASS: $|De|^2$ term $\to \langle e\rangle^2 A^2$, must eat χral multiplet!

EXTRA TOPIC II : $\boxed{\mathcal{N}=2 \quad d=4}$ SQED IN $F=1,2$

$\mathcal{N}=2$ VECTOR-PLET $\Rightarrow$ $\mathcal{N}=1$ VECTOR $\oplus$ $\mathcal{N}=1$ XRAL (Φ)

$\mathcal{N}=2$ HYPER-PLET $\Rightarrow$ $\mathcal{N}=1$ CHIRAL $\oplus$ ANTICHIRAL

Hyper has $U(N_F)$ flavor sym
while Chiral $\oplus$ Antichiral seems to have $U(N_F)^2$.
BUT BREAKS TO $U(N_F)$ BY REQUIRED W...

$\mathcal{N}=2$ ALSO IMPOSES $W = \sqrt{2}\, \Phi Q_r \tilde{Q}^r$ (w/ some normaliz of K)

i think $K = \frac{1}{g^2} \Phi^\dagger \Phi$

THEN FOR $N_f = 1$, MODULI SPACE IS $(V=0)$

$0 = D = |Q|^2 - |\tilde{Q}|^2 \longrightarrow$ satisfied by $M = Q\tilde{Q}$ (tautological)

$0 = F_\Phi^\dagger = Q\tilde{Q} \longrightarrow$ IMPOSES $M = 0$

$0 = F^\dagger = \Phi Q$

$0 = \tilde{F}^\dagger = \Phi \tilde{Q}$ $\Rightarrow$ $\forall \Phi$ allowed

$\langle\Phi\rangle$ GIVES $Q, \tilde{Q}$ MASS
BUT Φ ITSELF REMAINS MASSLESS.
($U(1)$ UNBROKEN)

MOD SPACE ONLY HAS A COULOMB BRANCH
(massless photon $\to$ $1/r$ potential, $U(1)$ unbroken)

Q. Why no Higgsing this time? (cf $\mathcal{N}=1$ CASE)

IF Higgsing, VEC-PLET MUST EAT A CHARGED-PLET (Xral)

IN $\mathcal{N}=2$ CASE $(N_f=1)$
THIS IS THE WHOLE HYPER.

$\Rightarrow$ NO MASSLESS FIELDS LEFT FOR MODULI!

NO MASSIVE PHOTON BRANCH.

EXTRA TOPIC III $\mathcal{N}=2$ $d=4$ SQED w/ $\boxed{N_F = 2}$

What does this change?

NOW WE HAVE MORE Xral-PLETS LYING AROUND.
IN PARTICULAR, AN EXTRA $\mathcal{N}=2$ HYPER-PLET.

NOW: ~~[illegible]~~ CAN STILL HAVE $\langle \phi \rangle \neq 0$

⇒ GIVES MASS TO $Q, \tilde{Q}$'s → QUAD POT @ ORIGIN PREVENTS $\langle Q \rangle, \langle \tilde{Q} \rangle \neq 0$ } SAME COULOMB BRANCH

NEW POSSIBILITY: FORGET $\langle \phi \rangle$. FIRST SUPPOSE $\boxed{\langle Q \rangle, \langle \tilde{Q} \rangle \neq 0.}$ HIGGS BRANCH

⇒ VECTOR-PLET BECOMES MASSIVE & U(1) BROKEN

⇓

QUAD POT. PREVENTS $\langle \phi \rangle \neq 0$
(so we're consistent w/ prev. case)

↓

SO VECTOR EATS A HYPER (= Xral ⊕ Anti-Xral)
BUT WE STILL HAVE A SECOND HYPER LEFT OVER
TO DESCRIBE A NONTRIVIAL MODULI SPACE!

CHECK

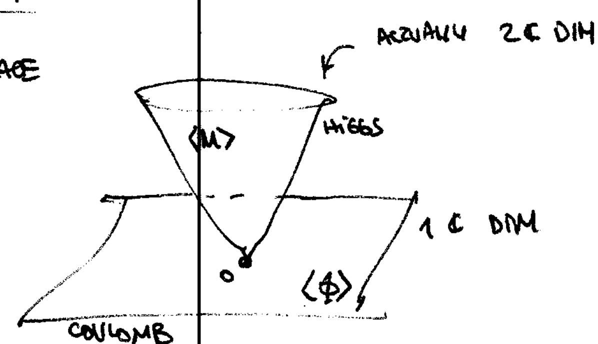
$$F_\phi^\dagger = Q_i \tilde{Q}_i = \mathrm{Tr}\, M = 0$$

$$\left.\begin{aligned} F_i^\dagger &= \phi Q_i \\ \tilde{F}_i^\dagger &= \phi \tilde{Q}_i \end{aligned}\right\} \quad \phi M_r{}^s = 0$$

