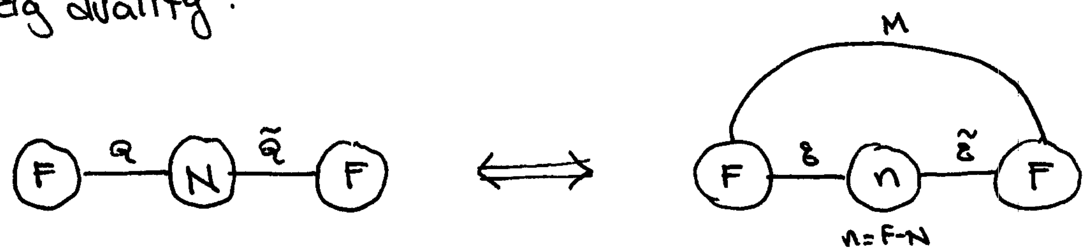
BSM JOURNAL CLUB: SPRING '13 ~~SUSY~~ across a duality 8 April '13

FLIP TANEDO pt267@cornell.edu

Motivation

Seiberg duality:



UV free IR free

Gives a handle for strong coupling → great for model building

most notable example: ISS (see Riccardo's fall '12 talk)
[OR PREVIOUS TALKS BY FLIP, DC, ...]

USES MAGNETIC DUAL TO DESCRIBE DYNAMICAL (metastable) ~~SUSY~~;
SM LIVES IN FLAVOR SECTOR. MEDIATE ~~SUSY~~ TO SM USING THIS FRAMEWORK.

natural question: CAN YOU REALIZE THE SM IN THE SU(n) } → or even just $SU(2)_L$?
MAGNETIC GAUGE GROUP?

the dream of technicolor
(BUT W/ POWER OF SUSY)

short answer: doesn't seem like it, maybe...
SEE CSÁKI, RANDALL, TERNING 1201.1293

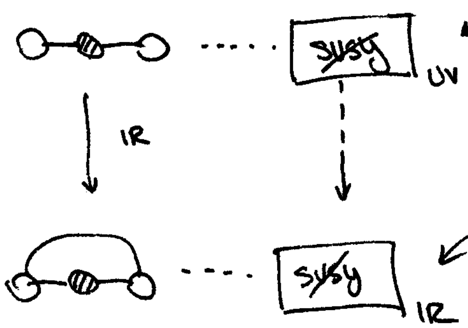
↳ same issue as technicolor, RS
if SM states are (strongly bound) composites,
why are their residual interactions so
weak?

eg NDA → ρ 〰< $_n^n$ $\sim \frac{4\pi}{\sqrt{N}} \approx 6$ vs. $g_{SM} \approx 0.65$

but anyway, that's not the topic for today.

IF YOU WANT TO PUT SM IN MAGNETIC GAUGE GROUP, YOU'LL EVENTUALLY WANT TO THINK ABOUT ~~SUSY~~.

ASSUMING THAT YOU DON'T WANT TO FUCK WITH THE SU(n) GROUP, ie ignore dynamical ~~SUSY~~ SOLUTIONS a la ISS, THEN ~~SUSY~~ OCCURS OUTSIDE THE SQCD MODULE.



EXPECT THAT IT LIVES IN THE UV

WE WANT THE EFFECT OF UV ~~SUSY~~ ON THE IR THEORY

the question: given $\boxed{\cancel{\text{susy}}}_{UV}$, what is $\boxed{\cancel{\text{susy}}}_{IR}$?

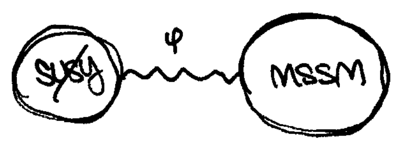
in other words: How do we map soft susy-breaking terms from the UV to the IR?