} either $\langle \phi \rangle \neq 0$ or $\langle M \rangle \neq 0$

Cartoon

MODULI SPACE



CLASSICAL $d=3$ $\mathcal{N}=2$ SQED : MODULI SPACE

OUR $d=3$ $\mathcal{N}=2$ IS A DIM REDUCTION OF $d=4$ $\mathcal{N}=1$.
SO IN SOME SENSE WE'RE ALREADY VERY FAMILIAR W/ THE DEGREES OF FREEDOM. ALL THAT CHANGES IS HOW THESE DOF ARE PACKAGED, IN PARTICULAR, THE COMPACTIFICATION GIVES US NEW SCALARS. (would be PSEUDO SCALARS, BUT NO PARITY IN ODD DIMENSIONS!)

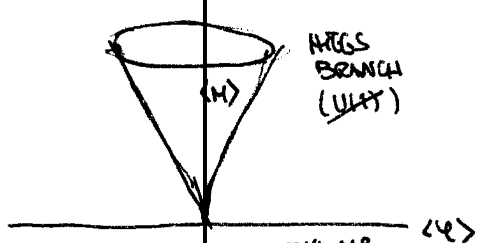
IN PARTICULAR, THE 3-COMPONENT OF THE GAUGE FIELD $A_3 \to \varphi$

Kinetic terms in $\mathcal{L}$: $|D_3 Q|^2 + |D_3 \tilde{Q}|^2 \longrightarrow \boxed{\varphi^2 (|Q|^2 + |\tilde{Q}|^2)}$

MODULI SPACE (CLASSICAL) ... naively

EITHER $\begin{cases} \varphi = 0 & \to \text{ MODULI SPACE PARAM BY } M \sim v^2 = \langle Q\tilde{Q} \rangle \\ Q = \tilde{Q} = 0 & \to \text{ MODULI SPACE PARAM BY } \varphi \end{cases}$

SO MODULI SPACE IS:



$\langle M \rangle$ gives MASS TO φ
BUT φ PART OF VECTOR
$\to A$ GETS MASS. ($\cancel{U(1)}$)

But: we're enlightened. We know that the theory REALLY contains an additional scalar field (& hence potential modulus), the DUAL POTENTIAL, σ.

WE KNOW THAT φ & σ COMBINE INTO A $\mathbb{C}$ SCALAR

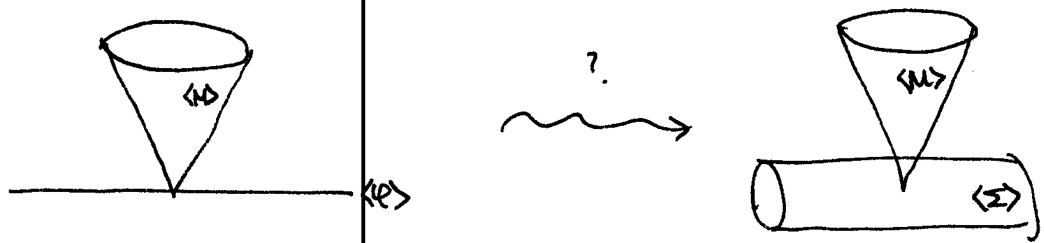
$$\Sigma = \varphi + i\sigma$$

the naive Moduli space for Σ depends on e^2.

(RECALL: )

SO LET'S STUDY RG OF e^2.

SO WE KNOW THAT THIS PICTURE ... ... must turn into something like this.



THE QUESTION IS: HOW DO WE PROPERLY TAKE INTO ACCOUNT σ?
(or alternately, how do we properly promote $\varphi \to \Sigma = \varphi + i\sigma$?)

WE KNOW THAT THE RADIUS OF THE Σ CYLINDER IS CONTROLLED BY THE STRENGTH OF THE ELECTRIC COUPLING, e.

↳ but this is a running quantity! (even classically)

LET'S STOP & STUDY THE RG OF e^2.

Relevant diagram:

{ aside: Why is this the relevant diagram? Doesn't e^2 control the coupling of fermions to gauge fields? Shouldn't we be calculating

THIS IS ACTUALLY SOMEWHAT SUBTLE! PHYSICALLY, WE MEASURE

GAUGE-DEPENDENT & CANCELS!
see e.g. Peskin ch. 12, eee 11.7.