<u>the approach</u>: SPURION ANALYSIS ON SUPERSPACE

→ based on "<u>analytic continuation into superspace</u>"

Giudice-Rattazzi & ARKANI-HAMED + LWTY

used to determine (higher) loop ~~susy~~ terms in, eg, Gauge mediation



spurion (from hidden sector): $\langle X \rangle = M + \theta^2 F$

mass to messengers

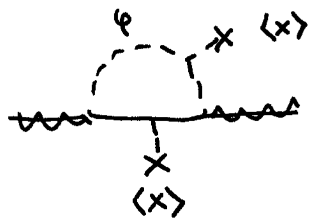
SUSY BREAKING

EFFECTIVE OPERATORS IN MSSM SECTOR:

$$\mathcal{L}_{eff} \supset c_1 \int d^2\theta \, \frac{X}{M} W^2 + c_2 \int d^4\theta \, \frac{X^\dagger X}{M^2} Q^\dagger Q$$

Usual ~~spurion~~ EFT analysis: have to do <u>matching</u> to get c_1 & c_2 -

HOW DO WE GET ~~SUSY~~ MASSES IN, eg. GAUGE MEDIATION?



+ 7 others

1 & 2 LOOP DIAGRAMS! annoying as hell.

but all is not lost!

Key observation: lowest order vevs are not ~~susy~~ing ... they're terms that we understand.

$$\text{eg: } \underbrace{\int d^2\theta \, \langle X\rangle|_{\theta^0} \frac{1}{M}}_{\sim \tau} W^2 \rightarrow \text{GAUGINO MASS}$$

$$\text{eg: } d^4\theta \, \underbrace{\langle X^\dagger X\rangle|_{\theta^0} \frac{1}{M^2}}_{Z_Q} Q^\dagger Q \rightarrow \text{SOFT SCALAR MASS}$$

We <u>Know</u> how these objects behave — in particular, their RG

IDEA: take this known behavior & <u>promote</u> into SUPERSPACE to understand the ~~susy~~ing terms.

↑ the power of holomorphy.

SUPPOSE WE HAVE $\tau(\mu, \Lambda)$, $Z(\mu, \Lambda)$ } from integrating out messengers @ scale M — so these have M dependence

@ scale (μ) — cutoff (Λ)

promote M dependence to X dependence — gives higher θ terms in $\theta \to$ ~~SUSY~~ spurions.

in general: let $f(M)$ be analytic.

PROMOTE $M \to X = M + \theta^2 F = \cancel{M(1+\frac{F}{M}\theta^2)}$ ~~expansion~~

$$f(\langle X\rangle) = f(M) + \underbrace{\frac{\partial f(M)}{\partial X}}\, F\, \theta^2$$

i'll drop $\langle\cdots\rangle$

nicer way to write (looks mysterious the first time)

$$f(X)\big|_{\theta^2} = \frac{\partial f(M)}{\partial X} F = \frac{\partial \ln f(M)}{\partial X} f(X) F$$

$$= \frac{\partial \ln f(M)}{\partial \ln X} f(M) \frac{F}{M} \quad \text{only } \theta^0 \text{ term} \qquad (*)$$

meaning of $\ln X$? TAYLOR EXP.

$$\ln X = \ln(M + \theta^2 F) = \ln M + \theta^2 \frac{F}{M}$$

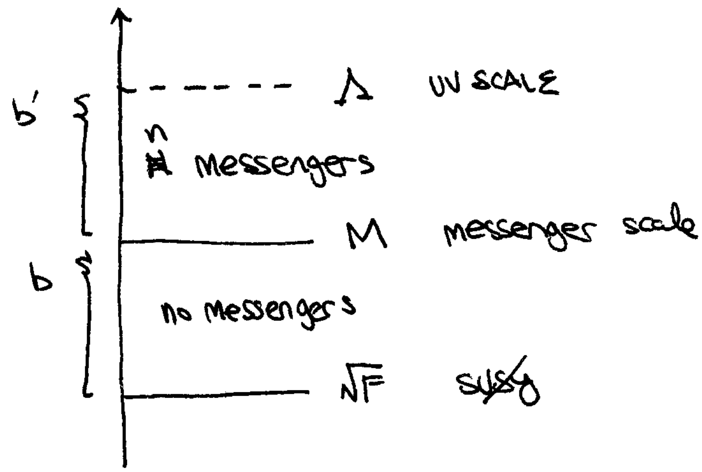
<u>example</u>: <u>Gaugino mass</u>

trick becomes: we know NSVZ β

for Gauge mediation (low scale SUSY): $F/M^2 \ll 1$
IN THIS LIMIT, CAN NEGLECT THRESHOLD EFFECTS

ie non-susy flow FROM $\sqrt{F} \to$ EW SCALE

RUNNING DEPENDS ON MESSENGER SECTOR BY MATCHING @ $\mu = M$
WHERE MESS. FIELDS ARE INTEGRATED OUT.



$$\frac{1}{g^2(\mu)} = \frac{1}{g^2(\Lambda)} + \frac{b'}{8\pi^2}\ln\frac{M}{\Lambda} + \frac{b}{8\pi}\ln\frac{\mu}{M}$$

$\downarrow$

$$\tau(\mu) = \tau(\Lambda) + \frac{ib'}{2\pi}\ln\frac{M}{\Lambda} + \frac{ib}{2\pi}\ln\frac{\mu}{M}$$

<u>FACT</u>: $b_0 = 3N - F$ ← holomorphic, not NSVZ

$\Rightarrow b - b' = n$ ← # messenger fields

the usual stuff

$$\tau(\mu) = \tau(\Lambda) + i\frac{b'-b}{2\pi}\ln(X) + \cdots \qquad (**)$$

$$\mathcal{L}_{sym} = \frac{1}{16\pi i}\int d^2\theta\, \tau(\mu)\, W^2 + h.c. + \cdots$$

$$\hat{m}_\lambda = \frac{-2\tau}{16\pi i}\Big|_{\theta^2} \longrightarrow \frac{-2g^2}{16\pi i}\tau(X)\Big|_{\theta^2}$$

↑ not yet canonically normalized

$$= -\frac{1}{2\tau}\,\tau(X)\Big|_{\theta^2}$$

$$= -\frac{1}{2}\frac{\partial\ln\tau}{\partial\ln X}\Big|_{X=M}\frac{F}{M}$$

PLUGGING IN (*) & (**)

$$M_\lambda = -\frac{1}{2}\frac{\partial \ln \tau}{\partial \ln X}\Big|_{X=M}\frac{F}{M}$$

$$= \frac{-i}{2\tau}\,\frac{b'-b}{2\pi}\,\frac{F}{M}$$

$$= \frac{ng^2}{16\pi^2}\,\frac{F}{M}$$

$$= \boxed{n\,\frac{\alpha}{4\pi}\,\frac{F}{M}}$$

exactly the LO contribution from a loop calculation

(IT IS A LOOP CALC... JUST THE LOOP FOR THE β FUNCTION, WHICH IS KNOWN)

<u>example II</u>: SOFT SCALAR MASS

$$Z = Z(M) \to Z(\sqrt{XX^\dagger})$$

for simplicity, assume MSSM = WZ

Wess-Zumino model (GAUGED)

$$Z(\mu) = 1 + |\lambda^2| \ln\left|\frac{\Lambda}{\mu}\right|$$

1-LOOP

doesn't involve messengers

not the quantity we care about

SUSY pres.

WE WANT WAVEFUNCTION REN. FROM GAUGE INT.

2-LOOP

$$\frac{\partial \ln Z}{\partial \ln \mu} = \frac{c_2(r)}{\pi}\,\underline{\underline{\alpha(\mu)}}$$

$$\alpha = \frac{i}{\tau}$$

integrate this: $$Z(\mu) = Z_0\left(\frac{\alpha(\Lambda)}{\alpha(M)}\right)^{\frac{2c_2}{b'}}\left(\frac{\alpha(M)}{\alpha(\mu)}\right)^{2c_2/b}$$

$$M \mapsto \sqrt{X^\dagger X}$$

not for talk (just for myself)

$$\frac{\partial \ln Z}{\partial t} = \frac{c}{\pi}\Big(\underbrace{\frac{1}{\alpha_0} + \frac{b}{4\pi}\ln\Big(\frac{\mu^2}{M^2}\Big)}_{\alpha^{-1}(\mu)}\Big)^{-1} \qquad \frac{b}{2\pi}\ln\Big(\frac{\mu}{M}\Big)$$