No! IN FACT, THE  DOES NOT GIVE A 1-LOOP CORRECTION TO e^2 AT ALL! (You can check this.)
↳ corrects magnetic moment

Why? the PARAMETER e^2 (OR g^2 IN GENERAL) "NATURALLY" WANTS TO LIVE AS A PREFACTOR TO THE GAUGE KINETIC TERM:

$$\mathcal{L}_{GAUGE} = -\frac{1}{4e^2} F^2$$

THIS IS NOT CANONICALLY NORMALIZED, BUT IT REMOVES e FROM THE COVARIANT DERIVATIVE. IN OTHER WORDS, WE CAN GO TO A NORMALIZATION WHERE e HAS NOTHING TO DO WITH FERMIONS. }

NDA :

CLASSICAL DIM.: (USE CANONICAL NORMALIZATION)

$$[\mathcal{L}] = d \Rightarrow 2[\partial A] = d \qquad [A] = d/2 - 1$$

$$[D_\mu] = 1 \Rightarrow [eA] = 1 \qquad [e] = 2 - d/2 = \boxed{1/2 \text{ for } d=3}$$

LOOP ESTIMATE

 $\sim d^3k \, \frac{1}{\not k}\gamma \frac{1}{\not k}\gamma \, e^2 \sim \Lambda \leftarrow$ linear divergence!

But, as we all know: Flip ~~[illegible]~~ Yuhsin never draw divergent diagrams!

Finite by ① Gauge invariance (Ward Identity)
② Lorentz invariance

same mechanisms that make $\mu \to e\gamma$ finite in $d=5$.

SKETCH CALCULATION: (k is shifted variable, after Feynman's trick)

$$\mathcal{M} \sim -N_F e^2 \int d^3k \, \frac{1}{(k^2+\Delta^2)^2}\left[\not k \gamma^\mu \not k \gamma^\nu + \not k \gamma^\mu (b\not p)\gamma^\nu + c\, \not p \gamma^\mu \not p \gamma^\nu\right]$$

N_F: # FLAVOR ENHANCEMENT! (hint of large N simplification!)

e^2: e^2 DISAPPEARS IF WE'RE USING "NATURAL" NORMALIZATION

$\not k \gamma^\mu \not k \gamma^\nu$: DIVERGENT, BUT VANISHES BY WARD IDENTITY (indep of p^μ)

$\not k \gamma^\mu (b\not p)\gamma^\nu$: ODD POWER OF $\not k$, vanishes by Lorentz.

$c\, \not p \gamma^\mu \not p \gamma^\nu$: FINITE

COUNTING POWERS:

$$\sim \frac{d^3k}{k^4} \to \frac{c}{\mu}$$

$$\Rightarrow \boxed{\frac{1}{e^2(\mu)} = \frac{1}{e_0^2} + c\frac{N}{\mu}}$$

DEFINE <u>DIMENSIONLESS</u> PARAMETER

$$\xi \equiv \frac{e^2(\mu)}{\mu} = \left(\frac{\mu}{e_0^2} + CN_F\right)^{-1} .$$

← GOES TO A CONST @ SMALL μ! (in IR)

$$\beta_\xi = \mu\frac{\partial \xi}{\partial \mu} = \frac{1}{\mu/e_0^2 + CN_F}\cdot\frac{\mu/e_0^2}{\mu/e_0^2 + CN_F}$$

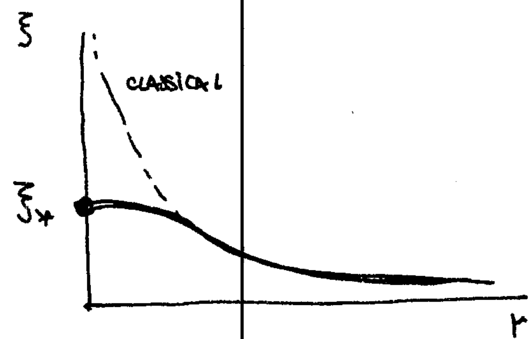
$$\frac{\mu/e_0^2}{\mu/e_0^2 + CN_F} = \frac{\overbrace{\mu/e_0^2 + CN_F}^{=1} - CN_F}{\mu/e_0^2 + CN_F}$$

$$= \boxed{\xi\left(1 - CN_F\,\xi\right)}$$

← ∃ FIXED POINT

$$\boxed{\xi_* = \frac{1}{CN_F}}$$

FP @ <u>WEAK</u> COUPLING FOR <u>LARGE</u> N_F!