$t = \ln \mu$

$$= \frac{c}{\pi}\left(\frac{\alpha_0}{1 + \alpha_0\, b/2\pi\; t}\right)$$

$$d\ln Z = \frac{c\alpha_0}{\pi}\left(\frac{dt}{1 + \alpha_0 b/2\pi\; t}\right)$$

$$\ln Z = \frac{c\alpha_0}{\pi}\cdot\frac{2\pi}{\alpha_0 b}\ln\Big(1 + \frac{\alpha_0 b}{2\pi}t\Big) \qquad + \text{const}\ldots$$

$$= \frac{2c}{b}\ln\Big(1 + \frac{\alpha_0 b}{2\pi}t\Big)$$

$$Z = \Big(1 + \frac{\alpha_0 b}{2\pi}t\Big)^{2c/b}$$

$$= \alpha_0^{2c/b}\Big(\underbrace{\frac{1}{\alpha_0} + \frac{b}{2\pi}t}_{\alpha(\mu)^{-1}}\Big)^{2c/b}$$

SO NOW Z HAS ~~SUSY~~ SPURIONS

$$\mathcal{L}_{kin} = \int d^4\theta\,\big(Z + F_Z\theta^2 + F_Z^*\bar\theta^2 + D_Z\theta^4\big)\,\Phi^\dagger\Phi$$

$$\frac{\partial Z}{\partial X}F \qquad \frac{\partial Z}{\partial X^\dagger}F^* \qquad \frac{\partial^2 Z}{\partial X\partial X^\dagger}FF^* \quad \Big\}\ \text{in terms of } X = M + F\theta^2$$

$\frac{1}{Z}\,\cancel{\frac{\partial \ln Z}{\partial Z/\partial X}}$

canonical normalization: $\Phi \mapsto \Phi' = Z^{1/2}\Big(1 + \frac{\partial \ln Z}{\partial X}F\theta^2\Big)\Phi$

then drop prime

$$\mathcal{L}_{kin} = \int d^4\theta\,\Big[\,1 - \underbrace{\frac{\partial \ln Z}{\partial X}\frac{\partial \ln Z}{\partial X^\dagger}|F|^2\theta^4 + \frac{1}{Z}\frac{\partial^2 Z}{\partial X\partial X^\dagger}|F|^2\theta^4}_{\text{THESE GIVE } \tilde m^2\Phi^\dagger\Phi \text{ SOFT SCALAR MASS}}\Big]\Phi^\dagger\Phi$$

subtract extra term

norm.

<u>claim:</u> $$\tilde{m}^2 = -\frac{\partial^2 \ln Z}{\partial \ln X \, \partial \ln X^\dagger} \frac{|F|^2}{M^2}$$

Pf/ (not ~~for talk~~)

$$\tilde{m}^2 = -\frac{\partial^2 \ln Z}{\partial X \partial X^\dagger} |F|^2$$

$$= -\left[\frac{\partial}{\partial X^\dagger}\left(\frac{1}{Z}\frac{\partial Z}{\partial X}\right)\right] |F|^2$$

$$= -\left[-\frac{1}{Z^2}\frac{\partial Z}{\partial X}\frac{\partial Z}{\partial X^\dagger} + \frac{1}{Z}\frac{\partial^2 Z}{\partial X \partial X^\dagger}\right] |F|^2$$

$$= -\left[-\frac{\partial \ln Z}{\partial X}\frac{\partial \ln Z}{\partial X^\dagger} + \frac{1}{Z}\frac{\partial^2 Z}{\partial X \partial X^\dagger}\right] |F|^2 \quad \checkmark$$

in fact, can go further & get A & B soft terms for free from the rescaling of Φ.

<u>not ~~for talk~~</u>

$$W(\phi_0) = W\left(\sqrt{Z}\left(1 - \frac{\partial \ln Z}{\partial \ln X}\frac{F}{M}\theta^2\right)\Phi\right)$$

$$\Delta \mathcal{L}_{SOFT} = \int d^2\theta \; \frac{\partial W}{\partial \Phi_0}\bigg|_{\Phi_0 = \phi_0} \sqrt{Z}\left(-\frac{\partial \ln Z}{\partial \ln X}\right)\frac{F}{M}\theta^2$$

↳ will give A term... B term is a touchy subject ... needn't be generated by MESSENGERS.

Now to the main point

Write the UV SUSY BREAKING TERMS AS SPURIONS

$$\mathcal{L} = \int d^4\theta \left(Q^\dagger Z e^V Q + \tilde{Q}^\dagger Z e^V \tilde{Q} \right) + \int d^2\theta \, S W^2 + h.c.$$

$$Z = 1 + \theta^2 B + \bar{\theta}^2 B^* - \theta^4 (M_{UV}^2 - |B|^2)$$

→ for future simplicity, ignore the e^V (JUST FOR NOTATION)

signs?

$$S = \frac{\tau}{16\pi i} = \frac{1}{4g^2} + \frac{\theta_{YM}}{32\pi i} + \theta^2 \frac{m_\lambda}{g^2}$$

[factors of z all over...]

A NICE WAY TO PACKAGE THIS IS TO USE THE HOLOMORPHIC STRONG COUPLING SCALE:

$$\Lambda_h = \mu e^{-2\pi i \tau/b} = \mu e^{-16\pi^2 S/b}$$

$$= \Lambda e^{i\theta_{YM}/b}$$

this is an <u>RG invariant</u>

eg. Λ_{QCD} is a physical value.

MAKES IT USEFUL WHEN TALKING ABOUT DEEP IR.

Anomalous axial U(1) symmetry:

$$\begin{aligned} &\overset{(\sim)}{Q} \to e^A \overset{(\sim)}{Q} \\ &Z \to e^{-A-A^\dagger} Z \\ &\Lambda_h \to e^{2F/bA} \Lambda_h \end{aligned}$$

(we've assumed $T = 1/2$)

$$\sim \quad S \to S + F \frac{T}{4\pi^2} A$$

Dynkin index $T(\square) = 1/2$

question: doesn't seem like W^2 is invariant?

↳ it is: Konishi anomaly gives shift in opposite direction

↑ SUSY GENERALIZATION OF AXIAL ANOMALY (incl. SCALE)

$$\mathcal{D}(e^{i\alpha}Q)\,\mathcal{D}(e^{+i\alpha}\bar{Q}) = \mathcal{D}Q\,\mathcal{D}\bar{Q}\; e^{-\frac{i}{4}\int d^2\theta\, \frac{T}{8\pi^2} 2i\alpha\, W^2 + h.c.}$$

Promote: $e^{i\alpha} \to z^{1/2}$ $\qquad\qquad$ $\frac{T}{8\pi^2}\ln z$

compare to shift in θ from anomalous U(1)

Λ_h not $U(1)_A$ invariant, construct a $U(1)_A$ & RG invariant scale

$$\Lambda^2 = \Lambda_h^\dagger Z^{2F/b} \Lambda_h \leftarrow \Lambda|_0 \text{ is really strong scale}$$

↑ idea: use this for dimensional analysis in IR

UV susy terms live in this:

$$\Downarrow \quad \log\frac{\Lambda}{\mu} = \frac{-8\pi^2}{bg^2} - \frac{8\pi^2 m_\lambda}{bg^2}(\theta^2+\bar{\theta}^2) - \frac{F}{b} m_{UV}^2 \theta^4$$

Magnetic theory:

$$M \sim Q\tilde{Q}$$
$$Q^N \sim q^{F-N}$$
$$\tilde{Q}^N \sim \bar{q}^{F-N}$$

} determines $U(1)_A$ charges

$$\overset{(\sim)}{q} \to e^{\frac{AN}{F-N}} \overset{(\sim)}{q}$$

$$M \to e^{2A} M$$

CAN NOW WRITE MAGNETIC SPURIONS USING

1. U(1) INVARIANCE
2. Λ to do DIMENSIONAL ANALYSIS (w/ UV dimensions)

$$\mathcal{L}_{\text{"UV"}} = \int d^4\theta \, \frac{M^\dagger Z^2 M}{\Lambda^2} + \overset{(\sim)}{q}{}^\dagger Z^{\frac{N}{F-N}} \overset{(\sim)}{q} \cdot \frac{1}{\Lambda^{\frac{4N-2F}{F-N}}}$$

$$+ \int d^2\theta \, \tilde{S} W^2 + \frac{y M q \tilde{q}}{\Lambda_h^{b/F-N}} + \text{h.c.}$$

~~then pick up susy vevs:~~

discussion:

e.g. $\frac{M^\dagger Z^2 M}{\Lambda^2}$ ← $M^\dagger M \to e^{2(A+A^\dagger)}$
need Z^2 spurion to soak up U(1)