<u>Facts</u>:

- in QED_3, there <u>really</u> is a FP ξ_* @ large N_F
- in $SQED_3$ w/ $\mathcal{N}=2$, ξ_* EXISTS AT 1-LOOP FOR <u>ALL</u> N_F!
- IN $SQED_3$ w/ $\mathcal{N}=4$, IT is <u>EXACT</u> (no higher loop corrections)!!

↑ very cool! ... but not the direction that we're interested in for this talk

<u>Back to the point</u>: WANT QUANTUM MODULI SPACE FOR $SQED_3$ $(\mathcal{N}=2)$

$\Sigma = \varphi + i\sigma$ w/ $\sigma \in (0, 2\pi e^2)$ compact + periodic

↑ DEPENDS ON $e^2(\mu)$

WE DISCUSSED $e^2(\mu)$ ABOVE, BUT DIDN'T MENTION ROLE OF THE <u>MASS</u> OF THE CHARGED MATTER. IN $SQED_3$,

$$\langle\varphi\rangle^2\left(|q|^2 + |\tilde{q}|^2\right) + \varphi\left(\bar{\psi}\psi + \bar{\tilde{\psi}}\tilde{\psi}\right)$$

So BELOW $\mu \sim \langle\varphi\rangle$, NO MORE MATTER TO GIVE RG LOOPS! $e^2(\mu)$ STOPS RUNNING FOR $\mu < \langle\varphi\rangle$.

LET US WRITE THE LOW-ENERGY COUPLING AS $e_L^2 \equiv e^2(\mu \to 0)$

THEN FOR $N_F \sim 1$,

- $$\frac{1}{e_L^2} \sim \frac{1}{e_0^2} + \frac{c}{\langle\varphi\rangle} \approx \frac{1}{e_0^2} \quad \text{for } \langle\varphi\rangle \text{ very large}$$

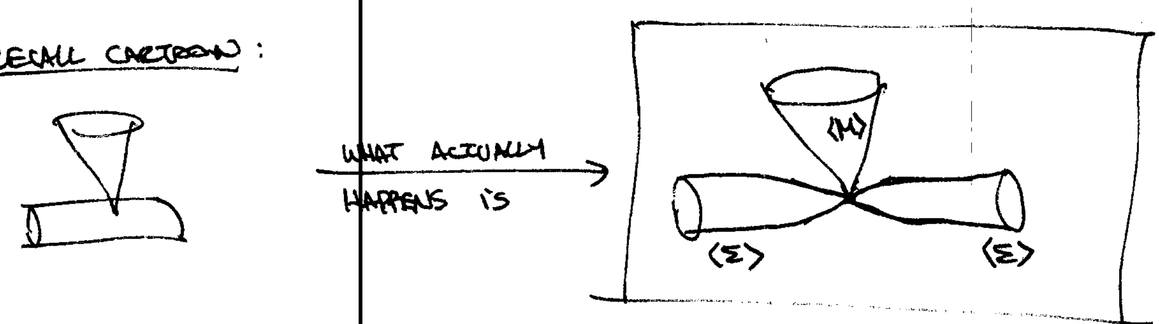
  $\Rightarrow$ radius of σ cylinder is $\mathcal{O}(e_0^2)$

$\boxed{\text{OR}}$ FOR $\langle\varphi\rangle$ VERY SMALL,

$$\frac{1}{e_L^2} \approx \frac{c}{\langle\varphi\rangle} \longrightarrow e_L^2 \approx \underbrace{\langle\varphi\rangle \to 0}_{} \Rightarrow \text{radius of cylinder } \underline{\text{vanishes}}.$$

this is where the Higgs branch meets the coulomb branch!

RECALL CARTOON:



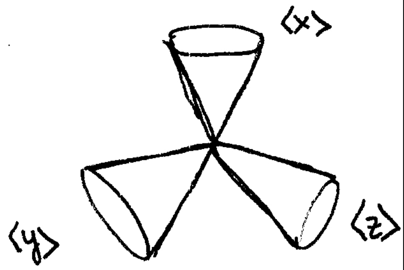
look this up on knowyourmeme.com

THIS IS THE "WHEN YOU SEE IT, YOU'LL SHIT BRICKS" MOMENT!