$[M]_{UV} = 2 \to$ need $1/\Lambda^2$

in IR $[M] \to 1$, so $[Z] \to 1$ since Λ is invariant

then: $Z^2 \Lambda^{-2}\big|_{\theta^4} = Z^2 \mu^{-2} e^{-2\log \Lambda/\mu}\big|_{\theta^4}$

soaked up by anomalous dimensions in IR when $[M] \to 1$

B terms cancel

$-2m_{UV}^2 + 2\frac{F}{b} m_{UV}^2$ ← $b = 3N - F$ ignore m_λ terms

$$\boxed{m_{\tilde{q}M}^2 = 2\,\frac{3N-2F}{b}\, m_{UV}^2}$$

Not for talk (too tedious)

$$d^4\theta \cdot \frac{q^\dagger Z^{\frac{N}{F-N}} q}{\Lambda^{\frac{4N-2F}{F-N}}} \rightsquigarrow Z^{\frac{N}{F-N}} e^{-\frac{4N-2F}{F-N}\log\frac{\Lambda}{\mu}} \cdot \mu^{\#}$$

soaked up by anomalous dimensions

$$-\frac{N}{F-N} m_{UV}^2 \qquad + \frac{4N-2F}{F-N}\frac{F}{b} m_{UV}^2$$

$$-\frac{3N^2-NF-4NF+2F^2}{(F-N)\underbrace{(3N-F)}_{b}} = +\frac{(3N-2F)\cancel{(F-N)}}{(3N-F)\cancel{(F-N)}}$$

$$\boxed{m_q^2 = -\frac{3N-2F}{b} m_{UV}^2}$$

observe: relative minus sign to m_M^2 ! tachyon!

"no-go"-ish for $3N-2F \neq 0$! ← bndy of conformal window

LIGHT STOPS: for composite 3rd gen, let this term vanish!

LO soft terms are higher O

$$m_{comp}^2 = m_{IR}^2 + m_{UV}^2\left(\frac{\mu}{\Lambda}\right)^{\gamma}$$

m_{IR}^2 ↑ WHAT WE CALC. $m_{UV}^2(\mu/\Lambda)^\gamma$ ↑ not zero for $\gamma > 0$

GAUGINO MASS: Λ is RG invariant

$$\Lambda^2 = \Lambda_h^\dagger Z^{2F/b} \Lambda_h \quad w/ \quad \Lambda_h \sim e^{\cdots \theta^2 m_\lambda / g^2}$$

$$\Rightarrow \frac{m_\lambda}{b g^2} = \frac{m_{\tilde\lambda}}{\tilde b \tilde g^2} \xrightarrow[\text{NORM}]{\text{CANONICAL}} m_{\tilde\lambda} = \frac{\tilde b}{b} = \frac{3(F-N)-F}{3N-F} = \boxed{-\frac{3N-2F}{3N-F} m_\lambda}$$

also $=0$ @ edge of conf. window → dual w,z ?

EXTRA STUFF – not for talk

$F < N$ ADS
$F = N$ q'm mod constr.
$F = N+1$ S-CONFINEMENT

$F > N+1$ duality

↳ $\frac{3}{2}N < F < 3N$ conformal window

free mag   free electric

Properties of Λ

$$\Lambda^2 = \Lambda_h^\dagger Z^{2F/b} \Lambda_h$$

IS RG INV'T & θ^0 COMP IS PHYSICAL STRONG SCALE: hep-th/9705189

Why? Holomorphic scale transforms when rescaling Q's.
(by Konishi)

Important for RG ⟷ 1. NDA RESCALING OF ALL DIM'FUL QUANTITIES (eg $\Lambda \to e^t \Lambda$)

here Λ_h changes ⟹ 2. RESCALE CUTOFF BACK TO ORIG CUTOFF, KEEPING IR PHYSICS FIXED

What about higher terms?

$$\Lambda_{\theta^2} = \lim_{M_{UV}\to\infty} \frac{16\pi^2}{b}\left(\frac{m_\lambda}{g^2}\right)$$

$$\Lambda_{\theta^4} = \frac{-2F}{b} \lim_{M_{UV}\to\infty} m_Q^2$$

UV fixed point limits