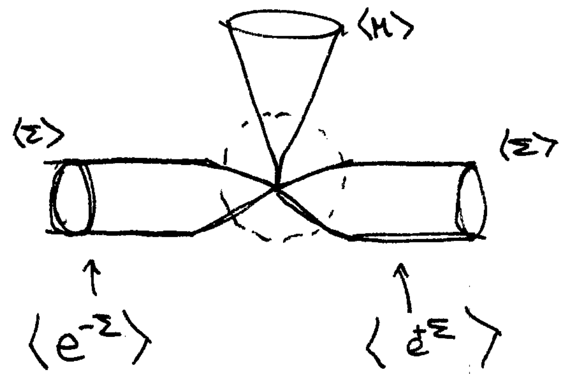
THIS IS THE SAME MODULI SPACE THAT WE DREW FOR THE CHIRAL XYZ MODEL!!

@ the intersection of the Higgs & Coulomb branches is a CFT which is DUAL to the XYZ model!

DUALITY



EXACT $\mathbb{Z}_3$ SYM



ACCIDENTAL $\mathbb{Z}_3$ (IR)

~~MASS DEFORMATION~~

$\Delta W = \xi X$

$Y = \sqrt{\frac{\xi}{h}}\, f(r)\, e^{i\theta}$

$Z = \sqrt{\frac{\xi}{h}}\, f(r)\, e^{-i\theta}$

SOLITONS!
(VORTICES

$\boxed{\text{not dual}}$
JUST AN ILLUSTRATION

$\langle M \rangle \neq 0$

PHASES OF $Q, \tilde{Q}$ MAY ALSO WRAP @ ∞ TO GIVE MASSIVE SOLITONS.

(GAUGE FIELDS CUT OFF LOG DIVERGENCE)

(this is a particle-vortex duality !)

HIGGS BRANCH MASSIVE SOLITONS CORRESPOND TO X & Y FIELDS WHICH GET MASSES WHEN $\langle x \rangle \neq 0$.

ie $X \longleftrightarrow M = Q\tilde{Q}$

$W = hXYZ + mX \longleftrightarrow W = mQ\tilde{Q}$

log confined vortices $\longleftrightarrow$ $Q, \tilde{Q}$ log confined (BY MASSLESS PHOTON)

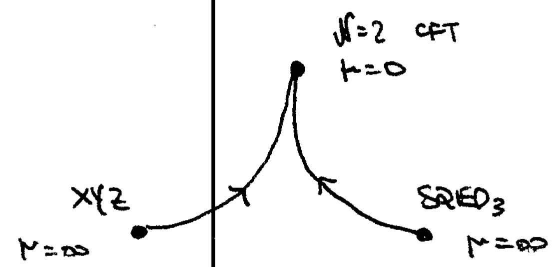
CLOSING THOUGHTS

THIS IS AN EXAMPLE OF AN IR DUALITY, THE KIND WE OFTEN SEE IN (SUPERSYMMETRIC) FIELD THEORY.

↳ eg. Seiberg Duality
eg. AdS/CFT

THIS MEANS THAT TWO MANIFESTLY DIFFERENT AND UNEQUAL THEORIES IN THE UV FLOW TO THE SAME FIXED POINT IN THE INFRA-RED.

(this is, of course, predicted by Wilsonian RG)



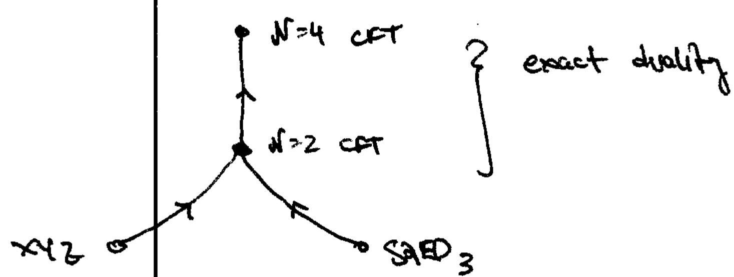
IT IS SOMETIMES SAID THAT QFT ONLY HAS IR DUALITIES (while string theory has exact dualities)

IT TURNS OUT THERE IS A DEFORMATION WHICH MAKES THIS AN EXACT DUALITY:

ADD NEUTRAL χSF Φ w/ $\Delta W = \sqrt{2}\,\Phi Q\tilde{Q}$

↳ PROMOTES THY TO $\mathcal{N}=\underline{4}$ $SQED_3$.

DESTABILIZES $\mathcal{N}=2$ CFT, CAUSES IT TO FLOW TO $\mathcal{N}=4$ CFT:



this is just $\Delta W = \Phi X$ in XYZ model. THIS DECOUPLES X, LEAVES Y & Z w/ NO SUPERPOTENTIAL ⟹ DUAL OF $\mathcal{N}=4$ CFT IS FREE!

So: FREE MASSLESS Y, Z PARTICLES ⟷ $\mathcal{N}=4$ $SQED_3$ VORTICES. □